國立政治大學經濟學系碩士論文

指導教授：莊委桐教授

不完全資訊和雙重改變下的分群模型

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謝辭

本論文得以完成，要深深感謝莊委桐老師在碩士班的指導、以及兩位

口委老師：毛維凌老師和袁國芝老師，以及支持我多年的家人們。

邱彥閔　謹誌

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Abstract

This paper first constructs a grouping model with heterogeneous[1] population under the setting of complete information. When player can observe other's type, the result is non-segregation: most players have no intention to move and they can match with the one who brings them the best payoff in the original group. The equilibrium state is always efficient.

We then construct another grouping model with incomplete information and double mutation. The result shows that, although non-segregation equilibria may emerge as stable equilibria in the short run, only segregation equilibria can be stochastically stable in the long run. This is because most of non-segregated states can switch to the others by the same resistance and some of them can easily switch to segregated state, but it is hard to switch back.

[1] Here we define heterogeneous population as a population where players have different types.

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1. Introduction and Literature Review

Grouping happens on every utility-maximizing people in the world. Some grouping have the characteristic of adherence, which means people can choose to match with the same one if he wants, just as marriage, friendship, and joint venture. Some grouping does not have adherent characteristic and thus people must rematch at every time, e.g. clubs (people cannot estimate the partners he will play with tonight), library studying (quiet student may be beset by other noisy students). The result of adherent grouping is intuitive: people finally find a partner to match with and continue the game until one of the participants dissatisfy. Thus, no matter what the initial group proportion is, if people can match with someone who brings him the best payoff (or payoff higher than his aspiration, see Borgers, 2000) then he will not move. If most people are lucky enough on matching or we prolong the times of re-match before moving, then the result of grouping is undoubtedly non-segregation. Since everyone can match with the best partners, they have no incentive to leave.

But once the matching is not adherent and randomized, then the result will vary depending on the existence of complete information. If information is complete, then player can easily find the best partner he needs and thus has no reason to move out. That is what we want to show in chapter two.

If information is incomplete, some people might consider the environment he lives in. If there is another better place to play the repeat random matching, Some people who dissatisfy the partners may play with might decide to move to another group. Consider the types of people as n and n groups, then the result will be a segregation because if non- segregation exists some people cannot have the best payoff. If their tolerance (see Foster and Young, 2006) are low and no transaction cost exists, then by try-error process people will select and move to the best environment.

But there still exist some other possible equilibrias: people may choose to stay if the environments of other groups is not better than his group or the information is obstructed such that people cannot know other groups' performance. Thus the equilibria may be non-segregated. That is, the static equilibria with incomplete information can be segregated or non-segregated.

In real world even without transaction cost e.g. move cost, there still exist some obstruction factors such that non-segregation may happen, like incomplete information. The type of players may be hard to know by others, and even hard for some investigation institutions due to large access cost. All of these factors will hinder the formation of segregation. In chapter 3, we can derive that equilibrium can be non-segregated or segregated due to the obstruction of information and failed estimation of another group from both aspects: player-based and institution-based estimation. However the equilibrium is a short-term and static equilibrium.

In further discussion, we try to construct a long-term and dynamic equilibrium and have a stochastic potential test just like adapt play (by Young, 2001). The stochastic stable state is undoubtedly segregated, because a little mutation can switch an non-segregated state to segregated state, but the switching back needs more mutations. In intuition it is not so hard to infer: once some players are not so rational, the non-segregated state can be broken due to worse performance for most players.

Another useful literature in this paper is social equilibrium (Jackson and Watts, 2008). In chapter 2 we construct a model based on social equilibrium, which will be non-segregated and almost social efficient, but this model has some drawbacks: too little move of players and lack of the process of being non-segregation. These drawbacks lead to an unrealistic result.

Most concepts of grouping in this paper originate from Schelling(1971) and his followers, such as Carrington et al.(1996) and Milchtaich(2002).

2. Grouping Equilibrium with Complete Information

Consider a population in which there are two types of players and two groups. Player can enjoy full payoffs with the partner of same type by playing the same strategies which are their favorite, or enjoy less payoffs by playing the same strategies which are not their favorite. If player matches with different type partner and both of them play the same strategies, then there must be one enjoy less payoffs and the other enjoy full payoffs. If players fail to cooperate and play the different strategies, both of them will get nothing. Every player can access the type information of other players, choose opponent in same group freely, and if they need to match with someone in another group they must move with moving cost, ϵ (small but positive). That is, move is a method to seek for better opponent. Thus, the reasonable decision of player is to match with the same type if possible. Since the information is clear, same type player of a group will firstly match with each other and all of them have no intension to move because they cannot better off by paying moving cost. Movement in this environment will be fewer.

We try to construct an analogous concept as social equilibrium (Jackson and Watts, 2010), and denote it by grouping equilibrium. Assume that two groups $\mathrm{G_1, G_2}$ and there are two types of players in this model: tennis-preferring and billiards-preferring, denoted by T.P. and B.P. player with strategy decision $\mathrm{s_i = \{T, B\}}$, partner decision $\mathrm{a_i}$, and decision and move decision $\mathrm{m_i = \{1,2\}}$. The partner decision is matching a partner in the group $\mathrm{m_i}$, that is, $\mathrm{a_i \in G_{m_i}}$. The initial population size of $\mathrm{G_1, G_2}$ is $\mathrm{n_1, n_2}$ and the corresponding proportion of T.P. player of $\mathrm{G_1, G_2}$ is p_1^{Te}, p_2^{Te}. There are three possible matching matrixes in this model:

For a T.P. player matched with T.P. player,

	Tennis	Billiard
Tennis	(a,a)	(0,0)
Billiard	(0,0)	(b,b)

For a T.P. player matched with B.P., T.P. chooses the row strategy and B.P. chooses the column.

	Tennis	Billiard
Tennis	(a,b)	(0,0)
Billiard	(0,0)	(b,a)

For a B.P. player matched with B.P.,

	Tennis	Billiard
Tennis	(b,b)	(0,0)
Billiard	(0,0)	(a,a)

Now we define the concept of grouping equilibrium in brief.

Definition 1.

Assume a_i *is the matching partner of player i and* $a_i \in G_{m_i}$*, strategy decision* $s_i = \{T, B\}$*, and move decision* $m_i = \{1,2\}$*,* $\forall player\ i$*. A strategy profile* $(s_i^*, s_{-i}^*, m_i^*, m_{-i}^*, a_i^*, a_{-i}^*)$ *is a grouping equilibrium if, for each player i,* (s_i^*, m_i^*, a_i^*) *is the best response to the strategies* $(s_{-i}^*, m_{-i}^*, a_{-i}^*)$ *for other players such that*

$$\pi_i(s_i^*, s_{-i}^*, m_i^*, m_{-i}^*, a_i^*, a_{-i}^*, n_1, n_2, p_1^{Te}, p_2^{Te})$$

$$\geq \pi_i(s_i, s_{-i}^*, m_i, m_{-i}^*, a_i, a_{-i}^*, n_1, n_2, p_1^{Te}, p_2^{Te})$$

for $s_i = \{T, B\}$, $m_i = \{1,2\}$, *and* $a_i \in G_{m_i}$.

The concept of social equilibrium (Jackson and Watts, 2010) is a state of player cannot better off by playing with another player ,or by changing strategy, or by both aspect in the presupposition that any other players will not be worse off. Thus the grouping equilibrium must be the state that player match with the same type as his in order to enjoy the best they can have. Any player in mix-match can better off by finding anyone who belongs to the same type in another mix-match in a group. In the end there will be zero to two players who cannot find the same type as his, and they can try to move to another group for pure-match. It is trivial to discuss these players who cannot pure-match if the population is large. Thus we can conclude that in the grouping equilibrium there are no movements and the proportion is approximately same as initial proportion because most players can find the same type partners thus have no reason to move out and pay the moving cost. Moreover, the grouping equilibria is efficient because all players match with the same type partners. Although sometimes some remaining players cannot match with the same type or match with no one because all other same type players have the best matching, this will not affect the claim that grouping equilibria is efficient.

Proposition 2.

Consider a two-group model with complete information. The initial population of G_1, G_2 *is* n_1, n_2 *and the corresponding proportions of T.P. player of* G_1, G_2 *are* p_1^{Te}, p_2^{Te}. *Payoffs are given above. Assume that* $p_1^{Te}n_1, p_2^{Te}n_2, (1-p_1^{Te})n_1, (1-p_2^{Te})n_2$ *are all even. The grouping equilibrium of two-group model with complete information is that proportion of* G_1, G_2 *are* p_1^{Te}, p_2^{Te} *and population are* n_1, n_2, *stay unchanged. All T.P. players match with T.P., play strategy T, and do not move. All B.P.*

players match with B.P. player, play strategy B, and do not move.

However the result may be unrealistic because moving between groups is a common phenomenon. Moreover, grouping equilibrium only shows the grouping state, but lack the process of grouping. In the next chapter we consider another model to explain the process of grouping and show a realistic result.

3. Two-Group Grouping Model with incomplete information

Consider a model mostly following the setting in chapter 2 but something different. In this population, players are divided into two groups initially and have the right to select to move to another group in the end of every period. Periods can be infinite and each period contains R rounds. In each round, player is matched with another one but he cannot know what set his opponent will bring. The only information he has is the statistics published by an independent institution, which will investigate the proportion of tennis-preferring and billiard- preferring, and what sport sets they brought in past rounds. Thus, in every round players make decision based on the institution information and we can infer that players which are belong to same type, that is, T.P. or B.P., will choose the same strategy unless they make a mistake in strategy making. After matching up R rounds, in the end of period all players have the chance to choose move or not due to their evaluation of the situation of two groups.

Moreover, there always exists information obstruction in real world such that players cannot access complete information of another group. We add this characteristic into model: player cannot access the type of others. That means complete information no longer exist but there are two ways to access partial information: player investigation and institution investigation. player can sample some players in another group to learn the type of sport they played in last period (notice that player can only sample one period, not m period), or he can know the expected average payoff of another group, which is investigated and calculated by the institution of another group. This is because all institutions will investigate the strategy players using in every period and the proportion of T.P. and B.P. players. In the end of phase they will calculate the expected average payoff (because it is costly to investigate the type of each players' opponents in every periods) and announce it to players of two groups. Thus player knows the proportion, strategy distribution of his group, and the approximate average payoff of another group, but he cannot learn the detailed proportion and strategy distribution of another group due to the information obstruction.

3.1 Setting of Incomplete Information

With complete information, grouping equilibrium demonstrates that the proportions and group sizes of equilibria are roughly as initial ones. However, if complete information no longer exists, the group state will not remain non- segregation state affected mostly by the initial state. Here we list some possible information and rule restriction of this model mentioned above:

i. Players don't know the type of their opponents and they are randomly matched with each other in each round of the period.

ii. There exists a fair institution in each group. The institution will investigate the proportion of players of each type in the group and the strategies they use in every round.

iii. One of the partial information channel, by which player can access the group proportion and the strategy distribution of each type players in his group, and the average expected payoff of another group, is the institution investigation. Because the actual average payoff of each player is costly to reach for two institutions. (especially when T.P. players choose tennis and B.P. player choose billiards, it is costly to investigate the type of every partner of each player matching up in all rounds because even players themselves cannot distinct the type of their partners.) Thus institution will infer the approximate average payoff by the proportion of players of each type and strategies they used in all rounds, and institution will announce the group expected average payoff to all players of two groups. The approximate average payoff also has another aspect: expectation. When players choose to move or stay they will take this expectation into consideration: how much will he gain in expectation if moving to another group?

iv. According to the group proportion and the strategy distribution (investigated by institution), player can choose the best strategy depending on his memory of past m rounds. The m length is determined by the memorial ability of players. For a player preferring tennis, given the proportion of T.P. players of the group in period t, $p^{Te,t}$, and the strategy profile of his opponents, $s_{-i}^{r,t} = \left(p_{-i,Te}^{r,t}, p_{-i,Bi}^{r,t}\right)$, where $p_{-i,Te}^{r,t}$ means the proportion of T.P. players choosing tennis despite i player himself in past m rounds and $p_{-i,Bi}^{r,t}$ means proportion of B.P. players choosing tennis despite i player himself in his group. Here we simply assume player redeems all rounds in his memory are equal weighted. The player's best response is

(a) Tennis, if $a \cdot p^{Te,t} \cdot p_{-i,Te}^{r,t} + a \cdot (1 - p^{Te,t}) \cdot p_{-i,Bi}^{r,t} > b \cdot p^{Te,t} \cdot \left(1 - p_{-i,Te}^{r,t}\right) + b \cdot (1 - p^{Te,t}) \cdot (1 - p_{-i,Bi}^{r,t})$.

(b) Billiards, otherwise.

v. Another information channel about the different group is player investigation. Player can choose to believe the investigation of his own more or the institution investigation more. Here we denote the relative weight by δ. As mention before, players have two information channels to access another group: by institution or themselves. The latter is that players will investigate the strategies of players of another group use by themselves. However, the investigation samples only a few not all players of another group in the end of period. Since all they know about the other group is the expected average payoff which is given by institution of another group and the self-investigation, they will use them to decide moving or not. This is player's rule of moving formula: For a player i in group 1,

$$\forall\delta \in [0,1], \quad \text{if } \delta EP_I^2 + (1-\delta)EP_{II}^2 - EP_i^1 > 0, \text{ then move,}$$

where $EP_I^2 = \frac{\sum_{h=1}^{H} \pi_i(s_h^2)}{H}$, which means the player sample average payoff

$\pi_i(x)$ is i's payoff with mixed strategy x;

s_h^2 is the sample from group2 by player i and sample size is H;

EP_{II}^2 is institution announcing expected payoff;

EP_i^1 is the player's expected payoff if staying in the next phase.

One more mention, if player moves to new group, he will replace his memory by the new group data in the past m rounds from the institution.

3.2 equilibrium with incomplete information

Now we can infer the stable strategy in one group model under the above rules. Assume that every period contains large enough round and players in a group randomly matched with each other and choose strategy under the last m rounds' information. If they are in initial period (t=1) and in the first m rounds (r=1~m), players choose strategy at their will. Noticing that if choosing at will in first m rounds there will be enormous possible profiles, however, profile in the next period must be (1,1), (1,0), or (0,0). There will be no such existence like (0,1) due to the preference contradiction. Thus there will be only three strategy profiles in the remaining rounds.

Moreover, these three strategy profiles are also stable if the players in the group adapt them.

Check the stability of $s^{r,t} = (1,1), (1,0), \text{or}(0,0)$ for group 1 or 2, $\forall t = 1,2,$ …. and $r \in \{1,2,\dots,R\}$. By appendix A we know if $F(r-1,t) = p^{Te,t}p^{r-1,t}_{-i,Te} + (1-p^{Te,t})p^{r-1,t}_{-i,Bi} > \frac{a}{a+b}$ then players' strategy distribution $s^{r,t} = (1,1)$. Thus the strategy $s^{r,t} = (1,1)$ is stable because $p^{Te,t} \cdot 1 + (1-p^{Te,t}) \cdot 1 > \frac{a}{a+b}$. We can find that $s^{r,t} = (0,0)$ is also stable by the above method. As for $s^{r,t} = (1,0)$, which means $p^{r-1,t}_{-i,Te} = 1$ and $p^{r-1,t}_{-i,Bi} = 0$, it follows that $(r-1,t) = p^{Te,t}$. Then we know (1,0) is stable if $\frac{a}{a+b} \geq p^{Te,t} \geq \frac{b}{a+b}$,and (1,0) will change to (1,1) if $p^{Te,t} \geq \frac{a}{a+b}$, change to (0,0) if $\frac{b}{a+b} \geq p^{Te,t}$.

Extend to the best response of T.P. and B.P. players in a group when the current round is r and current period is t, $BR^{r,t} = (p^{r,t}_{Te}, p^{r,t}_{Bi})$ where $p^{r,t}_{Te}$ means the proportion of T.P. players choosing tennis in past m rounds and , $p^{r,t}_{Bi}$ means the proportion of B.P. players choosing tennis in past m rounds. Given any strategy distribution in the last m rounds, denoted by $s^{m,r,t}$:

(a) If previous strategy $s^{m,r,t} = (1,1)$, then $BR^{r,t} = (1,1), \forall r \in \{1,2,\dots,R\}$ and $t = 1,2,\dots.$

(b) If $s^{m,r,t} = (1,0)$, then

$BR^{r,t} = (1,1)$, if $p^{Te,t} > \frac{a}{a+b}$

$BR^{r,t} = (1,0)$, if $\frac{a}{a+b} \geq p^{Te,t} \geq \frac{b}{a+b}$

$BR^{r,t} = (0,0)$, if $\frac{b}{a+b} > p^{Te,t}$

(c) If $s^{m,r,t} = (0,0)$, then $BR^{r}_{t} = (0,0)$.

Under above results, we can calculate the expected average payoff announced by institution in the end of every period, since they infer the payoff by proportion of each type and strategy profile. The form of expected average payoff is

Previous strategy $s^{m,r,t}$	(1,1)	(1,0)	(0,0)
$p^{Te,t} > \frac{a}{a+b}$	$ap^{Te,t}+b(1-p^{Te,t})$	$ap^{Te,t}+b(1-p^{Te,t})$	$a(1-p^{Te,t})+bp^{Te,t}$
$\frac{a}{a+b} > p^{Te,t} > \frac{b}{a+b}$	$ap^{Te,t}+b(1-p^{Te,t})$	$a-2ap^{Te,t}(1-p^{Te,t})$ #	$a(1-p^{Te,t})+bp^{Te,t}$
$\frac{b}{a+b} > p^{Te,t}$	$ap^{Te,t}+b(1-p^{Te,t})$	$a(1-p^{Te,t})+bp^{Te,t}$	$a(1-p^{Te,t})+bp^{Te,t}$

(#): $ap^{Te,t} \cdot p^{Te,t} + a(1-p^{Te,t}) \cdot (1-p^{Te,t}) = a - 2ap^{Te,t}(1-p^{Te,t})$

$ap^{Te,t} \cdot p^{Te,t}$ is the expected payoff of T.P. player and $a(1-p^{Te,t}) \cdot (1-p^{Te,t})$ is that of B.P. player.

To concern that, if $s^{m,r,t}$ is (1,0) and group proportion satisfy $p^t > \frac{a}{a+b}$ or $\frac{b}{a+b} > p^t$ the strategy of coming rounds will change just mentioned above.

Now we start to figure out two-group-equilibrium, especially the recurrent class. At the end of every period, player will decide whether to move depending on the expected payoffs he might gain in the future, just like the difference of potential energy which makes water flow. However some information will obstruct the flow and thus create non-segregation stable state or recurrent class.

Recalling the moving rule of player is

$\forall\delta \in [0,1]$, if $\delta \mathrm{EP}_{\mathrm{I}}^{2} + (1-\delta)\mathrm{EP}_{\mathrm{II}}^{2} - \mathrm{EP}_{\mathrm{i}}^{1} > 0$ then move, for player i in group 1;

$\forall\delta \in [0,1]$, if $\delta \mathrm{EP}_{\mathrm{I}}^{1} + (1-\delta)\mathrm{EP}_{\mathrm{II}}^{1} - \mathrm{EP}_{\mathrm{i}}^{2} > 0$ then move, for player i in group 2.

Thus what makes the recurrent classes different is the relative weight between investigated by institution and that by player himself. Here we list a lot of possible states and discuss the stability of each one. Assuming in a model there are two groups $\mathrm{G}_1, \mathrm{G}_2$ and corresponding group size is $\mathrm{n}_1, \mathrm{n}_2$, corresponding group proportion of players preferring tennis is $\mathrm{p}_1, \mathrm{p}_2$.

To confirm that the equilibrium of incomplete information is stable, we must assume the self-investigation of players will sample their favorite strategy as much as possible. Notice that stability of a player means that even if his self-investigation overestimates the number of those whose strategy decisions of last round are just as the preference of player himself (e.g. T.P. player of G_1 samples all tennis, $\mathrm{EP}_{\mathrm{I}}^{2} = \frac{\mathrm{H}\pi_{\mathrm{i}}(\mathrm{Tennis})}{\mathrm{H}} = \mathrm{a}$, even if the actual strategy distribution is mostly billiard strategy), the player still choose to stay in his original group. To find a stable state of a recurrent class (but not stable) we must assume all players are “lucky” enough to sample strategies which are just as his preference when we try to construct the regret formula of each player.

Now we precede the inference of equilibrium with incomplete information. Given the initial population size of T.P. players in the model is N^{Te^*} and the population size of B.P. players in the model is N^{Bi^*}. Denote a state by $(p_1^{Te}, p_2^{Te}, s_1^{m,r,t}, s_2^{m,r,t}, N_1, N_2)$, where p_i^{Te} is the proportion of group i, $s_i^{m,r,t}$ is the previous strategy distribution of group i in round r and period t, and N_i is the size of group i. It follows that the variables $p_1^{Te}, p_2^{Te}, N_1, N_2$ of any states must satisfy the condition:

(a) $p_1^{Te}N_1 + p_2^{Te}N_2 = N^{Te^*}$.

(b) $(1 - p_1^{Te})N_1 + (1 - p_2^{Te})N_2 = N^{Bi^*}$

We denote the condition as (#). The followings are derivations for equilibria:

i. We claim that $(p_1^{Te}, p_2^{Te}, (1,1), (1,1), N_1, N_2)$ and $(p_1^{Te}, p_2^{Te}, (0,0), (0,0), N_1, N_2)$ is a recurrent class, $\forall 1 > p_1^{Te}, p_2^{Te} > 0$, and $p_1^{Te}, p_2^{Te}, N_1, N_2$ satisfy (#).

If previous strategies of two groups are both (1,1), $s_1^{m,r,t} = s_2^{m,r,t} = (1,1)$, will this state be stable? For a tennis preferring player, since $a\delta + (1-\delta)\left(ap_k^{Te} + b\left(1 - p_k^{Te}\right)\right) - a < 0, \forall\delta, k \in \{1,2\}$, the regret will be always small than zero under any relative weight the expected payoff investigated by individual or institution. There is no incentive for tennis preferring player in two groups move to another one. However, for a billiard preferring player, his regret $a\delta + (1-\delta)\left(ap_k^{Te} + b\left(1 - p_k^{Te}\right)\right) - b > 0$, $\forall\delta \in [0,1], k \in \{1,2\}$. This means that T.P. player won't move but B.P. player will. The proportion of each group will change, but soon it will recover to the original proportion in first phase because T.P. player's regret is negative and that of B.P. player is still positive. Thus this state may not be stable, but must be recurrent. (it is stable state if $p_1^{Te}, p_2^{Te} \in$

$\{1,0\}$) The detailed proportion path of these two groups is $p^{t=1} = \left(p_1^{Te}, p_2^{Te}\right)$,

$p^2 = \left(\frac{p_1^{Te} n_1}{p_1^{Te} n_1 + (1-p_2^{Te}) n_2}, \frac{p_2^{Te} n_2}{p_2^{Te} n_2 + (1-p_1^{Te}) n_2}\right)$, $p^3 = \left(p_1^{Te}, p_2^{Te}\right)$, ……, loop. This inference can be applied if $s_1^{m,r,t} = s_2^{m,r,t} = (0,0)$, and $1 > p_1^{Te}, p_2^{Te} > 0$.

ii. We also claim that $(p_1^{Te}, p_2^{Te}, (1,0), (1,0))$ is a recurrent class and a equilibrium with incomplete information if $\frac{1-2p_i^{Te}(1-p_i^{Te})+p_j^{Te}}{2-2p_i^{Te}(1-p_i^{Te})} > \delta \geq 0$ holds, $\forall p_1^{Te}, p_2^{Te} \in [\frac{a}{a+b}, \frac{b}{a+b}]$ and $p_1^{Te}, p_2^{Te}, N_1, N_2$ satisfy (#).

If $s_1^{m,r,t} = s_2^{m,r,t} = (1,0)$, we can firstly exclude the possible recurrent classes if one of $p_1^{Te}, p_2^{Te} \in \left[1, \frac{a}{a+b}\right] \cup \left[\frac{b}{a+b}, 0\right]$, since any group in that threshold have unstable strategy (1,0) by the above discussion. If both groups' proportions are in the threshold of $[\frac{a}{a+b}, \frac{b}{a+b}]$ and for player in group i, the proportions of two group i,j satisfy the restriction $\frac{1-2p_i^{Te}(1-p_i^{Te})+p_j^{Te}}{2-2p_i^{Te}(1-p_i^{Te})} > \delta \geq 0 \cdots (*)$, then this state will be a recurrent class since $a\delta + (1-\delta)\left(a - 2ap_i^{Te}\left(1-p_i^{Te}\right)\right) - ap_j^{Te} < 0$. This means that if players trust the investigation of institution over the above level (specific weight), then there will be little difference between their group and the other group in their view. If the weight is not higher than the level, they might move due to the "lucky sample" such that they believe another group is better than original one, even if actually it is not. In a word, they are deceived by their own sampling, however the deceive may be good for all players because it help all players group to segregated state, $s_1^{m,r,t} = (1,1), s_2^{m,r,t} = (0,0)$, and $p_1^{Te} = 1, p_2^{Te} = 0$ (or group 1 and 2 exchange) and enjoy the best they can have, which is social efficient. [2]

iii. Consider the state of $s_1^{m,r,t} = (1,1), s_2^{m,r,t} = (1,0), p_1^{Te} \in (0,1), p_2^{Te} \in [\frac{a}{a+b}, \frac{b}{a+b}]$. It must be neither stable nor recurrent. Since under any size of δ, T.P. players in the group j and B.P. players in group i will move due to the difference of potential payoffs. Finally this will be a segregation state, namely $s_1^{m,r,t} = (1,1), s_2^{m,r,t} = (0,0)$, and $p_1^{Te} = 1, p_2^{Te} = 0$. The process also fits with states of $s_1^{m,r,t} = (1,1), s_2^{m,r,t} = (0,0), p_1^{Te} \in (0,1), p_2^{Te} \in (0,1)$ and states of $s_1^{m,r,t} = (0,0), s_2^{m,r,t} = (1,0), p_1^{Te} \in (0,1), p_2^{Te} \in [\frac{a}{a+b}, \frac{b}{a+b}]$.

iv. In the last part, we claim that all segregated states are recurrent classes and equilibrias with incomplete information, no matter what strategy distribution of the state is or what groups' size are.

It is obviously to find that segregated states, i.e. $p_1^{Te} = 1, p_2^{Te} = 0, (s_1^{m,r,t}, s_2^{m,r,t}) \in \{((1,1),(1,1)),((1,1),(0,0)),((0,0),(0,0))\}$, are stable states and recurrent classes.

Definition 3.

Consider a two-group model with group size n_1, n_2, *group proportion* p_1, p_2, *strategy decision* $s_i = \{T, B\}$, *and move decision* $m_i = \{1,2\}$, $\forall player\ i$. *A strategy profile* $(s_i^*, s_{-i}^*, m_i^*, m_{-i}^*)$ *is a two-group equilibrium if, for each player i,* (s_i^*, m_i^*) *is the best response to the strategies* (s_{-i}^*, m_{-i}^*) *for other players such that*

[2] Some segregated states may not be social efficient, e.g. $s_1^{t-1,m} = (1,1), s_2^{t-1,m} = (1,1)$ or $s_1^{t-1,m} = (0,0), s_2^{t-1,m} = (0,0)$.

$$\pi_i(s_i^*, s_{-i}^*, m_i^*, m_{-i}^*, , n_1, n_2, p_1, p_2) \geq \pi_i(s_i, s_{-i}^*, m_i, m_{-i}^*, n_1, n_2, p_1, p_2)$$

$$for \;\; s_i = \{T, B\},\;\; m_i = \{1,2\}.$$

Proposition 4.

Consider two types of players matching with each other in a two-group game with the chance to move in every end of period. From inference above, we can claim that for group 1,2, corresponding proportion p_1, p_2, *and corresponding previous strategy profile:*

(a) $(p_1^{Te}, p_2^{Te}, (1,1), (1,1), N_1, N_2)$ *and* $(p_1^{Te}, p_2^{Te}, (0,0), (0,0), N_1, N_2)$ *is a recurrent class,* $\forall 1 > p_1^{Te}, p_2^{Te} > 0$, *and* $p_1^{Te}, p_2^{Te}, N_1, N_2$ *satisfy (#).*

(b) $(p_1^{Te}, p_2^{Te}, (1,0), (1,0))$ *is a recurrent class and a equilibrium with incomplete information if* $\frac{1-2p_i^{Te}(1-p_i^{Te})+p_j^{Te}}{2-2p_i^{Te}(1-p_i^{Te})} > \delta \geq 0$ *holds,* $\forall p_1^{Te}, p_2^{Te} \in [\frac{a}{a+b}, \frac{b}{a+b}]$ *and* $p_1^{Te}, p_2^{Te}, N_1, N_2$ *satisfy (#).*

(c) For $p_1^{Te} = 1, p_2^{Te} = 0, (s_1^{m,r,t}, s_2^{m,r,t}) \in \{((1,1), (1,1)),((1,1),(0,0)),((0,0),(0,0))\}$, *the state is a recurrent class and equilibrium with incomplete information.*

We try to simplify proposition 3. as a table form with different block labels. These labels will be used in the following discussion::

$s_i^{m,r,t}$	(a) (1,1)	(b) (1,0)	(c) (0,0)
(A) $p_i^{Te} > \frac{a}{a+b}$	Recurrent if another group's strategy distribution is (1,1), stable if two groups are segregated. (Aa)	Neither stable nor recurrent. (Ab)	Recurrent if another group's strategy distribution is (0,0), stable if both groups are segregated. (Ac)
(B) $\frac{a}{a+b} > p_i^{Te} > \frac{b}{a+b}$	Same as above (Ba)	Recurrent if $\frac{1-2p_i^{Te}(1-p_i^{Te})+p_j^{Te}}{2-2p_i^{Te}(1-p_i^{Te})} > \delta \geq 0$ (Bb)	Same as above (Bc)
(C) $\frac{b}{a+b} > p_i^{Te}$	Same as above (Ca)	Neither stable nor recurrent (Cb)	Same as above (Cc)

4. Stochastic Stability with Double Mutations

This chapter we start to build a system of total potential of all recurrent classes to confirm which class is most stochastic stable under dynamic mutation model. For simplicity here we must set another assumption: there are two possible mutations in dynamic model, one is strategy mutation and the other is move mutation. Strategy mutation means an individual switch his strategy irrationally and by enough switching we can notice that the group's total strategy distribution (from now on) changes. Move mutation means two individuals of different group exchange their position, that is, if a move mutation happens between a T.P. player and a B.P. player then these two groups will maintain their size but change proportions. If we count an individual strategy mutation as 1, then the move mutation is 2 due to it needs two individual to make mistake.

Consider a model whose total size of T.P. players is N_1 and size of B.P. players is N_2. Define a recurrent class (1,0,(1,1),(0,0)) by typical segregation, that is, typical segregation shows all players of same type gathered in same group and choose their favorite strategy. Then the corresponding size of two groups of typical segregated state is N_1 and N_2. Following we construct all possible recurrent classes under these settings and calculate the total potential.

We must give an extinct boundary condition of the total size N_1 and N_2:

$\frac{a}{b}N_2 > N_1 > \frac{b}{a}N_2$ and $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$. This is because under the condition of $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$, namely the difference of two type population is large, one kind of recurrent class, $(p_1^{Te}, p_2^{Te}, (1,0), (1,0))$ with proper p_1^{Te}, p_2^{Te}, cannot exist.

4.1. Stochastic stability when type size difference is small

4.1.1. Specifications of recurrent classes

Assume that the difference of two type size is not so large, $\frac{a}{b}N_2 > N_1 > \frac{b}{a}N_2$. Despite typical segregation there will be two untypical segregations: (1,0,(1,1),(1,1)) and (1,0,(0,0),(0,0)), and we denote typical segregation as (i) and untypical ones as (ii)(a) and (ii)(b). Another kind of recurrent classes is in the block of (Bb), namely proportions of two groups is in the threshold $(\frac{b}{a+b}, \frac{a}{a+b})$ and satisfying the restriction (*), and the strategy distributions are all (T,B). We denote them as (iii). Another kind is classes in the block of (Ba) and (Ca): all players play tennis and thus the strategy distribution is (T,T), denoted by (iv). The final one is in the block of (Ac) and (Bc), denoted by (v). Here is the list:

(i) Typical segregation (1,0,(1,1),(0,0))

(ii) Untypical segregation (1,0,(1,1),(1,1)) and (1,0,(0,0),(0,0)). We denote (1,0,(1,1),(1,1)) by (ii)(a) and (1,0,(0,0),(0,0)) by (ii)(b).

(iii) Recurrent classes in which two group all in the block of (Bb).

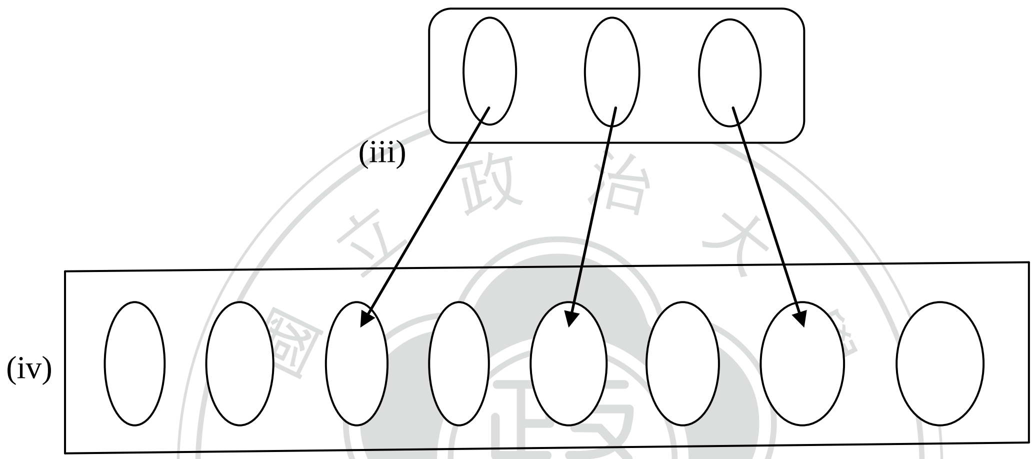
(iv) Recurrent classes in which two group all in the blocks of (Ba) and (Ca), or one is in (Aa) and the other is in (Ba) or (Ca).

(v) Recurrent classes in which two group all in the blocks of (Ac) and (Bc) ,or one is in (Cc) and the other is in (Ac) or (Bc).

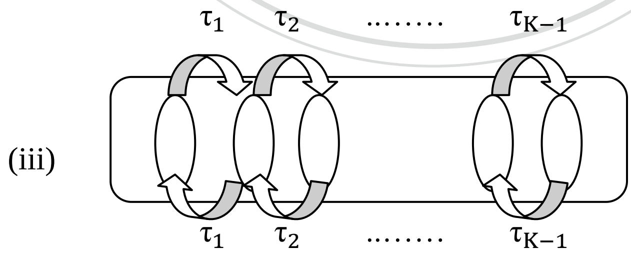
The reason why we eliminate recurrent classes that both groups in the block (Aa) of (iv) or both in the block (Cc) of (v) is that, the total size of T.P. and B.P. players are not disproportionate such that there will not the existence of "too much" T.P. players in both group (but there is still chance of "too much" T.P. players in one group but the other group is not). Thus there will not be such existence of too much B.P. players in both groups.

4.1.2. Transition between classes in the same specification

First, we claim that every class in (iii) have a corresponding class which has the same proportion distribution in (iv) and another one in (v). Notice that all the classes in (iii), whose strategy distribution is both groups play (T,B) and their proportions satisfy the restriction ($*$), can mutate to the classes of (iv) and (v) by mutate their strategy distribution. That means in (iv) and (v), there is a corresponding class for each class in (iii) and they have the same proportion distributions despite the strategy distributions are not the same. We call the class in (iv) and (v) "mirror class" of its corresponding class. Thus we can infer that all classes in (iii) can mutate to corre-sponding mirror class in (iv) or (v) by strategy mutation of both group.

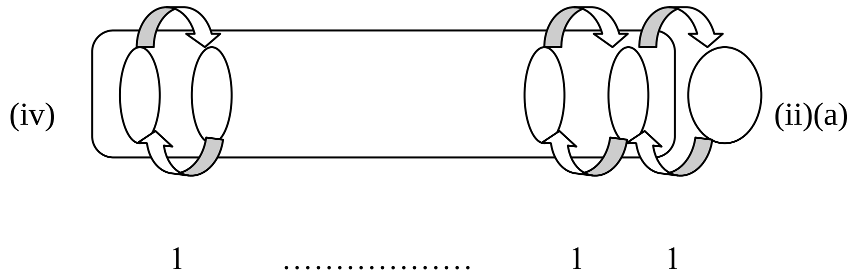


We then construct the transition resistances in the same specification. The following is specification (iii):

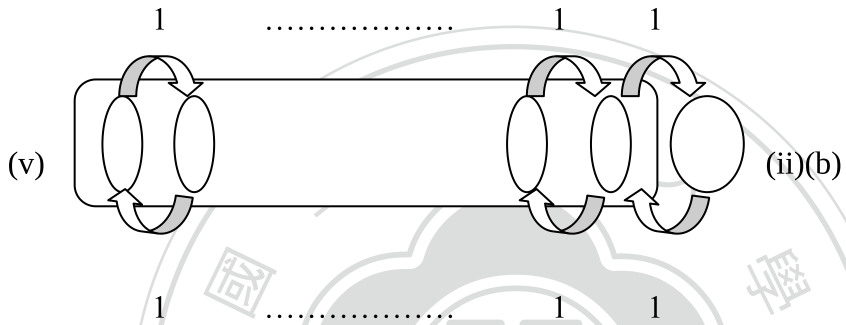


where the total recurrent classes in (iii) is K and $\tau_k \geq 1$. About specification (iv) and (ii)(a):

1 ……………… 1 1



This is specification (v) and (ii)(b), which are analogous to (iv) and (ii)(a):



The reason why we place (ii)(a) and (iv) together is that the strategy distribution of (ii)(a) is (T,T), same as (iv). That means an single individual move mutation can make (ii)(a) switch to (iv), and vice versa. This switching works on (ii)(b) and (v), too.

The reason why the inner resistance of (iii) and that of (iv) and (v) are not the same is, for every recurrent class in (iii) there is a corresponding class in (iv) and (v), however for every class in (iv) there is not always a corresponding class in (iii) due the restriction (*). Some classes of (iv) and (v) whose proportion distributions do not satisfy (*) cannot be a recurrent class when they mutate to (iii) by only strategy mutation. Thus we know the resistance between each recurrent class of (iii) may be larger than that of (iv) and (v), i.e. 1, due to the restriction ($*$). We denote all resistances between classes in (iii) by $\tau_1, \tau_2, \ldots, \tau_{K-1}$, where K is the number of recurrent classes in (iii) and $\tau_k \geq 1$.

Moreover, we can claim that all recurrent classes in (iii) have the same stochastic potential due to the same resistance, and that all recurrent classes in (iv) and (ii)(a) have the same stochastic potential due to the same resistance. So do all recurrent classes in (v) and (ii)(b).

lemma 5.

The following shows that some classes have the same stochastic potential:

(1) all recurrent classes in (iii).

(2) all recurrent classes in (iv) and (ii)(a).

(3) all recurrent classes in (v) and (ii)(b).

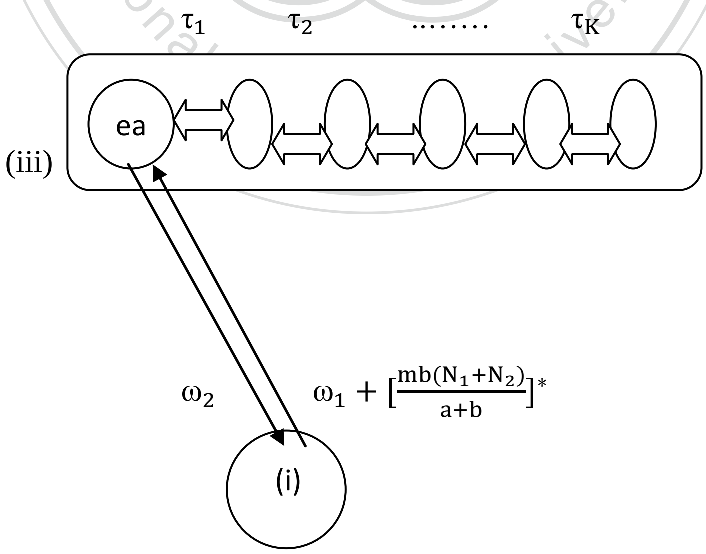
the resistance in (1) is $\tau_1, \tau_2, \ldots, \tau_{K-1}$*, where K is the number of recurrent classes in (iii) and* $\tau_k \geq 1$*, resistance in (2) and (3) is 1.*

proof: above.

4.1.3. Transition between specification (i) and (iii)

About (i) and (iii), from typical segregation to (iii), two groups must switch its proportion first and then switch to strategy distribution (T,B). We choose a class of (iii) whose group proportion is easiest for typical segregation to switch and denote the class as “easy-achieve”, “e.a.”. Define $p_{ea}^{Te} = \arg\min_h(\frac{a}{a+b} - p_h^{Te}, p_h^{Te} - \frac{b}{a+b})$, where p_h is the proportion which belongs to any recurrent class in (iii). Then p_{ea} is the possible proportion which is closest to $\frac{a}{a+b}$ or $\frac{b}{a+b}$ and the recurrent class which p_{ea} belongs to is e.a. class. We give a further definition of two proportions which belong to e.a. class, denoted by p_{ea1}^{Te} and p_{ea2}^{Te}. Of course, one of them is p_{ea}^{Te}. $p_{ea1}^{Te} = p_{ea}^{Te}$ or $p_{ea2}^{Te} = p_{ea}^{Te}$.

It needs ${\omega_1}^3$ individual move mutations for a proportion of segregated state to switch to p_{ea} and needs $[\frac{mb(N_1+N_2)}{a+b}]^*$ individuals strategy mutations, thus the total mutations we need is $\omega_1 + [\frac{mb(N_1+N_2)}{a+b}]^*$. However, the mutations needed for switching back is some move mutations such that one of group proportion is out of the threshold $(\frac{b}{a+b}, \frac{a}{a+b})$, denoted by $\omega_2 = \min((\frac{a}{a+b} - p_{ea1}^{Te})N_1, (p_{ea1}^{Te} - \frac{b}{a+b})N_1, (\frac{a}{a+b} - p_{ea2}^{Te})N_2, (p_{ea2}^{Te} - \frac{b}{a+b})N_2)$.



lemma 6.

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3 $\omega_1 = \min((1 - p_{ea1}^{Te})N_1, p_{ea1}^{Te}N_1, (1 - p_{ea2}^{Te})N_2, p_{ea2}^{Te}N_2)$.

The resistance from specification (i) to (iii) is

$$\omega_1 + [\frac{mb(N_1+N_2)}{a+b}]^* where\ \ \omega_1 = min((1-p_{ea1}^{Te})N_1, p_{ea1}^{Te}N_1, (1-p_{ea2}^{Te})N_2, p_{ea2}^{Te}N_2).$$

From (iii) to (i), the resistance is

$$\omega_2 = min((\frac{a}{a+b} - p_{ea1}^{Te})N_1, (p_{ea1}^{Te} - \frac{b}{a+b})N_1, (\frac{a}{a+b} - p_{ea2}^{Te})N_2, (p_{ea2}^{Te} - \frac{b}{a+b})N_2).$$

4.1.4. Transition between specification (ii), (iii), (iv), and (v)

About (ii), (iii), (iv), and (v): since in (iv) or (v) there are corresponding classes (mirror class) for all classes of (iii), the switching is easy: from (iii) to (v) or (iv) the groups only need to switch their strategy distribution from (T,B) to (T,T) or (B,B). The mutations needed to switch from (T,B) to (T,T) is

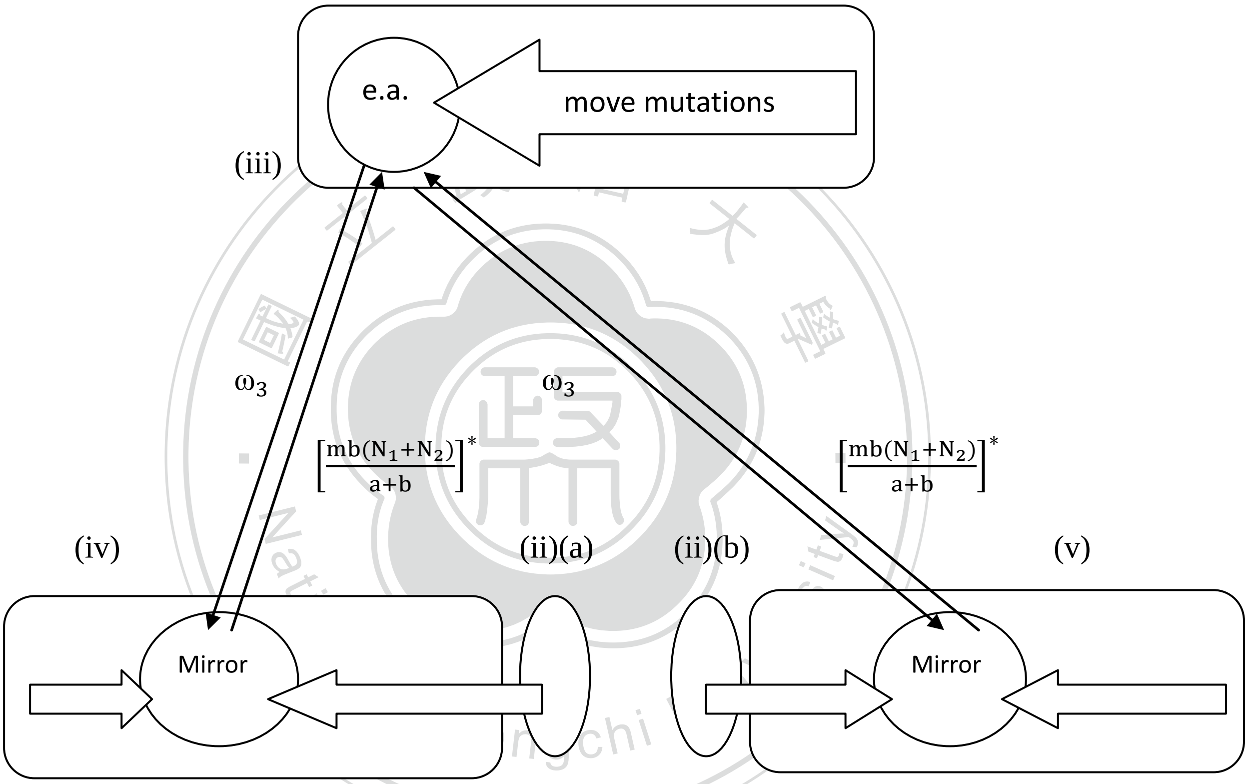
$$\left[mN_1 \cdot \min(\frac{a}{a+b} - p_i^{Te}, p_i^{Te} - \frac{b}{a+b}) + mN_2 \cdot \min(\frac{a}{a+b} - p_j^{Te}, p_j^{Te} - \frac{b}{a+b})\right]^*, \forall \text{group i, j}$$

Since in (iii), class "e.a" is the class whose group proportion is closest to $\frac{a}{a+b}$ or $\frac{b}{a+b}$, we can infer that e.a. class has the minimal resistance to mutate to (v) or (iv) in all recurrent classes of (iii). Then the resistance is

$$\left[mN_1 \cdot \min(\frac{a}{a+b} - p_{ea1}^{Te}, p_{ea1}^{Te} - \frac{b}{a+b}) + mN_2 \cdot \min(\frac{a}{a+b} - p_{ea2}^{Te}, p_{ea2}^{Te} - \frac{b}{a+b})\right]^*$$

Denoted by ω_3, where p_{ea1}^{Te} is the proportion of group 1 of e.a. recurrent class and p_{ea2}^{Te} is that of group 2 of e.a. class, just as we mentioned in 4.1.3.

For another aspect: from (iv) or (v) to (iii), all classes must switch its group proportions and strategy distribution from (T,T) or (B,B) to (T,B). This is a small strategy mutation and need $\left[\frac{\mathrm{mb}(\mathrm{N}_1+\mathrm{N}_2)}{\mathrm{a+b}}\right]^*$ mutations. By the way, the transition between (ii)(a) and (iii) must pass (iv)/(v), thus similar argument apply. So does (ii)(b). The graph is printed in the next page:



lemma 7.

The resistance from (iii) to (iv) or from (iii) to (v) is

$$\left[mN_1 \cdot min(\frac{a}{a+b} - p_{ea1}^{Te}, p_{ea1}^{Te} - \frac{b}{a+b}) + mN_2 \cdot min(\frac{a}{a+b} - p_{ea2}^{Te}, p_{ea2}^{Te} - \frac{b}{a+b})\right]^*,$$

which is denoted by ω_3.

The resistance from (iv) to (iii) or from (v) to (iii) is

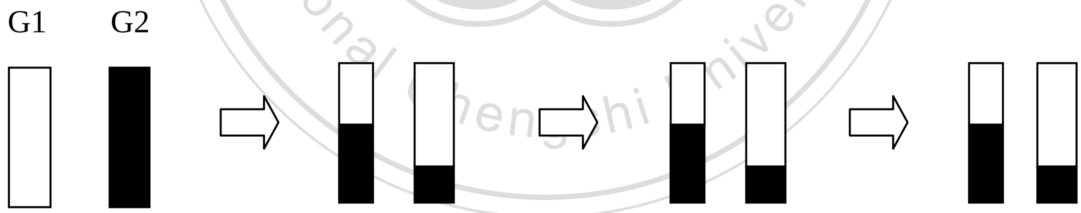
$$\left[\frac{mb(N_1+N_2)}{a+b}\right]^*.$$

proof: above.

4.1.5. Transition between (i), (ii), (iv), and (v)

The last part is (i), (ii), (iv), and (v). We firstly check the process from (i) and (v) to (iv).(the process from (iv) and (i) to (v) is the same) From the process for a typical segregation mutating to classes of (iii), we consider two path: 1st path is that (i) transit to (ii)(a) first and then transit to (iii); 2nd path is (i) directly transit to (iii) by the following method:

The 2nd path



(T,T) (B,B) (T,T) (B,B) (T,T) (T,B) (T,T) (T,T)

(1) (2) (3) (4)

Explanation:

(1) A typical segregation

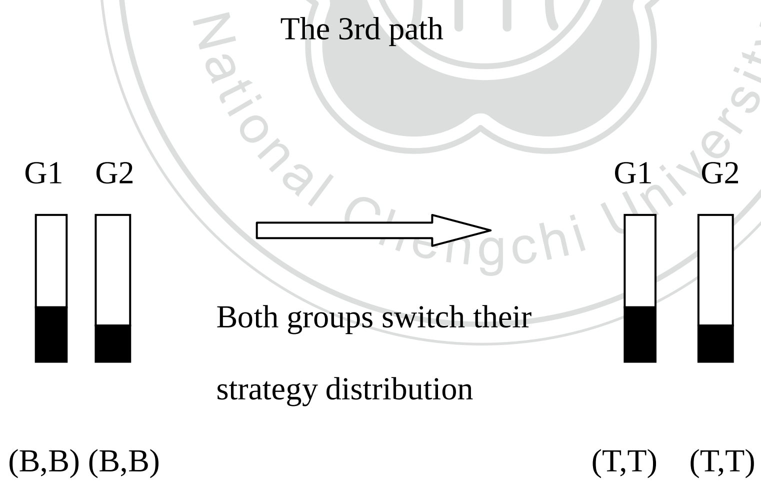
(2) Group 1 and 2 switch their proportions by some move mutations, $\frac{aN_2}{a+b}$, until group

2’s proportion is equal to $\frac{a}{a+b}$.

(3) Group 2 switches its strategy distribution from (B,B) to (T,B).

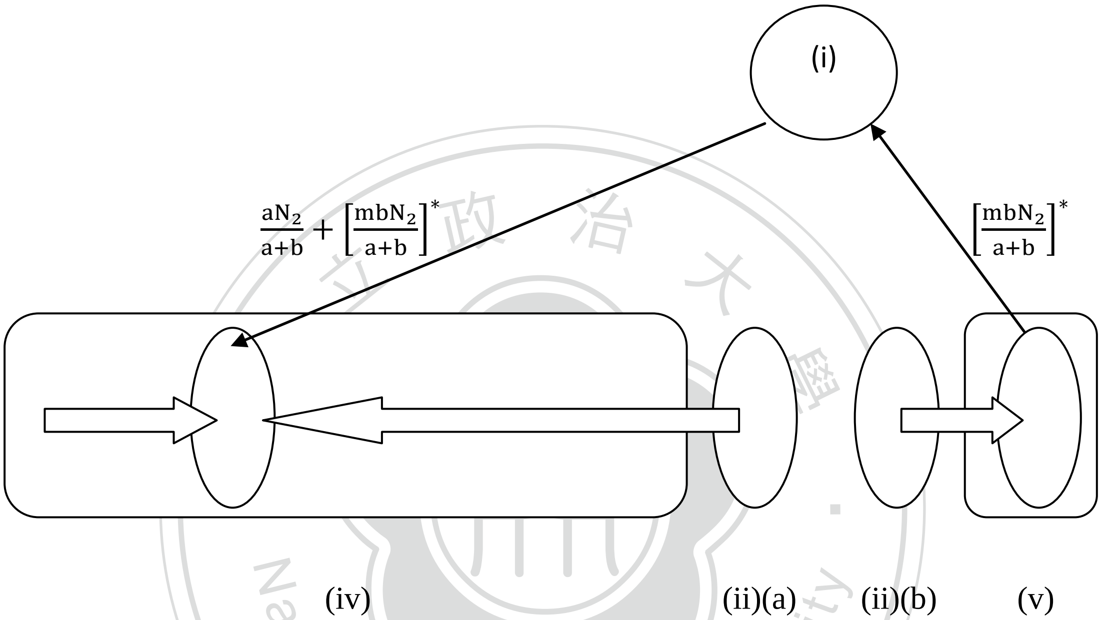
(4) Since in group 2 the proportion of T.P. players is large enough, soon the strategy distribution will switch to (T,T) automatically. We denote the class of (4) by relay class.

Moreover, from (ii)(b) and (v) to (iv), there will be three possible paths: first is mutate to typical segregation (i) by a small strategy mutation (if one group switch its strategy from (B,B) to (T,B) then the typical segregation happens) and then through 1^{st} path to mutate to (iv). Second is mutate to typical segregation (i) and then through 2^{nd} path to mutate to (iv). The last one is mutate directly to (iv) by strategy mutation:

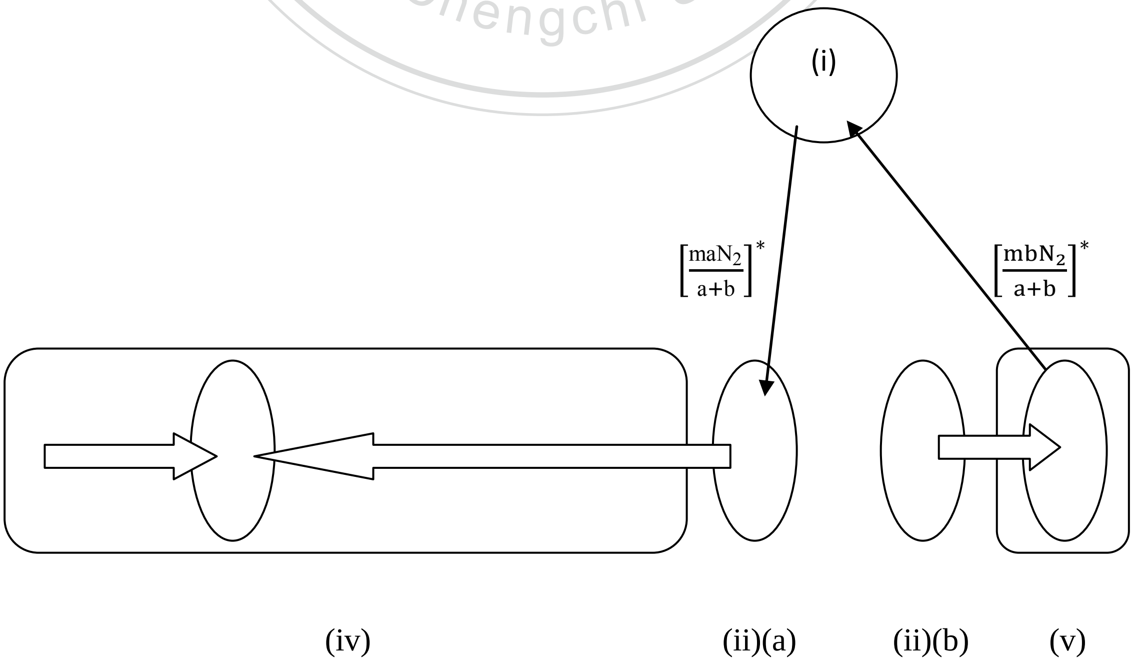


The resistance of 3^{rd} path is $\left[\frac{ma(N_1+N_2)}{a+b}\right]^*$ since the needed mutations for group 1 to switch its strategy distribution from (B,B) to (T,T) is $\left[\frac{maN_1}{a+b}\right]^*$ and that for group 2 to do so is $\left[\frac{maN_2}{a+b}\right]^*$.

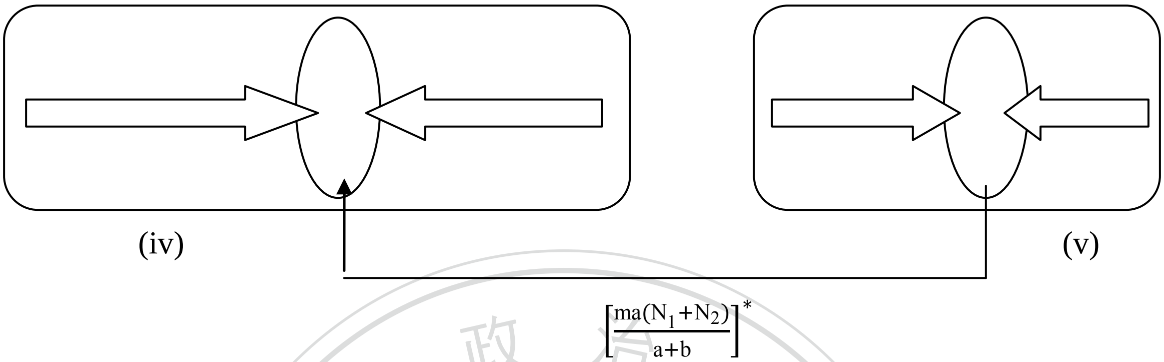
To compare easily, we give a graph below to describe the flows. One thing to notice is that from any class of (iv) or (v) transit to (i), the the resistance is $\left[\frac{mbN_2}{a+b}\right]^*$ because only by a small strategy mutation of a relative small size group the groups will soon switch to typical segregation. The following graph is rooted by the relay class of (iv), that is the process will use 1st path:



And this is by 2nd path:



The last one is 3rd path, rooted by any class of (iv):



By calculation we know that, under large enough m the path with minimal resistance is 1st path (Through relay class). This is very intuitive: once m is large, it means that a strategy distribution switching for a group is getting harder. Thus comparing with 2nd and 3rd path, 1st path needs less strategy mutations and of course it is the minimal resistance path.

lemma 7.

From the specification (v) to (iv), the transition will firstly pass typical segregation, (i), and then transit to the relay class of (iv). The total resistance is

$$\frac{aN_2}{a+b} + 2\left\lceil \frac{mbN_2}{a+b} \right\rceil^*$$

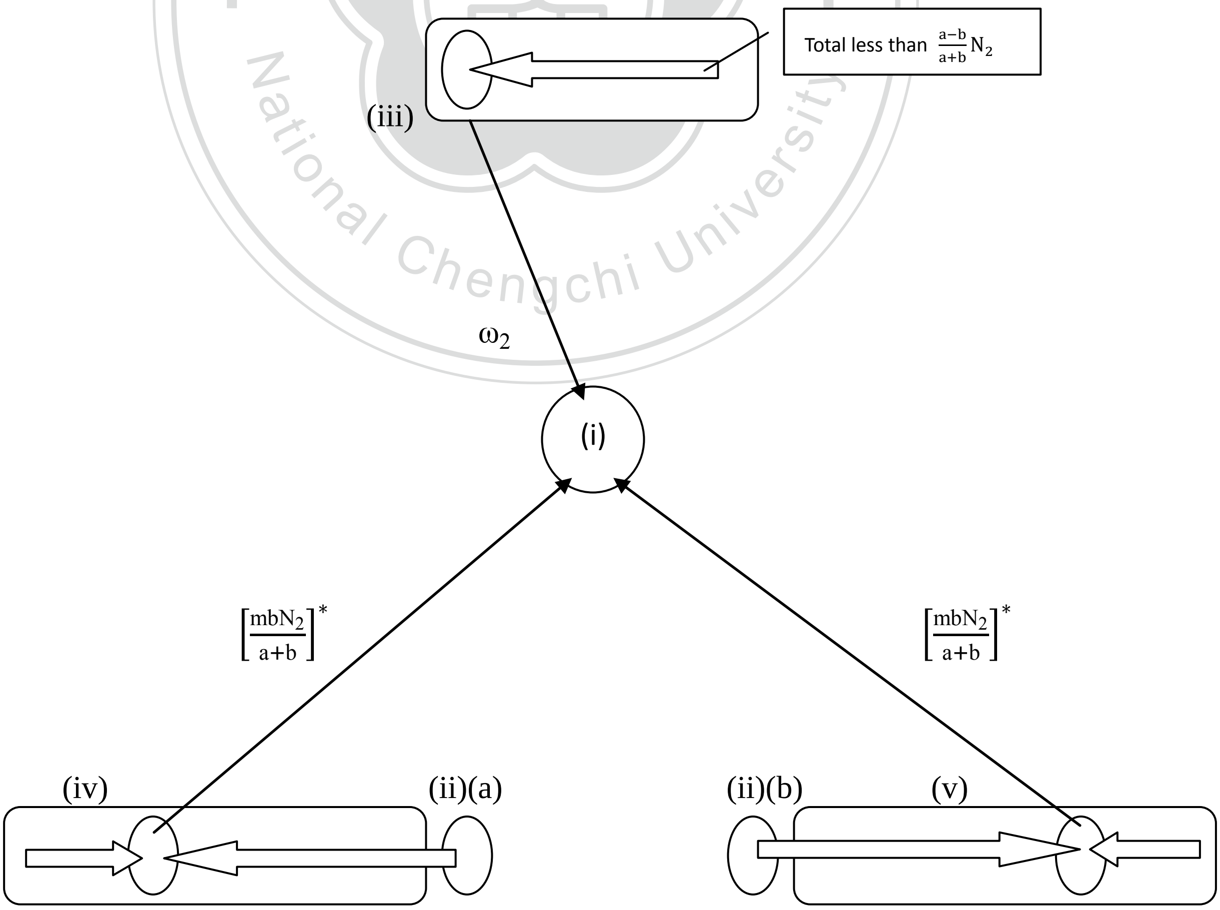
From (iv) to (v), the resistance is the same.

proof: above

4.1.6. The stochastic potential of classes in each specification

From lemma 5, 6, and 7, we have enough information about the resistance between each specification and can construct stochastic potential now. Because (v) is analogous to (iv) and the relay class is the class with minimal resistance of all class in (iv), we only need to compare (i), (ii), e.a. class of (iii), and relay class of (iv).

The (i)-tree:



Total $N_2 - 1$ Total $N_2 - 1$

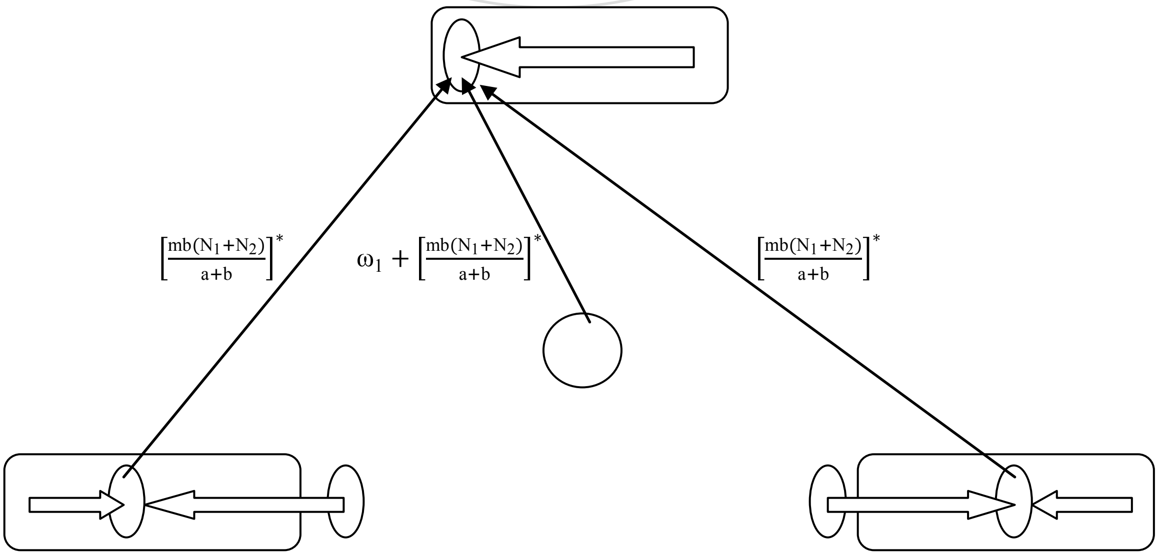
We claim that the stochastic potential of (i)-tree is $2\left[\frac{mbN_2}{a+b}\right]^* + \omega_2 + MR_T$, where MR_T is the sum of total move resistance in (ii), (iii), (iv), and (v); ω_2 is the minimal move mutations needed for e.a, equal to $\min(\left(\frac{a}{a+b} - p_{ea1}^{Te}\right)N_1, \left(p_{ea1}^{Te} - \frac{b}{a+b}\right)N_1, \left(\frac{a}{a+b} - p_{ea2}^{Te}\right)N_2, \left(p_{ea2}^{Te} - \frac{b}{a+b}\right)N_2)$. class to switch one of its group proportions out of the threshold $\left[\frac{b}{a+b}, \frac{a}{a+b}\right]$.

lemma 8.

The stochastic potential of (i)-tree is $2\left[\frac{mbN_2}{a+b}\right]^* + \omega_2 + MR_T$, *where* MR_T *is the sum of total move resistance in (ii), (iii), (iv), and (v). This means the typical segregated state's stochastic potential is* $2\left[\frac{mbN_2}{a+b}\right]^* + \omega_2 + MR_T$.

Now check the stochastic potential of e.a. class of (iii):

The (iii)-tree



We claim that the stochastic potential of (iii)-tree is $\omega_1 + 3\left[\frac{mb(N_1+N_2)}{a+b}\right]^* + MR_T$. Obviously the potential is larger than that of (i)-tree. This means all classes in (iii) have more stochastic potential than typical segregation.

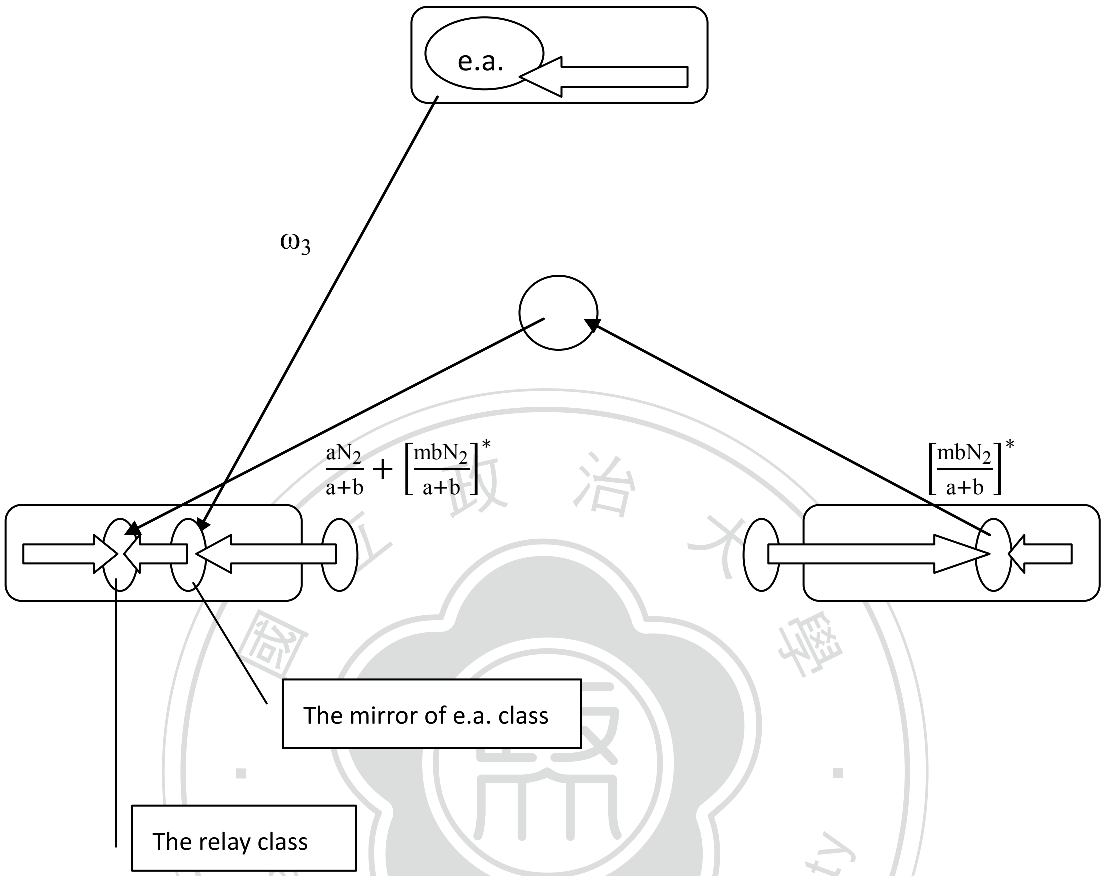
lemma 9.

The stochastic potential of (iii)-tree is $\omega_1 + 3\left[\frac{mb(N_1+N_2)}{a+b}\right]^* + MR_T$*, where* MR_T *is the sum of total move resistance in (ii), (iii), (iv), and (v). This means that all the recurrent classes in (iii) have the stochastic potential* $\omega_1 + 3\left[\frac{mb(N_1+N_2)}{a+b}\right]^* + MR_T$.

proof: above

About the relay class of (iv) and (ii)(a): we know that the resistance from (ii)(a) to (iv) is a single move mutation, and so is the inverse direction resistance. Thus the potential of (ii)(a) and all classes in (iv) are the same. (so are (ii)(b) and (v)) We can discuss them at same paragraph.

The (iv)-tree:



The stochastic potential of (iv)-tree is $\frac{aN_2}{a+b} + 2\left[\frac{mbN_2}{a+b}\right]^* + \omega_3 + MR_T$, where ω_3 is $\left[mN_1 \cdot \min(\frac{a}{a+b} - p^{Te}_{ea1}, p^{Te}_{ea1} - \frac{b}{a+b}) + mN_2 \cdot \min(\frac{a}{a+b} - p^{Te}_{ea2}, p^{Te}_{ea2} - \frac{b}{a+b})\right]^*$. Compare this with the potential of typical class, we can derive that when ω_2 is smaller than $\omega_3 + \frac{aN_2}{a+b}$ then the stochastic potential of typical class is the smallest one, and vice versa. Since both ω_2 and $\frac{aN_2}{a+b}$ are move mutations and $\omega_2 = \min(\left(\frac{a}{a+b} - p^{Te}_{ea1}\right)N_1, \left(p^{Te}_{ea1} - \frac{b}{a+b}\right)N_1, \left(\frac{a}{a+b} - p^{Te}_{ea2}\right)N_2, \left(p^{Te}_{ea2} - \frac{b}{a+b}\right)N_2)$, much smaller than $\frac{aN_2}{a+b}$, we can infer that the stochastic stable state of the dynamic model is typical class.

One more thing to notice is that, since both ω_2 and $\frac{aN_2}{a+b}$ are move mutations and $\omega_2 < \frac{aN_2}{a+b}$, it means the stable state will not change even if we impose a multiplier φ on all move mutations and claim that move mutation is much easier or harder than strategy mutation(i.e. φ is much smaller or larger than 1).

Since (ii)(a) has the same stochastic potential as (iv) and the (v)-tree is analogous to the (iv) tree, we can infer that all recurrent classes in (iv) and (v), and untypical segregated states (ii)(a) and (b), have the same stochastic potential which is equal to $\frac{aN_2}{a+b} + 2\left[\frac{mbN_2}{a+b}\right]^* + \omega_3 + MR_T$.

lemma 10.

The stochastic potential of (iv)-tree is $\frac{aN_2}{a+b} + 2\left[\frac{mbN_2}{a+b}\right]^* + \omega_3 + MR_T$, *where* ω_3 *is* $\left[mN_1 \cdot min(\frac{a}{a+b} - p_{ea1}^{Te}, p_{ea1}^{Te} - \frac{b}{a+b}) + mN_2 \cdot min(\frac{a}{a+b} - p_{ea2}^{Te}, p_{ea2}^{Te} - \frac{b}{a+b})\right]^*$. *Thus all recurrent classes in (iv) and (v), and untypical segregated states (ii)(a) and (b), have the same stochastic potential which is equal to* $\frac{aN_2}{a+b} + 2\left[\frac{mbN_2}{a+b}\right]^* + \omega_3 + MR_T$.

proof: above.

By lemma 8, 9, and 10. We know that the typical segregated state, (i), has the minimal stochastic potential. Moreover, if move mutations is much easier than strategy mutations, the stable state is still typical segregated state. This is because there is no more move mutations than MR_T in the stochastic potential of (iii)-tree (lemma 9). Another reason is that in the stochastic potential of (iv)-tree and (v)-tree, both ω_2 and $\frac{aN_2}{a+b}$ are move mutations but $\omega_2 < \frac{aN_2}{a+b}$ (lemma 10).

Proposition 11.

If $\frac{a}{b}N_2 > N_1 > \frac{b}{a}N_2$, then the stochastic stable state of model with incomplete information is typical segregated state, (1,0,(1,1),(0,0)).

proof: above

4.2. Stochastic stability when type size difference is large

However, proposition 11 is based on the size of two group is close, i.e. $\frac{a}{b}N_2 > N_1 > \frac{b}{a}N_2$, will the stochastic stable state still be typical segregation if $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$?

We can deduce that if $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$ then the specification distribution will change: the existence of (Bb) will distinguish and thus (iii) no longer exist. That means the model is just like the simplified version of above model. The only difference is that we remove the specification: recurrent classes in which two group all in the block of (Bb). Moreover, the classes of (iv) and (iv) will change a little. Here we replace the list:

(i)Typical segregation (1,0,(1,1),(0,0))

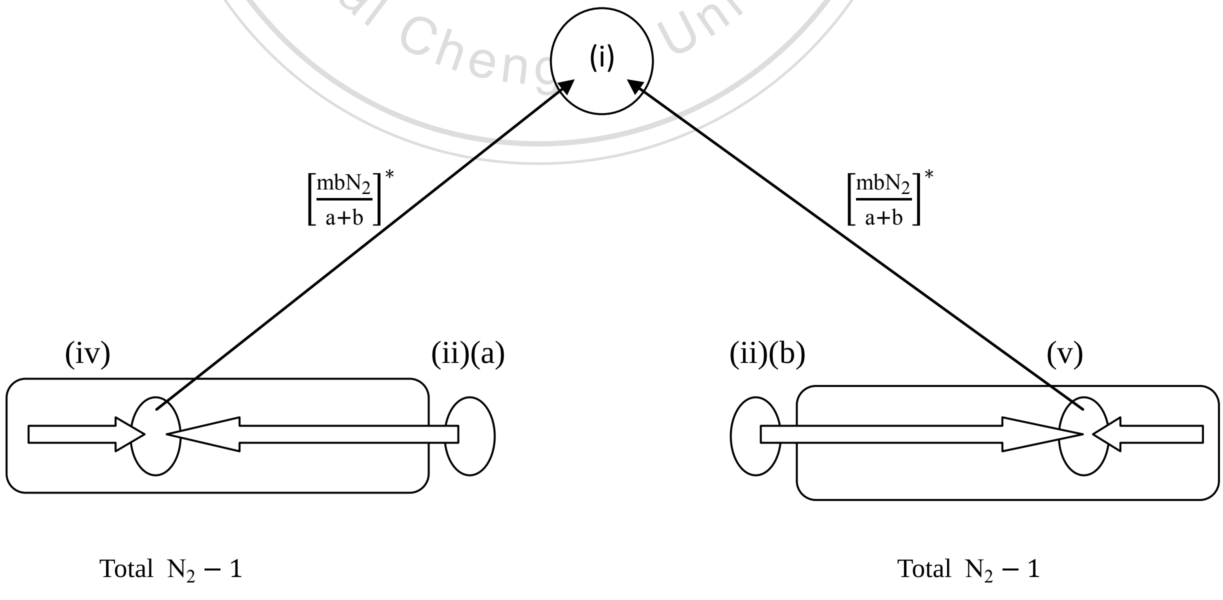
(ii)Untypical segregation (1,0,(1,1),(1,1)) and (1,0,(0,0),(0,0)). We denote (1,0,(1,1),(1,1)) by (ii)(a) and (1,0,(0,0),(0,0)) by (ii)(b).

(iv)Recurrent classes in which two group all in the blocks of (Aa) and (Ba), or one is in (Ca) and the other is in (Aa) and (Ba).

(v)Recurrent classes in which two group all in the blocks of (Bc) and (Cc) ,or one is in (Ac) and the other is in (Bc) and (Cc).

If we set (i) as the root:

The (i)-tree

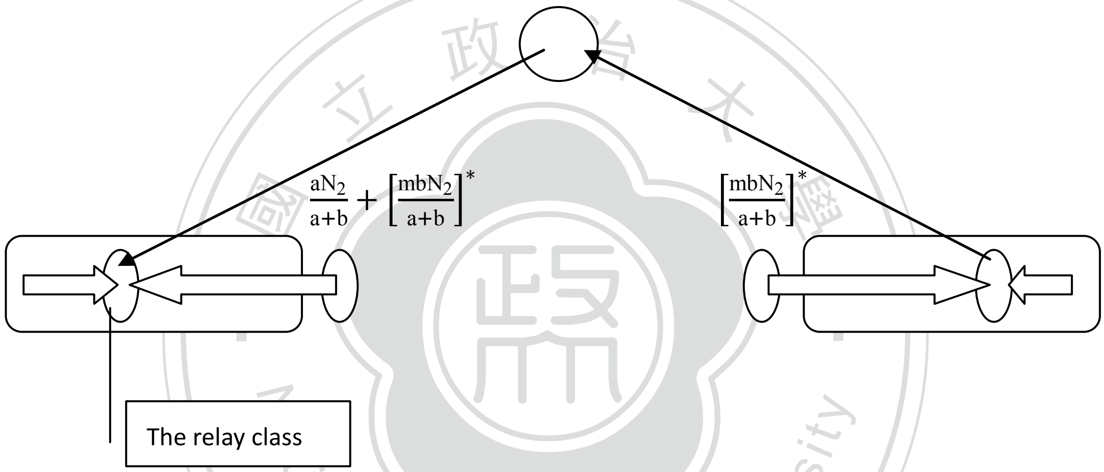


lemma 12.

The stochastic potential of (i)-tree is $2\left[\frac{mbN_2}{a+b}\right]^* + MR_T^*$*, where* MR_T^* *is the sum of move resistance of (iv) and (v).*

If rooted by the relay class of (iv):

The (iv)-tree



lemma 13.

The stochastic potential of (ii), (iv), and (v) are all $\frac{aN_2}{a+b} + 2\left[\frac{mbN_2}{a+b}\right]^* + MR_T^*$*, where* MR_T^* *is the sum of move resistance of (iv) and (v).*

To note that lemma 12 and 13 is based on the assumption $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$. Obviously the stochastic stable state is typical segregation, no matter how small the chance of move mutation is, i.e. φ does not matter, because there is no more move mutations than MR_T^* in the (i)-tree.

Propostion 14.

If $N_1 > \frac{a}{b}N_2$ or $\frac{b}{a}N_2 > N_1$, *then the stochastic stable state of model with incomplete information is typical segregated state, (1,0,(1,1),(0,0)).*

proof: above.

4. Conclusion

Two-group model shows another result, far from the grouping equilibrium. Equilibrium of two-group model may be segregated or non- segregate depending on the obstruction of information and misconception of players, and it also shows that with double mutations the stochastic stable state is the typical segregated state. Moreover, it solves the drawback of grouping equilibrium: move is easily to happen if people are dissatisfying on the payoff, and two-group model also shows the process of grouping formation. We can find that in the random and inherent match game, people will try to find another group to maximize his payoff. But when infor- mation is not clear on some aspects: players cannot distinct the type of each other and those in another group, or people might be "deceived" by the investigation of his or a fair institution, then non-segregation equilibrium will happen. However in a dynamic process, we will notice that in most situations, typical segregation is stable because only a small strategy change, the equilibrium of non-segregation will easily be broken by strategy mutations.

Since we know that in the dynamic model, after periods the equilibrium will be segregated as a typical segregation, which means efficiency for every player. Thus, what else a government should do is to increase the speed of process to form a typical segregation. Government can increase the speed by increase the number of re-matches in a given period (such like a week, a month). Once the number of match is increased, the probability of making mutation in strategy decision in a given period is increased. From Chapter 4 we know typical segregation is more stable than other recurrent classes, since the probability of mutation is increased we can infer the model is easier to derive a typical segregation, namely efficient equilibria.

Another method to solve the problem of inefficiency is removing the obstruction of information. However, it is costly for government to investigate the type of all players, and sometimes the information is always incomplete. In this situation, method of increasing the number of re-matches in a given period is a better choice.

Appendix A.

Assume that in the same group, the proportion of players who choose to bring tennis in T.P. type is p_T and that in B.P. type is p_B. The proportion of T.P. players in this group is p and the size of the group is N_i.

Then for a T.P. player, he will choose tennis if

$$app_T + a(1-p)p_B > ap\left(1-p_T\right) + b(1-p)(1-p_B)$$

For a T.P. player, he will choose tennis if

$$bpp_T + b(1-p)p_B > bp\left(1-p_T\right) + a(1-p)(1-p_B)$$

Simplify we yield

$$pp_T + (1-p)p_B > \frac{b}{a+b}$$

$$pp_T + (1-p)p_B > \frac{a}{a+b}$$

Thus for $s_S^{r-1} = (1,1)$ or $(0,0)$ to mutate to $(1,0)$, we need the proportion of players who choose to bring tennis in T.P. type to change, that is, $\frac{b}{a+b}\frac{1}{p}$, or the proportion of players who choose to bring tennis in T.P. type to change, $\frac{b}{a+b}\frac{1}{1-p}$. (select the minimum) Consider the corresponding proportion, total population, and m memory periods, the minimum mutations are $[\frac{mbN_i}{a+b}]^*$. For (1,1) to (0,0) or (0,0) to (1,1), the minimum mutations are $[\frac{maN_i}{a+b}]^*$. For (1,0) to (1,1) or (0,0), the minimum mutations are $[mN_i \cdot \min(\frac{a}{a+b} - p, p - \frac{b}{a+b})]^*$.

Reference

1. Borgers, T. and Sarin, R. "Naive Reinforcement Learning with Endogenous Aspirations," *International Economic Review*, 2000, vol. 41, no. 4, pp. 921-950.
2. Carrington, William J., Detragiache, Enrica, and Vishwanath, Tara. "Migration with Endogenous Moving Costs," *American Economic Review*, 1996, vol. 86, pp. 909-930.
3. Castillo, Marco and Petrie, Ragan. "Discrimaination in The Lab: Does Information Trump Appearance," *Games and Economic Behavior*, 2010, vol. 68, pp. 50-59.
4. Currarini, Sergio, Jackson,Matthew O., and Pin, Paolo. "An Economic Model of Friendship: Homophily, Minorities, and Segregation," *Econometrica*, 2009, vol. 77, No. 4, pp.1003-1045.
5. Foster, D.P., Young H.P. "Regret testing: learning to play Nash equilibrium without knowing you have an opponent," *Theoretical Economics*, 2006, pp. 341-367.
6. Hart, Sergiu and Mas-Colell, Andreu. "A Simple Adaptive Procedure Leading to Correlated Equilibrium," *Econometrica*, 2000, vol. 68, no. 5, pp. 1127-1150.
7. Hoff, Karla and Sen, Arijit. "Homeownership, Community Interactions, and Segreagation," *American Economic Review*, 2005, vol.95, pp. 1167-1189.
8. Jackson, Matthew O. and Watts, Alison. "Social Games: Matching and The Play of Infinity Repeated Games," *Games and Economic Behavior*, 2010, vol. 70, pp. 170-191.
9. Milchtaich, Igal. "Stability and Segregation in Group Formation." *Games and Economic Behavior*, 2002, vol. 38, pp. 318-346.
10. Schelling, Thomas C. "Models of Segregation," *American Economic Review*, 1971, vol. 59, pp.488-493.
11. William T. Bielby and James N. Baron, "Sex Segreagation With Occupations," *American Economic Review*, 1998, vol. 76, pp. 43-47.
12. Young H.P. “*Individual strategy and social structure: An evolutionary theory of institutions."* Prinston, 2001.