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碩士學位論文

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最小成本下，規格及 $\bar{X}-S$ 管制圖之設計

The Design of Specification and $\bar{X}-S$ Charts with Minimal Cost

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ABSTRACT

The design of economic statistical control charts and specification are both crucial research areas in industry. Furthermore, the determination of consumer and producer specifications is important to producer. In this study, we consider eight cost models including the consumer loss function and/or the producer loss function with the economic statistical $\overline{X}$ and S charts or Shewhart-type economic $\overline{X}$ and S charts. To determine the design parameters of the $\overline{X}$ and S charts and consumer tolerance and/or producer tolerance, we using the Genetic Algorithm to minimizing expected cost per unit time. In the comparison of examples and sensitivity analyses, we found that the optimal design parameters of the Shewhart-type economic $\overline{X}$ and S charts are similar to those of economic statistical $\overline{X}$ and S control charts, and the expected cost per unit time may lower than the actual cost per unit time when the cost model only considering consumer loss or producer loss. When considering both consumer and producer tolerances in the cost model, the design parameters of the economic $\overline{X}$ and S charts are not sensitive to the cost models. If the producer tolerance is smaller than the consumer tolerance, and the producer loss is smaller than the consumer loss, the optimal producer tolerance should be small.

Keywords: Economic statistical control charts; Consumer tolerance; Producer tolerance; $\overline{X}$ and S charts; Loss function

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1. INTRODUCTION AND LITERATURES REVIEW

Control charts are widely used in statistical quality control. They monitor processes and determine whether a process is in-control or out-of-control. To use a control chart, the engineer must specify the sample size *n*, the sampling period *h*, and the coefficient of control limits *k*. In practice, these parameters are usually chosen by the engineer's experience and by considering statistical criteria such as type I and type II errors.

Duncan (1956) first recommended economic design of control charts. Duncan proposed a cost model to design the parameters of the $\bar{X}$ control chart, which assumes that the assignable cause occurs according to the Poisson process. This cost model includes the cost of sampling and testing, the cost of finding the assignable cause, and the cost of process correction. Duncan's work has been extensively studied and extended by many others.

After Duncan (1974) indicated that simultaneous employment of $\bar{X}$ and R charts to control process mean and variability "will give reasonably good control of the whole process," joint economic design research has been conducted. Rahim et al. (1988) presented the use of joint economic designs of $\bar{X}$ and S^2 charts when the sample size is moderately large. Collani and Sheil (1989) proposed the economic design of an S chart when only a single assignable cause may influence process variability.

When using economic design, the parameters of control charts should be determined by minimizing the expected cost from the process. This does not consider statistical properties such as Type I or Type II error rates. Woodall (1986, 1987) indicated that the Type I error rate of many economic control charts is higher than those of statistical design. Saniga (1979) proposed a method to design control charts that have bounds on Type I and Type II error probabilities and the average time to signal (ATS), but still allow for low-process variability and long-term quality. Although a design with these statistical constraints is more costly than economic design, it is more effective and reduces false alarm rates. Saniga called this design "economic statistical design."

Elsayed and Chen (1994) found that the practical applications of economic design are limited because of difficulties in estimating costs. Quality loss is considered a cost when quality characteristic is not within the specification limits. Quality loss was defined by Taguchi (1984) as "the loss to society caused by the product after it is shipped out." Taguchi proposed a quadratic loss function to estimate the quality loss of the manufactured product when it deviates from its target. Elsayed and Chen (1994) proposed the economic design of $\overline{X}$ charts based on the Taguchi loss function with continuous operations. Yang (1997) presented a joint economic design of $\overline{X}$ and S charts with two assignable causes using the Taguchi quadratic loss function and presented a statistical constrained economic model that considers the Taguchi quadratic loss function for the optimal design of the S control chart for controlling process variability.

In a complete inspection plan, every outgoing item is subjected to inspection, and items failing to meet the specifications are reworked. For items following a normal distribution, Tang (1988) presented an economic model to determine the most profitable specification for a complete inspection plan by considering quality loss. Fathi (1990) discussed producer tolerance and consumer tolerance, and proposed a graphical method to determine the producer tolerance for a given consumer tolerance by minimizing per-unit cost. Maghsoodloo and Li (2000) proposed an economic model for asymmetric tolerance design by minimizing the expected loss per unit. Because the design of the process mean is an issue for engineering, Kapura (1988) proposed an economic model to determine process mean and tolerance simultaneously by minimizing per-unit costs under the symmetric Taguchi quadratic loss function. Lee, Kim, Kwon and Hong (2004) presented an economic model for a filling process to determine the process mean under specification is known by maximizing expected profit per unit. Furthermore, Feng and Kapura (2006) proposed an economic model to determine the mean and tolerance by minimizing expected cost per unit using an asymmetric quadratic Taguchi loss function and a piecewise linear loss function.

In a practical example, like pad is important for the Chemical Mechanical Polishing (CMP) of wafer. When the consumer gives an order for pad producer, they also give a consumer specification. Typically, producer specification is set with the same or less than consumer specification. If the nonconforming rate of wafer is high by using the pad of the CMP, the customer will ask to modify specification and the producer should re-determine their specification. Since that, the determination of consumer specification and producer specification is an important issue for producer.

The design of the control chart and specifications are both essential to quality control. Previous research has not discussed how to design them simultaneously. In this study, we propose economic cost models to design $\overline{X}$ and S charts, consumer tolerance, and producer tolerance together. These models include the cases “only with consumer tolerance,” “only with producer tolerance,” and “with both producer and consumer tolerance.” We show the differences in the optimal costs, and differences in the optimal design parameters of $\overline{X}$ and S charts and producer and/or consumer specifications.

Section 2 of this study presents a discussion of an economic cost model without tolerance. This model is from Montgomery (1980) and Yang (1997). Sections 3 and 4 introduce a cost model with only consumer tolerance and a cost model with only producer tolerance. When we consider producer tolerance and consumer tolerance simultaneously, we assume that consumers and producers have different loss functions. Section 5 provides a discussion on different cost models for different relationships between consumer and producer loss functions. Section 6 shows a comparison of examples and sensitivity analysis for each model. Section 7 offers a conclusion.

2. DESIGN OF ECONOMIC STATISTICAL $\bar{x}$ AND S CHARTS WITHOUT TOLERANCE

2.1 Derivation of Cost Models

We assume that the process begins in a statistical in-control state with mean μ and standard deviation σ. The time ($T_{s.c.}$) until the occurrence of assignable causes is exponential, with a mean of 1/λ. A single assignable cause shifts the mean from μ to μ + $\delta_1\sigma$ ($\delta_1 \neq 0$) and a shift of standard deviation from σ to $\delta_2\sigma$ ($\delta_2 > 1$). Without loss of generality, we only consider the case of $\delta_1 > 0$ in this study. The quality characteristic is assumed to follow a normal distribution.

This study assumes the following:

$$X \sim N(\mu, \sigma^2), \text{ if process is in-control.}$$
$$X \sim N(\mu + \delta_1\sigma, \delta_2^2\sigma^2), \delta_1 > 0, \delta_2 > 1, \text{ if process is out-of-control.}$$

The samples of size *n*, unit time *h*, and the limits of the $\bar{X}$ and S charts are set as

$$UCL_{\bar{X}} = \mu + k_1 \frac{\sigma}{\sqrt{n}},$$
$$LCL_{\bar{X}} = \mu - k_1 \frac{\sigma}{\sqrt{n}}$$

$$UCL_S = k_2\sigma$$
$$LCL_S = k_3\sigma,$$

where $0 < k_3 < k_2$.

If at least one plotted point falls outside the control limits of the $\bar{X}$ and S charts, the process is assumed to be out-of-control and engineers must search for an assignable cause. If the process is out-of-control, corrective action is taken and the process continues.

The probability (α) that at least one plotted point falls outside the control limits of the $\bar{X}$ and S charts when the process is in-control is calculated as follows:

$$\alpha = \alpha_{\bar{X}} + \alpha_S - \alpha_{\bar{X}}\alpha_S,$$

where

$$\alpha_{\bar{X}} = P\left(\bar{X} < \mu - k_1\sigma/\sqrt{n} \,\middle|\, \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)\right) + P\left(\bar{X} > \mu + k_1\sigma/\sqrt{n} \,\middle|\, \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)\right)$$
$$= 2\Phi(-k_1),$$

$$
\begin{aligned}
\alpha_S &= P\left(s > k_2\sigma \middle| X \sim N\left(\mu,\sigma^2\right)\right) + P\left(s < k_3\sigma \middle| X \sim N\left(\mu,\sigma^2\right)\right) \\
&= P(Y > (n-1)k_2^2) + P(Y < (n-1)k_3^2) \\
&= 1 - F_y((n-1)k_2^2) + F_y((n-1)k_3^2)
\end{aligned}
,
$$

where $\mathrm{Y} \sim \chi^2_{\text{n-1}}$.

The probability (β) that no sample points fall outside the control limits of the $\overline{X}$ and S charts when the process is out-of-control is calculated as follows:

$$
\beta = \beta_{\overline{X}}\beta_S,
$$

where

$$
\begin{aligned}
\beta_{\overline{X}} &= P\left(\mu - k_1\sigma/\sqrt{n} < \overline{X} < \mu + k_1\sigma/\sqrt{n} \middle| \overline{X} \sim N\left(\mu + \delta_1\sigma, \frac{\delta_2^2\sigma^2}{n}\right)\right) \\
&= \Phi\left(\frac{k_1 - \sqrt{n}\delta_1}{\delta_2}\right) - \Phi\left(\frac{-k_1 - \sqrt{n}\delta_1}{\delta_2}\right)
\end{aligned}
,
$$

$$
\begin{aligned}
\beta_S &= P\left(k_3\sigma < s < k_2\sigma \middle| X \sim N\left(\mu + \delta_1\sigma, \delta_2^2\sigma^2\right)\right) = P\left(\frac{(n-1)k_3^2}{\delta_2^2} < Y < \frac{(n-1)k_2^2}{\delta_2^2}\right) \\
&= F_y\left(\frac{(n-1)k_2^2}{\delta_2^2}\right) - F_y\left(\frac{(n-1)k_3^2}{\delta_2^2}\right)
\end{aligned}
,
$$

where $\mathrm{Y} \sim \chi^2_{\text{n-1}}$.

The expected number of false alarms that occur before a shift is α times the expected number of samples taken before a shift ($1/\lambda h$).

The cycle time is defined as the time starts from in-control state and ends with the assignable cause is happened and repaired. A process is consisted of a series of independent and identical cycles. And the accumulated cost over the cycle is the expected cost. The process is called renewal reward process (Ross, 1993). In this article, the renewal approach is used to derive the expected cost per unit time.

Let ET be the expected cycle time, and let EC be the expected cost per cycle. And the optimal design parameters can be determined by minimizing the expected cost per unit time EA = EC / ET.

The production cycle consists of three periods: (1) in-control period $E(T_{s.c.}) = \dfrac{1}{\lambda}$, (2) time to signal and out-of-control period $h\left(\dfrac{1}{1-\beta} - \dfrac{1}{2} + \dfrac{\lambda h}{12}\right)$, and (3) time of searching and repairing assignable cause $T_{s.r.}$. Therefore, the expected cycle time is:

$$ET = \frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \tau\right) + T_{s.r.} \tag{2.1}$$

where the expected time of occurrence of the shift between j and j+1 sample is:

$$\tau = E(T_{s.c.} \mid T_{s.c.} < h) = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \cong \frac{h}{2} - \frac{\lambda h^2}{12} \tag{2.2}$$

(Duncan, 1956)

The expected quality loss per unit product can be easily estimated by the process variance and the deviation of process mean from the target, since we assume loss function (Figure 2.1) is

$$L = k(X - T)^2 \tag{2.3}$$

where k is the coefficient of the loss function and T is the target value.

That is, the expected cost per unit for in-control process is

$$L_I = k\left[\sigma^2 + (\mu - T)^2\right] = k\sigma^2\left[1 + \delta^2\right] \tag{2.4}$$

where $\mu - T = \delta\sigma$ is the deviation of a process mean from a target .

The expected quality cost per unit of product during the out-of-control period is

$$L_O = k\left[\delta_2^2\sigma^2 + (\mu + \delta_1\sigma - T)^2\right] = k\sigma^2\left[\delta_2^2 + (\delta + \delta_1)^2\right] \tag{2.5}$$

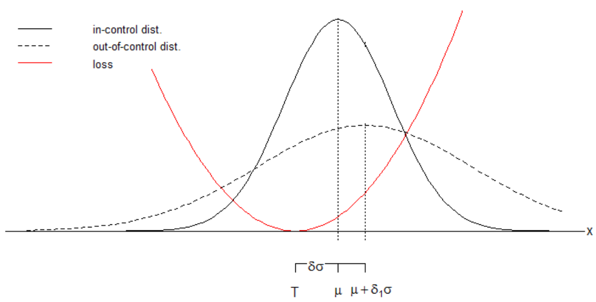

Figure 2.3. Loss Function, In-control and Out-of-control Distributions

The process costs, such as sampling and testing costs, false alarm costs, and assignable cause repair costs, are incurred in the cost model. We denote the costs for a cost model as follows:

a = fixed cost of sampling and testing.

b = cost per unit of sampling and testing.

C_f = the cost of investigating a false alarm.

C_{sr} = the cost of finding and repairing an assignable cause.

The expected cost per cycle is the sum of (1) the total expected quality cost for the in-control period $RL_I \frac{1}{\lambda}$, (2) the total expected quality cost for the out-of-control period $RL_O\left(\frac{h}{1-\beta}-\frac{h}{2}+\frac{\lambda h^2}{12}\right)$, (3) the total cycle cost of sampling and testing $(\mathrm{a}+\mathrm{bn})\left(\frac{1}{\lambda \mathrm{h}}+\frac{1}{1-\beta}\right)$, (4) the expected cost of false alarms during the cycle $C_f \frac{\alpha}{\lambda h}$, and (5) C_{sr}.

That is,

$$EC = RL_I \frac{1}{\lambda} + RL_O\left(\frac{h}{1-\beta}-\frac{h}{2}+\frac{\lambda h^2}{12}\right) + (\mathrm{a}+\mathrm{bn})\left(\frac{1}{\lambda \mathrm{h}}+\frac{1}{1-\beta}\right) + C_f \frac{\alpha}{\lambda \mathrm{h}} + C_{sr} \quad (2.6)$$

where R = expected output per unit time.

Hence, the expected cost per unit time is

$$EA = \frac{EC}{ET} = \frac{RL_I \frac{1}{\lambda} + RL_O\left(\frac{h}{1-\beta}-\frac{h}{2}+\frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h}+\frac{1}{1-\beta}\right) + C_f \frac{\alpha}{\lambda h} + C_{sr}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right) + T_{s.r.}} \quad (2.7)$$

The economic statistical design parameters can be determined by minimizing the cost function (2.7). A subroutine "DEoptim" in R program (Ardia et al., 2011), which is an optimization method based on a differential evolution algorithm, is used to solve the object.

The upper bound of α is set to α_U, and the upper bound of β is set to β_U. The lower and upper bounds of n, h, k_1, k_2, and k_3 are set to n_U, h_U, k_{1U}, and k_{2U}. The upper bound of k_2 is determined by the same cumulative probability of k_1. Therefore, the optimization model is expressed as

$$\begin{aligned} &\min\ \mathrm{EA}(n, h, k_1, k_2, k_3) \\ &\text{s.t.}\ \ n_L \le n \le n_U, \\ &\qquad 0 < h \le h_U, \\ &\qquad 0 < k_1 \le k_{1U}, \\ &\qquad 0 < k_3 < k_2 \le k_{2U}, \\ &\qquad \alpha \le \alpha_U, \\ &\qquad \beta \le \beta_U. \end{aligned}$$

2.2 An Example and Numerical Analysis

2.2.1 Example

In this section, we give an example to show the application of the economic statistical $\overline{X}$ and S control charts without tolerance. We compare the optimal solutions and the expected costs of three types of $\overline{X}$ and S control charts: (1) Shewhart-type economic $\overline{X}$ and S control charts with design *h*, (2) economic statistical $\overline{X}$ and S control charts with a given *n*, and (3) economic statistical $\overline{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

This study uses data from Montgomery (2009). The data were the standardized filling heights of soft drinks. Data were obtained from 15 subgroups of size 10 (= *n*), with a process mean of 0, a variance of 1, and a target value of 0. Other input parameters of the cost function were $\delta_1 = 1.5$, $\delta_2 = 2$, k = 4, R = 30, $\lambda = 0.01$, $T_{sr} = 3$, a = 0.5, b = 0.1, $C_{sr} = 35$, and $C_f = 50$.

(1) Shewhart-type economic $\overline{X}$ and S control charts with design *h*

To construct the Shewhart-type economic $\overline{X}$ and S charts when $n = 10$ and $\alpha = 0.00539$ ($\alpha_{\overline{X}} = \alpha_S = 0.0027$), we calculated that $k_1 = 3$, $k_2 = 1.735$, $k_3 = 0.371$, and $\beta = 0.06502$. The expected cost per unit time of the optimal Shewhart-type economic $\overline{X}$ and S charts is

$$\begin{aligned} &\min EA(h) \\ &\text{s.t. } 0 < h \le 8 \cdot \end{aligned}$$

The EA^* is 110.903, and h^* is 8. The optimal Shewhart-type economic $\overline{X}$ and S charts are constructed as follows.

$$\begin{aligned} UCL_{\overline{X}} &= 3 \\ LCL_{\overline{X}} &= -3 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 1.735 \\ LCL_S &= 0.371 \end{aligned}$$

Plotting the data in Shewhart-type economic $\overline{X}$ and S control charts shows whether they are in-control. Figure 2.2 shows that no points fall outside the limits of Shewhart-type economic $\overline{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process.

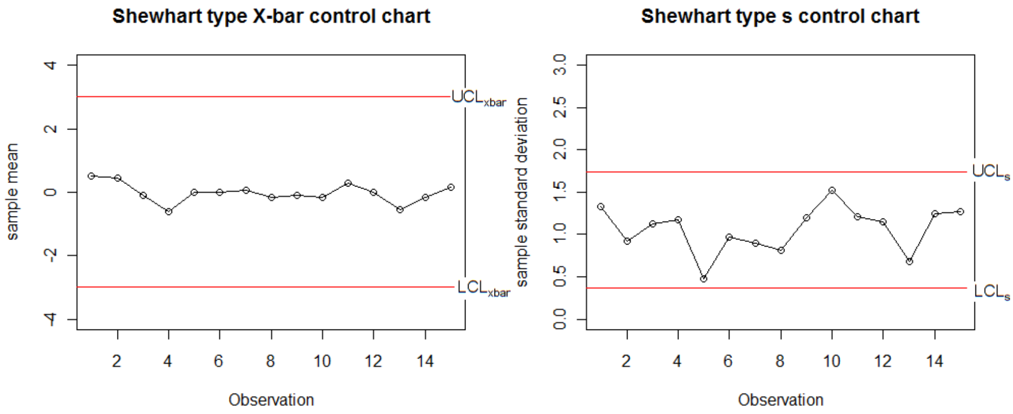

Figure 4.2. Shewhart-type Economic $\bar{X}$ and S Control Charts

(2) Economic statistical $\bar{X}$ and S control charts with a given *n*

The design parameters are determined with a given *n* by minimizing the cost function to construct the economic statistical $\bar{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$\begin{aligned} &\min \mathrm{EA}(h, k_1, k_2, k_3) \\ &\text{s.t. } 0 < \mathrm{h} \le 8, \\ &\quad 0 < \mathrm{k}_1 \le 4, \\ &\quad 0 < \mathrm{k}_3 < \mathrm{k}_2 \le 4.2, \\ &\quad \alpha \le 0.01, \\ &\quad \beta \le 0.2. \end{aligned}$$

The optimal design parameters are $h^* = 8$, $k_1{}^* = 3.581$, $k_2{}^* = 2.194$, $k_3{}^* = 0.017$, $\alpha^* = 0.00013$, and $\beta^* = 0.2$. The EA^* is 109.572. The optimal economic statistical $\bar{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\bar{X}} = 3.581 \\ LCL_{\bar{X}} = -3.581 \end{matrix} \quad \text{and} \quad \begin{matrix} UCL_S = 2.194 \\ LCL_S = 0.0017 \end{matrix}$$

Figure 2.3 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts..

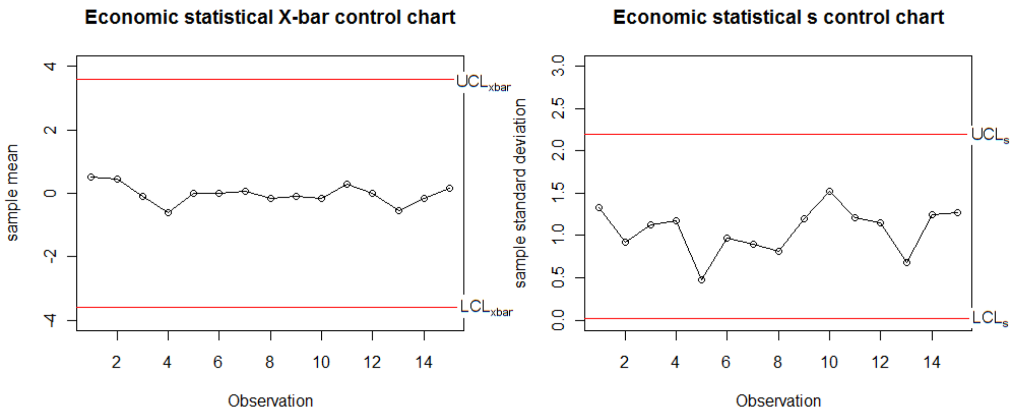

Figure 2.3. Optimal Economic Statistical $\overline{X}$ and S Control Charts and with a Given *n*

(3) Economic statistical $\overline{X}$ and S control charts with all determined design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimal economic statistical $\overline{X}$ and S charts is

$$\begin{aligned} &\min \mathrm{EA}(n, h, k_1, k_2, k_3) \\ &\text{s.t. } 2 \le n \le 25, \\ &\quad 0 < h \le 8, \\ &\quad 0 < k_1 \le 4, \\ &\quad 0 < k_3 < k_2 \le 4.2, \\ &\quad \alpha \le 0.01, \\ &\quad \beta \le 0.2. \end{aligned}$$

The parameters are $n^* = 7$, $h^* = 8$, $k_1^* = 3.300$, $k_2^* = 1.949$, $k_3^* = 0.0003$, $\alpha^* = 0.00184$, $\beta^* = 0.2$, and the EA^* is 109.556. The optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$\begin{aligned} UCL_{\overline{X}} &= 3.300 \\ LCL_{\overline{X}} &= -3.300 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 1.949 \\ LCL_S &= 0.0003 \end{aligned}$$

Finally, we compare “Shewhart-type economic charts” and “economic statistical charts with a given *n*” leads to finding (i) and (ii). Comparing economic statistical chart with all design parameters and with a given *n* leads to finding (iii) (See Table 2.1):

(i) If producer can design the chart, k_1^* and k_2^* should increase, k_3^* should decrease and EA^* will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart without design n, EA* could save about 1.2%. And the false alarm rate of economic statistical chart without design n will decrease, but the true alarm rate will decrease.

(iii) If producer can decide all design parameter of control chart, *n** should decrease from 10 to 7, k_1* and k_2* should be decrease and *EA** will reduce.

Table 2.1. Comparison of Three Types Design Charts under the Model without Tolerance

	n	h^*	k_1	k_2	k_3	α	β	EA^*
(1) Shewhart-type economic $\overline{X}$ and S control charts	10	8	3	1.735	0.371	0.00539	0.06502	110.903
	n	h^*	k_1^*	k_2^*	k_3^*	α^*	β^*	EA^*
(2) Economic statistical $\overline{X}$ and S control charts with a given n	10	8	3.581	2.194	0.017	0.00034	0.20000	109.572
	n^*	h^*	k_1^*	k_2^*	k_3^*	α^*	β^*	EA^*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	7	8	3.300	1.949	0.000	0.00184	0.20000	109.556

2.2.2 The Effects of Optimal Design Parameters under Different Combinations of δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Input parameters are from Montgomery (1985), and other input parameters of the cost function are $T = 0$, $\delta_1 = 1.5$, $\delta_2 = 2$, $k = 4$, $R = 30$, $\lambda = 0.01$, $T_{s.r.} = 3$, $a = 0.5$, $b = 0.1$, $C_{sr} = 35$, and $C_f = 50$.

The results of these objects are shown in Table 2.2. Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 75%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 75%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be a little larger, and *EA** will reduce about 47%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be a little larger, and *EA** will reduce about 47%.

Comparing economic statistical control charts with Shewhart-type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart-type economic $\overline{X}$ and S charts'.

(vi) The false alarm rate of Economic Statistic $\overline{X}$ and S chart is smaller, but its true alarm rate is smaller, too.

According to the findings (i)-(iv), if a producer can only improve mean or variance, it should decrease variance first because this can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\overline{X}$ and S charts. If a producer improves the deviation of the mean and target, the producer should increase the width of the $\overline{X}$ and S charts. According to findings (v)-(vi), The expected costs of the two charts are similar.

Producers are advised to use the Shewhart-type economic $\overline{X}$ or S charts depending on the convenience of using the chart.

Table 2.2. The Optimal Solution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-Type Economic $\overline{X}$ and S Charts" without Tolerance

Economic Statistical $\overline{X}$ and S Control Charts									
	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1^*)	$LCL^*_{\overline{X}}$ (k_1^*)	UCL^*_S (k_2^*)	LCL^*_S (k_3^*)	α^*	β^*	EA^*
(1) δ=0 and σ=2	7	8	6.572 (3.286)	-6.572 (3.286)	3.91 (1.955)	0.004 (0.002)	0.00184	0.20000	437.239
(2) δ=0 and σ=1	7	8	3.3 (3.3)	-3.3 (3.3)	1.949 (1.949)	0 (0)	0.00184	0.20000	109.556
(3) δ=1 and σ=2	7	8	7.59 (3.295)	-5.59 (3.295)	3.902 (1.951)	0.002 (0.001)	0.00184	0.20000	830.977
(4) δ=1 and σ=1	7	8	4.296 (3.296)	-2.296 (3.296)	1.951 (1.951)	0 (0)	0.00184	0.20000	207.990
Shewhart-Type Economic $\overline{X}$ and S Control Charts									
	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA^*
(1) δ=0 and σ=2	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	438.851
(2) δ=0 and σ=1	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	109.977
(3) δ=1 and σ=2	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	836.533
(4) δ=1 and σ=1	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	209.397

2.2.3 Determining Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(n, h, k_1, k_2, k_3, \delta) \\
&s.t.\ 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (δ_1, δ_2, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (1.5, 2, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(n, h, k_1, k_2, k_3, \delta, \sigma) \\
&s.t.\ 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad 0.5 \le \sigma \le 4, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine “DEoptim” in R program.

Table 2.3 shows the optimal solutions of 2 situations and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 2.2, when μ is unknown, n^* increases, h^* decreases, the width of the $\overline{X}$ chart decreases, and the width of the S chart increases, but EA^* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 2.2, when μ and σ are unknown, n^* increases, h^* decreases, the width of the $\overline{X}$ chart decreases, the width of the S chart increases, and EA^* is smaller.

Table 2.3. The Optimal Solution and In-control Distribution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-type economic $\overline{X}$ and S Charts" without Tolerance

Economic Statistical $\overline{X}$ and S Control Charts											
situation	δ*	σ*	n^*	h^*	$UCL_{\overline{X}}^*$ (k_1^*)	$LCL_{\overline{X}}^*$ (k_1^*)	UCL_S^* (k_2^*)	LCL_S^* (k_3^*)	α^*	β^*	EA^*
(1) σ is known, δ is unknown (σ=2)	4.778E -16	--	9	0.311	6.502 (3.251)	-6.502 (3.251)	3.59 (1.795)	0.006 (0.003)	0.00230	0.10701	475.800
(2) δ and σ are unknown	4.942E -16	0.500	9	1.250	1.6265 (3.253)	-1.6265 (3.253)	0.8975 (1.795)	0.002 (0.004)	0.00228	0.10729	31.801
Shewhart-Type economic $\overline{X}$ and S Control Charts											
situation	δ*	σ*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA^*
(1) σ is known, δ is unknown (σ=2)	1.394E -15	--	9	0.333	6 (3)	-6 (3)	3.56 (1.780)	0.682 (0.341)	0.00539	0.08853	476.087
(2) δ and σ are unknown	1.512E -16	0.500	9	1.338	1.5 (3)	-1.5 (3)	0.89 (1.780)	0.1705 (0.341)	0.00539	0.08853	31.872

2.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 11 costs and process parameters. This section uses an orthogonal array to study the effects and sensitivities of the input parameters (δ, σ, δ_1, δ_2, R, λ, T_{sr}, a, b, C_{sr}, and C_f) on the design parameters and expected cost per unit time. Table 2.4 shows three levels of each parameter. Because the cost of false alarms and cost of searching and repairing an assignable cause are correlated, these two parameters' levels are set in combination. Other parameters are assumed to be independent.

Table 2.4. Input Parameters' Levels Used in the Orthogonal Array

	Level		
Input parameter	1	2	3
δ	0	1	2
σ	1	2	2.5
δ_1	1	1.5	2.5
δ_2	1	1.5	2
R	30	100	500
λ	0.01	0.05	0.1
T_{sr}	3	2	1
a	0.5	50	100
b	0.1	1	5
(C_{sr}, C_f)	(35,50)	(50,25)	(100,40)

An orthogonal array table $L_{27}(3^{13})$ is used for sensitivity analysis. Table 2.5 shows the optimal solutions for these 27 combinations of input parameters.

Table 2.5. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model without Tolerance

No	δ	σ	δ_1	δ_2	R	λ	Tsr	a	b	Csr	Cf	n*	h*	k1*	k2*	k3*	EA*
1	0	1	1	2	500	0.05	2	50	0.1	100	40	24	0.51	3.11	1.39	0.07	2007.86
2	0	1	1	1.5	100	0.1	1	100	0.1	50	25	25	1.51	2.74	1.39	0.11	480.74
3	0	2	2.5	1.5	500	0.1	3	50	1	50	25	6	0.13	2.59	2.10	0.00	6797.82
4	2	2	1	2	100	0.1	2	0.5	5	30	50	8	0.20	2.93	1.67	0.00	6998.05
5	0	2.5	1.5	1	500	0.01	1	50	5	30	50	8	0.77	2.58	3.87	0.00	12609.61
6	0	2	2.5	1	100	0.01	2	100	1	30	50	5	1.45	2.66	4.20	0.00	1709.85
7	1	1	2.5	2	100	0.01	1	50	5	50	25	5	1.56	2.69	2.01	0.00	887.00
8	1	1	2.5	1	500	0.1	2	0.5	5	100	40	3	0.11	2.58	4.20	0.00	3574.64
9	1	2.5	1	2	30	0.01	2	100	1	50	25	17	2.25	2.98	1.44	0.00	1571.71
10	0	2	2.5	2	30	0.05	1	0.5	1	100	40	3	0.16	2.68	2.47	0.00	508.05
11	1	2	1.5	1	30	0.1	1	100	0.1	100	40	16	0.91	2.86	3.22	0.05	1075.38
12	0	1	1	1	30	0.01	3	0.5	0.1	30	50	17	1.84	2.79	3.76	0.06	119.37
13	0	2.5	1.5	2	100	0.05	3	100	5	100	40	10	0.65	2.83	1.61	0.00	2574.03
14	2	2.5	2.5	2	500	0.1	1	100	0.1	30	50	12	0.09	3.17	1.71	0.01	58837.58
15	1	2	1.5	1.5	100	0.05	2	50	0.1	30	50	21	0.44	2.95	1.52	0.05	3121.83
16	2	2.5	2.5	1.5	100	0.01	3	0.5	0.1	100	40	5	0.07	3.25	2.58	0.00	12168.15
17	0	2.5	1.5	1.5	30	0.1	2	0.5	5	50	25	7	0.44	2.64	1.88	0.00	759.81
18	2	1	1.5	1	100	0.05	1	0.5	1	50	25	6	0.25	2.58	4.18	0.00	1958.69
19	2	1	1.5	2	30	0.1	3	50	1	30	50	12	0.92	2.84	1.55	0.01	563.11
20	1	2	1.5	2	500	0.01	3	0.5	0.1	50	25	8	0.06	3.04	1.78	0.00	15581.57
21	2	1	1.5	1.5	500	0.01	2	100	1	100	40	15	1.09	2.65	1.56	0.01	10011.49
22	1	1	2.5	1.5	30	0.05	3	100	5	30	50	5	1.82	2.59	2.24	0.00	323.74
23	2	2	1	1	500	0.05	3	100	5	50	25	17	0.38	2.58	3.69	0.04	35615.47
24	1	2.5	1	1.5	500	0.05	1	0.5	1	30	50	13	0.09	2.73	1.56	0.01	24123.07
25	1	2.5	1	1	100	0.1	3	50	1	100	40	20	0.40	2.58	2.82	0.06	4121.12
26	2	2	1	1.5	30	0.01	1	50	5	100	40	16	2.67	2.73	1.50	0.06	2472.47
27	2	2.5	2.5	1	30	0.05	2	50	0.1	50	25	6	0.40	2.95	4.15	0.00	3636.60

Table 2.6 shows the main effects of the optimal solutions and optimal values for three input parameter levels and it shows the following findings:

(i) δ_1 is significant to average sample size *n**. When δ_1 increases, average *n** decreases.

(ii) R, λ and a are significant to average sampling interval *h**. When R, λ, or a increase, average *h** decreases.

(iii) δ_2 is significant to average k_1* and k_2*. When δ_2 increases, optimum average k_1* increases and average k_2* decreases.

(iv) All input parameters are significant to average *EA**. When δ, σ, R, or λ increase, average *EA** increases. When b increases, average *EA** decreases. When δ_1, δ_2, T_{sr}, or a increase, average *EA* *decreases first and then increases.

Table 2.6. Main Effect of the Optimal Solutions and Optimal Values under the Cost Model without Tolerance

	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{n}$	1	11.67	12.44	17.44	10.89	11.00	10.67	11.11	7.78	14.89	11.22
	2	12.00	11.11	11.44	12.56	11.67	11.67	11.78	13.11	10.78	10.78
	3	10.78	10.89	5.56	11.00	11.78	12.11	11.56	13.56	8.78	12.44
	diff	1.22	1.56	**11.89**	1.67	0.78	1.44	0.67	**5.78**	**6.11**	1.67
$\bar{h}$	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.83	1.07	1.09	0.72	1.27	1.31	0.70	0.36	0.65	0.85
	2	0.85	0.71	0.61	0.92	0.73	0.52	0.77	0.87	0.75	0.78
	3	0.67	0.57	0.64	0.71	0.36	0.52	0.89	1.13	0.96	0.73
	diff	0.17	0.49	0.48	0.21	**0.91**	**0.78**	0.19	**0.77**	0.31	0.12
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	2.74	2.73	2.80	2.68	2.78	2.82	2.79	2.80	2.99	2.80
	2	2.78	2.78	2.77	2.76	2.80	2.78	2.83	2.78	2.70	2.75
	3	2.85	2.85	2.80	2.92	2.78	2.77	2.75	2.78	2.68	2.81
	diff	0.12	0.12	0.02	**0.24**	0.02	0.05	0.08	0.02	**0.30**	0.05
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	2.52	2.48	2.14	3.79	2.47	2.52	2.46	2.68	2.39	2.45
	2	2.31	2.46	2.35	1.81	2.44	2.54	2.45	2.32	2.43	2.51
	3	2.51	2.40	2.85	1.74	2.43	2.28	2.43	2.34	2.52	2.37
	diff	0.21	0.07	0.71	**2.05**	0.04	0.25	0.03	0.35	0.13	0.14

	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{k}_3$	1	0.03	0.03	0.05	0.02	0.02	0.01	0.02	0.01	0.04	0.01
	2	0.02	0.02	0.01	0.03	0.02	0.02	0.02	0.03	0.01	0.02
	3	0.01	0.01	0.00	0.01	0.02	0.03	0.03	0.02	0.01	0.03
	diff	0.01	0.02	0.04	0.02	0.01	0.01	0.01	0.02	0.03	0.01
	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{\alpha}$	1	0.0088	0.0091	0.0091	0.0078	0.0080	0.0076	0.0079	0.0079	0.0046	0.0077
	2	0.0081	0.0078	0.0081	0.0083	0.0081	0.0081	0.0079	0.0082	0.0097	0.0087
	3	0.0074	0.0074	0.0071	0.0081	0.0082	0.0085	0.0085	0.0083	0.0100	0.0079
	diff	0.0014	0.0017	0.0020	0.0005	0.0001	0.0009	0.0006	0.0004	0.0054	0.0010
	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{\beta}$	1	0.0596	0.0434	0.0754	0.0454	0.0645	0.0546	0.0492	0.1224	0.0325	0.0591
	2	0.0491	0.0693	0.0590	0.0632	0.0561	0.0669	0.0449	0.0295	0.0622	0.0622
	3	0.0629	0.0589	0.0372	0.0630	0.0510	0.0500	0.0774	0.0198	0.0769	0.0503
	diff	0.0139	0.0259	0.0382	0.0178	0.0134	0.0169	0.0325	0.1026	0.0445	0.0119
	Level	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{EA}$	1	3063.02	2214.07	8612.21	7157.86	1225.58	6347.91	8651.60	7310.16	10781.01	12045.13
	2	6042.23	8208.94	5361.72	6695.46	3779.94	8207.70	3710.20	4024.16	5707.21	7476.60
	3	14695.73	13377.97	9827.05	9947.66	18795.46	9245.36	11439.18	12466.67	7312.76	4279.24
	diff	**11632.72**	**11163.89**	**4465.33**	**3252.20**	**17569.87**	**2897.45**	**7728.97**	**8442.51**	**5073.80**	**7765.89**

In Table 2.6, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation "+", if the input parameter is significant and their relationship is linear and negative, we use notation "-", and if the input parameter is significant and their relationship is quadratic, we use notation "q"; otherwise, we use notation "N".

Table 2.7. The Significant Input Parameters of Each Design Parameter and EA

Optimal Design parameters and EA	Input parameters									
	δ	σ	δ_1	δ_2	R	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{n}$	N	N	-	N	N	N	N	+	-	N
$\vec{h}$	N	N	N	N	-	-	N	+	N	N
$\bar{k}_1$	N	N	N	+	N	N	N	N	-	N
$\bar{k}_2$	N	N	N	-	N	N	N	N	N	N
$\bar{k}_3$	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	q	q	-	-

3. DESIGN OF CONSUMER TOLERANCE AND ECONOMIC STATISTICAL $\bar{\mathbf{X}}$ AND S CHARTS

3.1 Derivation of Cost Models

The assumptions of process distributions and economic statistical $\bar{X}$ and S charts continue to hold in this section. A consumer tolerance d_c is assumed to exist such that if $|X-T|> d_c$, the product is nonconforming for consumers. T+ d_c and T- d_c are the consumer specification limits (Fathi, 1990).

Let L_c denote the consumer quadratic loss function (Figure 3.1).

$$L_c = \begin{cases} k_c (X-T)^2 & if\ |X-T| \le d_c \\ A_c & if\ |X-T| > d_c \end{cases} \tag{3.1}$$

where k_c is the coefficient of the consumer loss function for a conforming product, and A_c is the cost of correcting a nonconforming product after its shipment to the consumer.

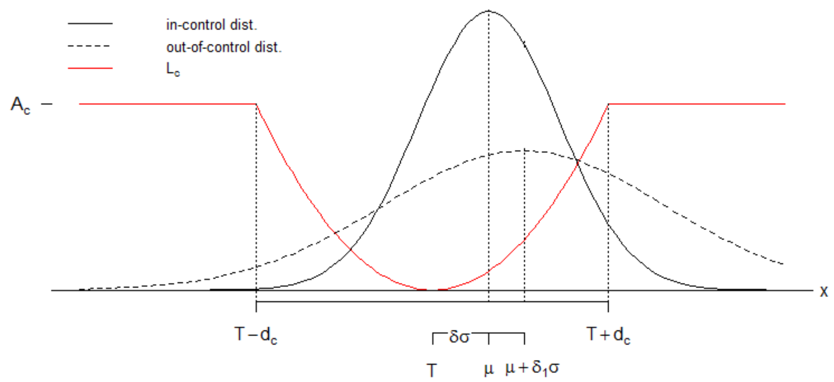

Figure 3.1. Consumer Loss Function, In-control and Out-of-control Distributions

The expected cost per unit while the process is in-control is shown in (3.2), and the expected per unit cost while the process is out-of-control is shown in (3.3):

$$\begin{aligned} L_I &= A_c\left[1 - P(T-d_c \le X \le T-d_c)\right] + k_c \int_{T-d_c}^{T-d_c} k_c (x-T)^2 f_X(x)dx \\ &= A_c\left[\int_{-\infty}^{-\delta-\frac{d_c}{\sigma}} \phi(z)dz + \int_{-\delta+\frac{d_c}{\sigma}}^{\infty} \phi(z)dz\right] + k_c\sigma^2 \int_{-\delta-\frac{d_c}{\sigma}}^{-\delta+\frac{d_c}{\sigma}} (z+\delta)^2 \phi(z)dz \end{aligned} \tag{3.2}$$

$$
\begin{aligned}
L_O &= A_c\left[1-P(T-d_c \le X \le T+d_c)\right]+\int_{T-d_c}^{T-d_c} k_c(x-\mu)^2 f_X(x)dx \\
&= A_c\left[\int_{-\infty}^{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_c}{\sigma}\right)}\phi(z)dz+\int_{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_c}{\sigma}\right)}^{\infty}\phi(z)dz\right] \\
&\quad + k_c\sigma^2\int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_c}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_c}{\sigma}\right)}(\delta_2 z+\delta+\delta_1)^2\phi(z)dz
\end{aligned}
\tag{3.3}
$$

Hence, the expected cost per unit time is

$$
EA=\frac{R\dfrac{L_I}{\lambda}+RL_O\left(\dfrac{h}{1-\beta}-\dfrac{h}{2}+\dfrac{\lambda h^2}{12}\right)+(\mathrm{a}+\mathrm{bn})\left(\dfrac{1}{\lambda \mathrm{h}}+\dfrac{1}{1-\beta}\right)+C_f\,\dfrac{\alpha}{\lambda \mathrm{h}}+C_{sr}}{\dfrac{1}{\lambda}+h\left(\dfrac{1}{1-\beta}-\dfrac{1}{2}+\dfrac{\lambda h}{12}\right)+T_{sr}}
\tag{3.4}
$$

The design parameters can be determined by minimizing the cost function (3.4). A subroutine "DEoptim" in R program is used to solve the object. The upper bound of α is set to α_U, and the upper bound of β is set to β_U. The upper bounds of d_c, n, h, k_1, k_2, and k_3 are set to d_{cU}, n_U, h_U, k_{1U}, and k_2. The lower bound of d_c and n are d_{cL}, n_L . Let the rate of nonconforming products for consumers be smaller than 0.1, the bounds of d_c can be determined. Therefore, the optimization model is expressed as follows:

$$
\begin{aligned}
&\min \mathrm{EA}\left(d_c, n, h, k_1, k_2, k_3\right) \\
&\text{s.t.}\ \ d_{cL} \le d_c \le d_{cU}, \\
&\qquad n_L \le n \le n_U, \\
&\qquad 0 < h \le h_U, \\
&\qquad 0 < k_1 \le k_{1U}, \\
&\qquad 0 < k_3 < k_2 \le k_{2\mathrm{U}}, \\
&\qquad \alpha \le \alpha_U, \\
&\qquad \beta \le \beta_U.
\end{aligned}
$$

3.2 An Example and Numerical Analysis

3.2.1 Example

In this section, we give an example to show the application of the economic statistical $\overline{X}$ and S control chart only with consumer tolerance. We compare the optimal solutions and the expected costs per unit time of three types of $\overline{X}$ and S control charts: (1) Shewhart-type economic $\overline{X}$ and S control charts with design h and d_c, (2) economic statistical $\overline{X}$ and S control charts with a given n, and (3) economic statistical $\overline{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_c=100, R=30, λ=0.01, T_{sr}=3, a=0.5, b=0.1, C_{sr}=35, and C_f=50.

(1) Shewhart-type economic $\overline{X}$ and S control charts with design h and d_c

To construct the Shewhart-type economic $\overline{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\overline{X}} = \alpha_S$ = 0.0027), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time of the optimum Shewhart-type economic $\overline{X}$ and S charts is

$$\begin{aligned} &\min EA(d_c, h) \\ &\text{s.t.}\ 0.8 < d_c \le 25, \\ &\quad 0 < h \le 8. \end{aligned}$$

The EA* is 4.817, h^* is 8, and d_c^* is 25. Under the d_c^*, the calculated rate of nonconforming product is 0, and the optimal Shewhart-type economic $\overline{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\overline{X}} = 3 & \text{and} & UCL_S = 1.735 \\ LCL_{\overline{X}} = -3 & & LCL_S = 0.371 \end{matrix}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 3.2 shows that no points fall outside the limits of Shewhart-type $\overline{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 3.3 shows that all products fall into the consumer specification.

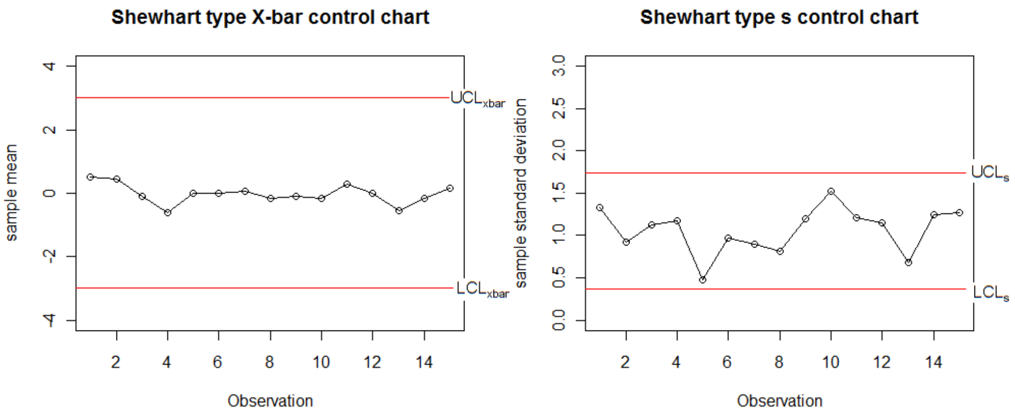

Figure 3.2. Shewhart type Economic $\overline{X}$ and S Control Charts with Consumer Tolerance

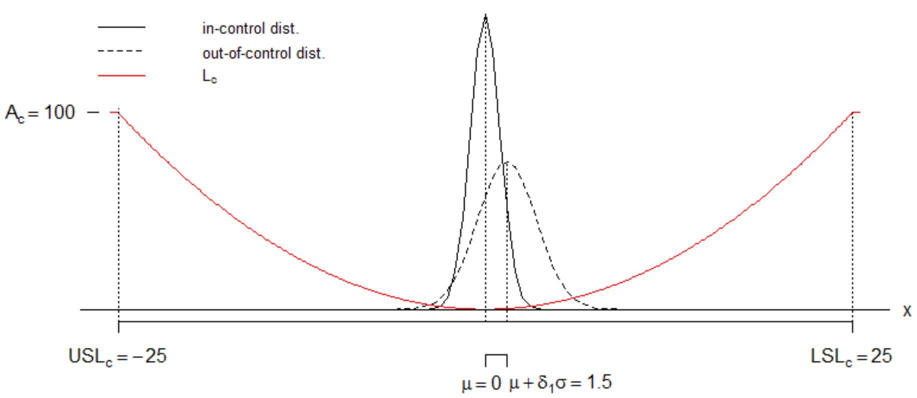

Figure 3.3. Optimal Consumer Loss Function with In-control and Out-of-control Distributions

(2) Economic statistical $\overline{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\overline{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\overline{X}$ and S charts is

$$
\begin{aligned}
&\min \mathrm{EA}(d_c, h, k_1, k_2, k_3)\\
&\text{s.t. } 0.8 < d_c \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}
$$

The optimal design parameters are $d_c^* = 25$, $h^* = 8$, $k_1^* = 4$, $k_2^* = 2.001$, $k_3^* = 0.005$, $\alpha^* = 0.00001$, and $\beta^* = 0.2$. The EA^* is 4.712. The optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 4 \quad \text{and} \quad UCL_S = 2.001$$
$$LCL_{\bar{X}} = -4 \qquad\qquad LCL_S = 0.005 \, .$$

Figure 3.4 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts. Because the optimal consumer tolerance is the same as the previous, all in-control and out-of-control products are in the consumer specification limits.

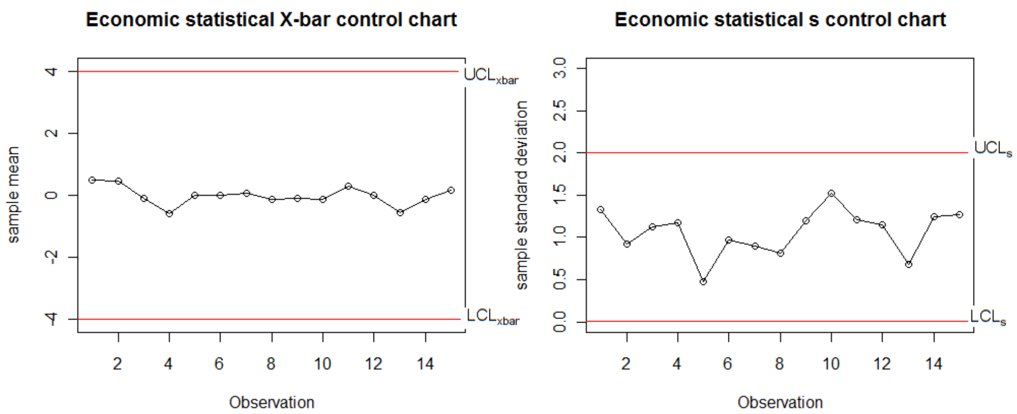

Figure 3.4. Optimal Economic Statistical $\bar{\mathrm{X}}$ and S Control Charts with Consumer Tolerance and with a Given *n*

(3) Economic statistical $\bar{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$\begin{aligned}
&\min \mathrm{EA}(d_c, n, h, k_1, k_2, k_3) \\
&\text{s.t. } 0.8 < d_c \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

The parameters are d_c* = 25, n* = 7, h* = 8, k_1* = 3.300, k_2* = 1.949, k_3* = 0.001, α* = 0.00186, and β* = 0.2. The *EA** is 4.697. Under the d_c*, the rate of nonconforming product is 0, and the optimal economic statistical $\bar{X}$ and S charts are

$$UCL_{\bar{X}} = 3.300 \quad \text{and} \quad UCL_S = 1.949$$
$$LCL_{\bar{X}} = -3.300 \qquad\qquad LCL_S = 0.001$$

Finally, we compare the optimal solutions and the *EA** of these three types design charts (See Table 3.1).

Comparing with "Shewhart-type" and "economic statistical chart with a given *n*" leads to following findings:

(i) If producer can design the chart, k_1* and k_2* should increase, k_3* should decrease and *EA** will reduce.

(ii) Using economic statistical $\overline{X}$ and S charts with given *n*, EA* could save about 2.1%. And the false alarm rate of economic statistical chart with given *n* will decrease, but the true alarm rate will decrease.

(iii) The optimal consumer tolerance all equals to 25.

Comparing economic statistical chart with design n and with given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, *n** should decrease, k_1* and k_2* should be decrease and *EA** will reduce.

Because the *EA** cannot be saved a lot when we use economic statistical control chart with consumer tolerance, we advise that it is more convenience for using Shewhart-type economic control chart and let consumer tolerance equal to 25.

Table 3.1. Comparison of Three Types Design Charts under the Model with Consumer Tolerance

	d_c*	n	h*	k_1	k_2	k_3	α	β	*EA**
(1) Shewhart-type economic $\overline{X}$ and S control charts	25	10	8	3.000	1.735	0.371	0.00539	0.06502	4.817
	d_c*	n	h*	k_1*	k_2*	k_3*	α*	β*	*EA**
(2) Economic statistical $\overline{X}$ and S control charts with a given *n*	25	10	8	4.000	2.001	0.005	0.00010	0.20000	4.712
	d_c*	n*	h*	k_1*	k_2*	k_3*	α*	β*	*EA**
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	25	7	8	3.300	1.949	0.001	0.00184	0.20000	4.697

3.2.2 The Effects of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are A_c = 100, T = 0, δ_1 = 1.5, δ_2 = 2, k = 4, R = 30, λ = 0.01, T_{sr} = 3, a = 0.5, b = 0.1, C_{sr} = 35, and C_f = 50.

The results of these objects are shown in Table 3.2.

Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_c*, n* and h* will not change, the width of $\overline{X}$ and S charts will be smaller, and EA* will reduce about 74%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_c*, n* and h* will not change, the width of $\overline{X}$ and S charts will be smaller, and EA* will reduce about 74%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_c*, n*, h* and the width of $\overline{X}$ and S charts will not change, EA* will reduce about 46%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_c*, n*, h* and the width of $\overline{X}$ and S charts will not change, and EA* will reduce about 47%.

Comparing with economic statistical control charts and Shewhart-type economic control charts base on same combination of process mean and variance leads to the following findings

(v) EA* of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart-type economic $\overline{X}$ and s charts'.

(vi) The α* of Economic Statistic $\overline{X}$ and s chart is smaller, but its β* is smaller, too.

According to the findings (i)-(iv), decreasing variance can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\overline{X}$ and S charts. In all situations, optimal consumer tolerance equals to 25 and all products are in the consumer specification limits. According to findings (v)-(vi), the expected costs of the two charts are similar. Producers are advised to use the Shewhart-type economic $\overline{X}$ or S charts depending on the convenience of using the chart.

Table 3.2. The Optimal Solution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-type Economic $\overline{X}$ and S Chart" with Consumer Tolerance

Economic Statistical $\overline{X}$ and S Control Charts										
	d_c*	n*	h*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	25	7	8	6.6 (3.3)	-6.6 (3.3)	3.898 (1.949)	0.004 (0.002)	0.00184	0.2	17.804
(2) δ=0 and σ=1	25	7	8	3.3 (3.3)	-3.3 (3.3)	1.949 (1.949)	0.001 (0.001)	0.00184	0.2	4.697
(3) δ=1 and σ=2	25	7	8	7.6 (3.3)	-5.6 (3.3)	3.898 (1.949)	0.004 (0.002)	0.00184	0.2	33.554
(4) δ=1 and σ=1	25	7	8	4.3 (3.3)	-2.3 (3.3)	1.949 (1.949)	0.001 (0.001)	0.00184	0.2	8.634
Shewhart-Type Economic $\overline{X}$ and S Control Charts										
	d_c*	n*	h*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	17.892
(2) δ=0 and σ=1	25	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	4.737
(3) δ=1 and σ=2	25	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	33.799
(4) δ=1 and σ=1	25	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	8.714

3.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, R, A_c, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 30, 100, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \text{EA}(d_c, n, h, k_1, k_2, k_3, \delta)\\
&\text{s.t.}\, 0.8 < d_c \le 25\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (δ_1, δ_2, R, A_c, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (1.5, 2, 30, 100, 4, 0.01, 3, 0.5, 0.1, 35, 50)

$$\begin{aligned}
&\min \text{EA}(d_c, n, h, k_1, k_2, k_3, \delta, \sigma)\\
&\text{s.t.}\, 0.8 < d_c \le 25,\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad 0.5 \le \sigma \le 4,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 3.3 shows the optimal solutions of two situations and leads to the following findings:

(i) In situation (1), δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 3.2, when μ is unknown, d_c^* and the design parameters of charts are the same, but the *EA** is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 3.2, when μ and σ are unknown, the width of the $\bar{X}$ chart decreases, and the width of the S chart increases, and the *EA** is smaller.

Table 3.3. The Optimal Solutions and In-control Distribution of "Economic Statistic $\bar{X}$ and S Charts" and "Shewhart-type Economic $\bar{X}$ and S chart" with Consumer Tolerance

Economic Statistical $\bar{X}$ and S Control Charts												
situation	δ*	σ*	d_c^*	n^*	h^*	$UCL_{\bar{X}}^*$ (k_1^*)	$LCL_{\bar{X}}^*$ (k_1^*)	UCL_S^* (k_2^*)	LCL_S^* (k_3^*)	α^*	β^*	EA^*
(1) σ is known, δ is unknown (σ=2)	3.200E-16	--	25	7	8	6.6 (3.3)	-6.6 (3.3)	3.898 (1.949)	0.004 (0.002)	0.00230	0.2	17.804
(2) δ and σ are unknown	2.199E-15	0.5	25	7	8	1.65 (3.3)	-1.65 (3.3)	0.975 (1.949)	0.0005 (0.001)	0.00228	0.2	1.420
Shewhart-Type Economic $\bar{X}$ and S Control Charts												
situation	δ*	σ*	d_c^*	n^*	h^*	$UCL_{\bar{X}}$ (k_1)	$LCL_{\bar{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA^*
(1) σ is known, δ is unknown (σ=2)	1.604E-16	--	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	17.892
(2) δ and σ are unknown	6.029E-16	0.5	25	7	8	1.5 (3)	-1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1.448

3.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 12 cost and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), A_c = (100,200,300), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1), a = (0.5,50,100), b = (0.1,1,5), and ($C_{s.r.}$,C_f) = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using an orthogonal array table $L_{27}(3^{13})$ (Table 3.4).

Table 3.4. Orthogonal Array $L_{27}(3^{13})$ for 27 Combinations of Input Parameters

No	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	C_{sr}	C_f
1	0	1	1	2	500	300	0.05	2	50	0.1	100	40
2	0	1	1	1.5	100	200	0.1	1	100	0.1	50	25
3	0	2	2.5	1.5	500	100	0.1	3	50	1	50	25
4	2	2	1	2	100	100	0.1	2	0.5	5	30	50
5	0	2.5	1.5	1	500	200	0.01	1	50	5	30	50
6	0	2	2.5	1	100	300	0.01	2	100	1	30	50
7	1	1	2.5	2	100	100	0.01	1	50	5	50	25
8	1	1	2.5	1	500	200	0.1	2	0.5	5	100	40
9	1	2.5	1	2	30	200	0.01	2	100	1	50	25
10	0	2	2.5	2	30	200	0.05	1	0.5	1	100	40
11	1	2	1.5	1	30	100	0.1	1	100	0.1	100	40
12	0	1	1	1	30	100	0.01	3	0.5	0.1	30	50
13	0	2.5	1.5	2	100	100	0.05	3	100	5	100	40
14	2	2.5	2.5	2	500	300	0.1	1	100	0.1	30	50
15	1	2	1.5	1.5	100	200	0.05	2	50	0.1	30	50
16	2	2.5	2.5	1.5	100	200	0.01	3	0.5	0.1	100	40
17	0	2.5	1.5	1.5	30	300	0.1	2	0.5	5	50	25
18	2	1	1.5	1	100	300	0.05	1	0.5	1	50	25
19	2	1	1.5	2	30	200	0.1	3	50	1	30	50
20	1	2	1.5	2	500	300	0.01	3	0.5	0.1	50	25
21	2	1	1.5	1.5	500	100	0.01	2	100	1	100	40
22	1	1	2.5	1.5	30	300	0.05	3	100	5	30	50
23	2	2	1	1	500	200	0.05	3	100	5	50	25
24	1	2.5	1	1.5	500	100	0.05	1	0.5	1	30	50
25	1	2.5	1	1	100	300	0.1	3	50	1	100	40
26	2	2	1	1.5	30	300	0.01	1	50	5	100	40
27	2	2.5	2.5	1	30	100	0.05	2	50	0.1	50	25

Table 3.5 shows the optimal solutions for these 27 combinations of input parameters.

Table 3.5. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with Conusmer Tolerance

No	d_c^*	k_c^*	n*	h*	k_1*	k_2*	k_3*	α^*	β^*	EA*
1	25	0.48	10	8	3.49	1.75	0.00	1.57E-03	0.20	173.07
2	25	0.32	25	8	3.74	3.5	0.09	1.86E-04	0.20	16.59
3	25	0.16	10	8	2.66	1.81	0.02	8.28E-03	0.00	316.57
4	25	0.16	14	8	3.84	1.84	0.01	1.55E-04	0.20	134.84
5	25	0.32	6	8	2.83	4.11	0.00	4.62E-03	0.20	807.13
6	25	0.48	3	8	3.49	4.2	0.00	4.86E-04	0.20	191.56
7	25	0.16	3	8	2.93	2.65	0.00	4.28E-03	0.20	37.19
8	25	0.32	3	8	3.49	4.2	0.00	4.86E-04	0.20	266.87
9	25	0.32	8	8	3.08	1.72	0.00	6.13E-03	0.20	107.51
10	25	0.32	7	8	2.7	1.81	0.00	1.00E-02	0.01	46.46
11	25	0.16	10	8	3.9	3.74	0.01	9.55E-05	0.20	22.02
12	25	0.16	14	8	2.9	3.33	0.01	3.73E-03	0.20	3.91
13	25	0.16	6	8	3.02	1.93	0.00	4.85E-03	0.20	84.5
14	25	0.48	5	8	3.91	4.07	0.00	9.22E-05	0.20	4936.37
15	25	0.32	9	8	3.24	3.82	0.01	1.21E-03	0.20	166.8
16	25	0.32	3	8	3.07	3.54	0.00	2.13E-03	0.20	840.99
17	25	0.48	12	8	3.93	4.19	0.02	8.36E-05	0.20	55.95
18	25	0.48	6	8	2.83	4	0.00	4.62E-03	0.20	124.33
19	25	0.32	14	8	3.93	4.15	0.02	8.52E-05	0.20	23.43
20	25	0.48	6	8	3.01	1.93	0.00	4.85E-03	0.20	1661.72
21	25	0.16	7	8	2.71	4.14	0.00	6.81E-03	0.20	321.28
22	25	0.48	3	8	3.07	3.64	0.00	2.14E-03	0.20	28.15
23	25	0.32	12	8	2.62	3.82	0.03	8.73E-03	0.20	1426.94
24	25	0.16	13	8	2.85	1.58	0.02	7.27E-03	0.20	557.14
25	25	0.48	23	8	3.95	2.44	0.01	7.68E-05	0.20	243.62
26	25	0.48	13	8	3	1.52	0.02	8.62E-03	0.20	228.4
27	25	0.16	3	8	3.49	4.16	0.00	4.86E-04	0.20	98.54

Table 3.6 shows the main effect of the optimal solutions and optimal values. It shows following findings:

(i) δ_1 is significant to average sample size *n**. When δ_1 increases, average *n** decreases.

(ii) λ is significant to average k_1*. When λ increases, average k_1* increases.

(iii) δ_1 and δ_2 are significant to average k_2*. When δ_1 increases, average k_2* increases then decreases. When δ_2 increases, average k_2* decreases.

(iv) All input parameters are significant to average *EA**. When δ, σ, δ_1, R, or A_c increase, average *EA** increases. When b increases, average *EA** decreases. When δ_2, λ, or $T_{s.r.}$, or a increases, average *EA** decreases first then increases.

(5) All input parameters are not significant to average producer tolerance d_c*. Average d_c* equals to 25 for all levels of input parameters.

Table 3.6. Main Effect of the Optimal Solutions and Optimal Values under the Cost Model with Conusmer Tolerance

	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	1	25	25	25	25	25	25	25	25	25	25	25
	2	25	25	25	25	25	25	25	25	25	25	25
	3	25	25	25	25	25	25	25	25	25	25	25
	diff	0	0	0	0	0	0	0	0	0	0	0
$\overline{n}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	10.33	9.44	14.67	8.89	9.33	8.89	7.00	10.11	8.67	9.44	9.00
	2	8.67	9.33	8.44	10.56	10.22	9.67	7.67	7.67	10.11	10.11	9.44
	3	8.56	8.78	4.44	8.11	8.00	9.00	12.89	9.78	8.78	8.00	9.11
	diff	1.78	0.67	**10.22**	2.44	2.22	0.78	**5.89**	2.44	1.44	2.11	0.44
$\overline{h}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	2	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	3	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\overline{k_1}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.20	3.23	3.28	3.28	3.33	3.14	3.00	3.14	3.18	3.42	3.34
	2	3.28	3.16	3.27	3.14	3.35	3.19	3.04	3.42	3.28	3.13	3.14

	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	3	3.27	3.35	3.20	3.32	3.06	3.41	3.71	3.19	3.28	3.19	3.26
	diff	0.08	0.19	0.07	0.18	0.28	0.27	**0.71**	0.28	0.10	0.28	0.20
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	2.96	3.49	2.39	3.78	3.14	2.80	3.02	2.95	2.94	3.32	3.42
	2	2.86	2.72	3.56	3.08	3.10	3.41	2.95	3.34	2.94	2.87	3.09
	3	3.47	3.08	3.34	2.43	3.05	3.08	3.33	3.00	3.42	3.10	2.79
	diff	0.61	0.77	**1.17**	**1.35**	0.10	0.61	0.38	0.38	0.48	0.44	0.63
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.02	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
	2	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.02
	3	0.01	0.01	0.00	0.00	0.01	0.01	0.02	0.02	0.01	0.01	0.01
	diff	0.01	0.01	0.02	0.01	0.00	0.01	0.02	0.01	0.01	0.01	0.01
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0038	0.0027	0.0041	0.0026	0.0035	0.0040	0.0046	0.0039	0.0037	0.0016	0.0022
	2	0.0029	0.0047	0.0030	0.0041	0.0020	0.0037	0.0045	0.0019	0.0032	0.0049	0.0042
	3	0.0035	0.0029	0.0032	0.0036	0.0047	0.0025	0.0011	0.0044	0.0033	0.0038	0.0038
	diff	0.0008	0.0021	0.0010	0.0015	0.0027	0.0015	0.0036	0.0025	0.0005	0.0033	0.0020
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.1568	0.2000	0.2000	0.2000	0.1790	0.1778	0.2000	0.1778	0.1790	0.2000	0.2000
	2	0.2000	0.1568	0.2000	0.1778	0.2000	0.1790	0.1790	0.2000	0.1778	0.1568	0.1778
	3	0.2000	0.2000	0.1568	0.1790	0.1778	0.2000	0.1778	0.1790	0.2000	0.2000	0.1790
	diff	0.0432	0.0432	0.0432	0.0222	0.0222	0.0222	0.0222	0.0222	0.0222	0.0432	0.0222
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	188.41	110.53	321.33	353.88	68.26	175.11	466.63	514.43	410.25	880.00	761.04
	2	343.45	466.15	363.02	281.32	204.49	411.41	300.66	168.49	232.75	214.65	427.26
	3	903.90	859.08	751.41	800.56	1163.01	849.24	668.47	752.85	792.77	341.11	247.47
	diff	**715.49**	**748.55**	**430.07**	**519.25**	**1094.75**	**674.13**	**367.81**	**584.36**	**560.02**	**665.35**	**513.57**

In Table 3.7, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation "+", if the input parameter is significant and their relationship is linear and negative, we use notation "-", and if the input parameter is significant and their relationship is quadratic, we use notation "q"; otherwise, we use notation "N".

Table 3.7. The Significant Input Parameters of Each Design Parameter and EA under the Cost Model with Conusmer Tolerance

Optimal design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	N	N	N	N	N	N	N	N	N	N	N
$\bar{n}$	N	N	-	N	N	N	+	N	N	N	N
$\bar{h}$	N	N	N	N	N	N	N	N	N	N	N
$\bar{k}_1$	N	N	N	N	N	N	+	N	N	N	N
$\bar{k}_2$	N	N	q	-	N	N	N	N	N	N	N
$\bar{k}_3$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	+	q	+	+	q	q	q	-	-

4. DESIGN OF PRODUCER TOLERANCE AND ECONOMIC STATISTICAL $\bar{X}$ AND S CHARTS

4.1 Derivation of Cost Models

The assumptions of process distributions and economic statistical $\bar{X}$ and S charts continue to hold in this section. This section assumes that producer tolerance d_p exists, such that if |X-T|> d_p the product should be reworked using the same production equipment. Let T+ d_p and T- d_p be the producer specification limits, and p_p be the rate of nonconforming product for producers.

$$p_p = 1 - P(T - d_p \le X \le T + d_p) = 1 - P\left(-\delta - \frac{d_p}{\sigma} \le z \le -\delta + \frac{d_p}{\sigma}\right)$$

L_p denotes the producer loss function as a quadratic function (Figure 4.1):

$$L_p(X) = \begin{cases} k_p (X - T)^2 & \text{if } |X - T| \le d_p \\ A_p & \text{if } |X - T| > d_p \end{cases} \tag{4.1}$$

where k_p is the coefficient of the producer loss function for a conforming product, and A_p is the cost of rework a nonconforming product prior to its shipment.

The value of k_p is determined by $k_p = A_p / d_p^2$, since $A_p = k_p d_p^2$.

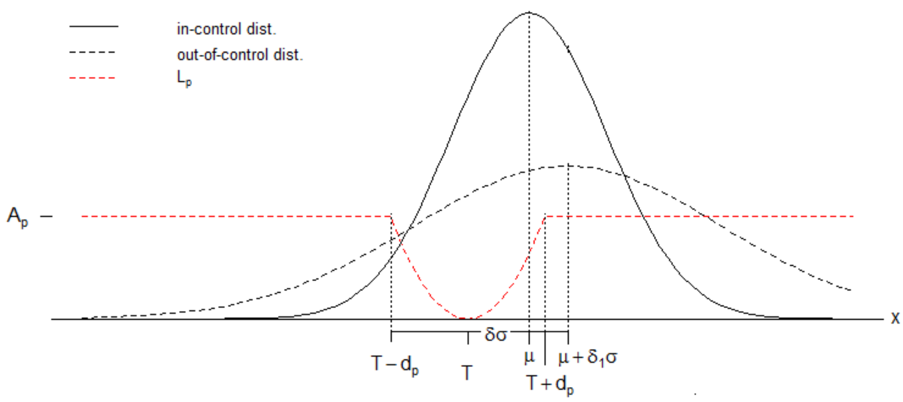

Figure 4.1. Producer Loss Function, In-Control and Out-Of-Control Distributions

If the producer implements a complete inspection plan in which all products are inspected before they ship to the consumer. When the process is in-control, the expected cost of a nonconforming product is the cost of rework plus the expected cost per unit when process is in-control.

$$L_I = IC + [1 - P(T - d_p \le X \le T + d_p)](A_p + L_I) + \int_{T-d_p}^{T+d_p} k_p (x - T)^2 f_X(x)dx \tag{4.2a}$$

where IC is the cost of inspection.

Writing L_I as a function of d_p:

$$L_I = \frac{IC + A_p - A_p \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz + k_p\sigma^2 \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} (z+\delta)^2 \phi(z)dz}{\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz} \tag{4.2b}$$

When the process is out-of-control, the expected cost of nonconforming product is the cost of rework plus the expected cost per unit when process is in-control.

$$\begin{aligned} L_O &= IC + \left[1 - P(T - d_p \le X \le T + d_p)\right](A_p + L_I) \\ &+ \int_{T-d_p}^{T+d_p} k_p (x-T)^2 f_X(x)dx \end{aligned} \tag{4.3a}$$

Writing L_O as a function of d_p:

$$\begin{aligned} L_O &= IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} \phi(z)dz\right](A_p + L_I) \\ &+ k_p\sigma^2 \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} (\delta_2 z + \delta + \delta_1)^2 \phi(z)dz \end{aligned} \tag{4.3b}$$

The expected cost per unit time is

$$EA = \frac{R\frac{L_I}{\lambda} + RL_O\left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h} + \frac{1}{1-\beta}\right) + C_f\frac{\alpha}{\lambda h} + C_{s.r.}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + T_{s.r.}} \tag{4.4}$$

The design parameters can be determined by minimizing the cost function (4.4). A subroutine "DEoptim" in R program is used to solve the object. The upper bounds of α, β, d_p, n, h, k_1, k_2, and k_3 are set to α_U, β_U, d_{pU}, n_U, h_U, k_{1U}, and k_2. The lower bounds of d_p and n are d_{pL} and n_L. Let the rate of nonconforming products for producers be smaller than 0.1, the bounds of d_p can be determined. Therefore, the optimization model is expressed as follows:

$$\begin{aligned} &\min \ \mathrm{EA}(d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t.} \ \ d_{pL} \le d_p \le d_{pU}, \\ &\quad\ \ n_L \le n \le n_U, \\ &\quad\ \ 0 < h \le h_U, \\ &\quad\ \ 0 < k_1 \le k_{1U}, \\ &\quad\ \ 0 < k_3 < k_2 \le k_{2\mathrm{U}}, \\ &\quad\ \ \alpha \le \alpha_U, \\ &\quad\ \ \beta \le \beta_U. \end{aligned}$$

4.2 An Example and Numerical Analysis

4.2.1 Example

In this section, we give an example to show the application of the economic statistical $\bar{X}$ and S control chart only with producer tolerance. We compare the performance and the expected cost per unit time of three types of $\bar{X}$ and S control charts: (1) Shewhart-type economic $\bar{X}$ and S control charts with design h and d_p, (2) economic statistical $\bar{X}$ and S control charts with a given n, and (3) economic statistical $\bar{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_p=20, R=30, λ=0.01, $T_{s.r.}$=3, a=0.5, b=0.1, $C_{s.r.}$=35, and C_f=50.

(1) Shewhart-type economic $\bar{X}$ and S control charts with design h and d_p

To construct the Shewhart-type economic $\bar{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\bar{X}} = \alpha_S$ = 0.0027), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time for the optimal Shewhart-type economic $\bar{X}$ and S charts is

$$\begin{aligned} &\min EA(d_p, h) \\ &\text{s.t.}\ 0.8 < d_p \le 25, \\ &\quad 0 < h \le 8. \end{aligned}$$

The EA^* is 1123.083, h^* is 8, and d_p^* is 25. Under the d_p^*, the rate of nonconforming product is 0, and the optimal Shewhart-type economic $\bar{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\bar{X}} = 3 \\ LCL_{\bar{X}} = -3 \end{matrix} \quad \text{and} \quad \begin{matrix} UCL_S = 1.735 \\ LCL_S = 0.371 \end{matrix}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 4.2 shows that no points fall outside the limits of Shewhart-type $\bar{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 4.3 shows that all product fall into the producer specification.

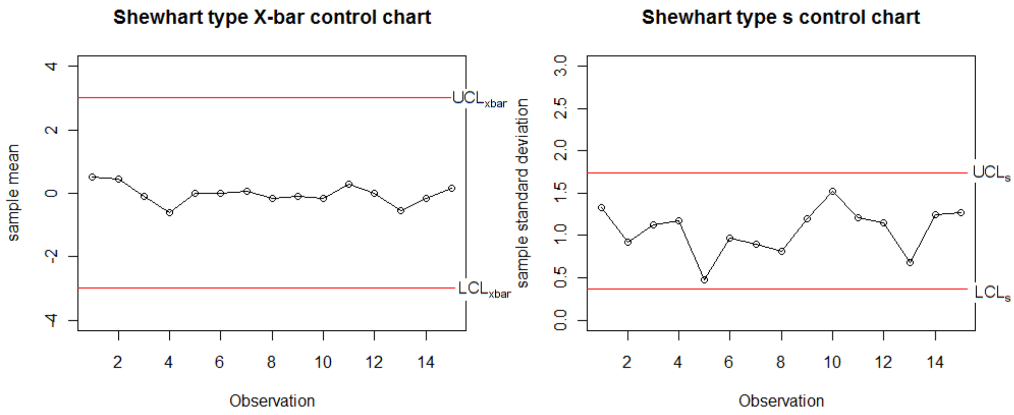

Figure 4.2. Shewhart-Type Economic $\bar{X}$ and S Control Chart with Producer Tolerance

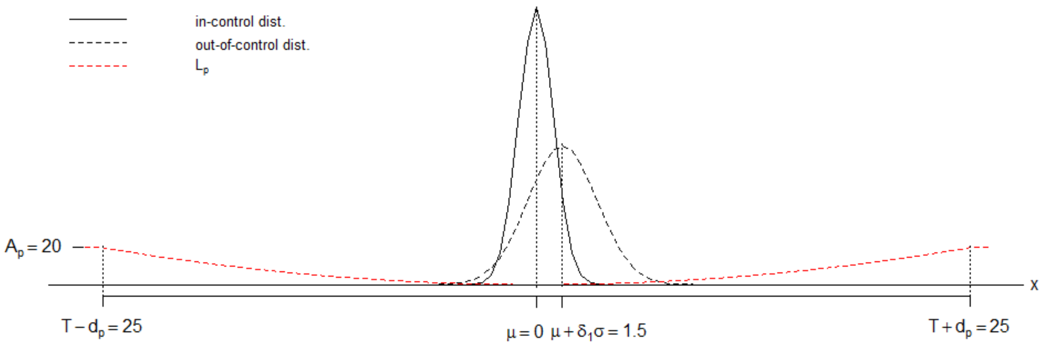

Figure 4.3. Optimal Producer Loss Function with In-Control and Out-Of-Control Distributions

(2) Economic statistical $\bar{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\bar{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$\begin{aligned}
&\min \mathrm{EA}(d_p, h, k_1, k_2, k_3) \\
&\text{s.t. } 0.8 < d_p \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

The optimal design parameters are d_p* = 25, h* = 8, k_1* = 3.061, k_2* = 3.881, k_3* = 0.00003, α* = 0.00221, and β* = 0.2. The EA* is 1051.767. The optimal economic statistical $\bar{X}$ and S charts are constructs as follows.

$$\begin{aligned} UCL_{\bar{X}} &= 3.061 \\ LCL_{\bar{X}} &= -3.061 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 3.881 \\ LCL_S &= 0.00003 \end{aligned}$$

Figure 4.4 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts. Because the optimal producer tolerance is the same as the previous, all in-control and out-of-control products are in the producer specification limits.

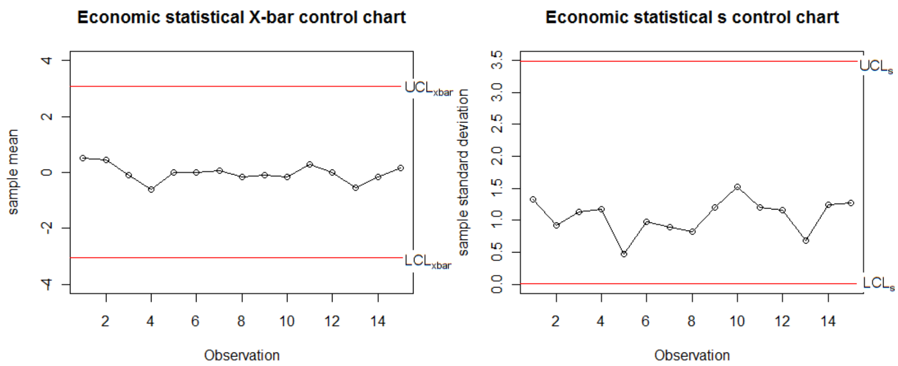

Figure 4.4. Optimal Economic Statistical $\bar{X}$ and S Control Charts with Producer Tolerance and with a Given *n*

(3) Economic statistical $\bar{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$\begin{aligned} &\min \mathrm{EA}(d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t. } 0.8 < d_p \le 25, \\ &\quad 2 \le n \le 25, \\ &\quad 0 < h \le 8, \\ &\quad 0 < k_1 \le 4, \\ &\quad 0 < k_3 < k_2 \le 4.2, \\ &\quad \alpha \le 0.01, \\ &\quad \beta \le 0.2. \end{aligned}$$

The parameters are $d_p* = 25$, $n* = 7$, $h* = 8$, $k_1* = 3.574$, $k_2* = 1.851$, $k_3* = 0.0001$, $\alpha* = 0.00186$, and $\beta* = 0.2$. The $EA*$ is 1156.458. Under the d_p*, the rate of nonconforming product is 0 and the optimal economic statistical $\bar{X}$ and S charts are constructed as follows.

$$\begin{aligned} UCL_{\bar{X}} &= 3.574 \\ LCL_{\bar{X}} &= -3.574 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 1.851 \\ LCL_S &= 0.0001 \end{aligned}$$

Finally, we compare the optimal solutions and expected cost of these 3 types design charts (Table 4.1).

Comparing with “Shewhart-type” and “economic statistical chart with a given *n*” leads to following findings:

(i) If producer can design the chart, k_1* should increase, k_2* should increase and EA* will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart without design n, EA* could save about 6%. And the false alarm rate of economic statistical chart without a design n will decrease, but it's true alarm rate will decrease.

(iii) The optimal producer tolerance all equal to 25.

Comparing economic statistical chart with all design parameters and with a given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, *n** should decrease, k_1* and k_2* should be decrease and EA* will reduce.

Because the expected cost per unit time cannot be saved a lot when we use economic statistical control chart with producer tolerance, we advise that it is more convenience for using Shewhart-type control chart and let producer tolerance equal to 25.

Table 4.1. Comparison of Three Type Design Charts under the Model with Producer Tolerance

	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β
(1) Shewhart-type economic $\overline{X}$ and S control charts	25	0	10	8	3	1.735	0.371	0.00539	0.06502
	d_p*	p_p	n	h*	k_1*	k_2*	k_3*	α*	β*
(2) Economic statistical $\overline{X}$ and S control charts with a given *n*	25	0	10	8	3.061	3.881	0.000	0.00221	0.20000
	d_p*	p_p	n*	h*	k_1*	k_2*	k_3*	α*	β*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	25	0	7	8	3.574	1.851	0.000	0.00254	0.20000

4.2.2 The Effects of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are $A_p = 20$, $T = 0$, $\delta_1 = 1.5$, $\delta_2 = 2$, $k = 4$, $R = 30$, $\lambda = 0.01$, $T_{sr} = 3$, $a = 0.5$, $b = 0.1$, $C_{sr} = 35$, and $C_f = 50$

The results of these objects are shown in Table 4.2. Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_p*, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 0.24%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_p*, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 0.29%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, *EA** will reduce about 0.1%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, and *EA** will reduce about 0.1%.

Comparing with economic statistical control charts and Shewhart type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart type economic $\overline{X}$ and s charts'.

(vi) The false alarm rate of Economic Statistic $\overline{X}$ and s chart is smaller, but its true alarm rate is smaller, too.

According to the findings (i)-(iv), decreasing variance can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\overline{X}$ and S charts. In all situations, optimal producer tolerance equals to 25 and the rate of nonconforming product equals to 0. According to findings (v)-(vi), the expected costs of the two charts are similar. Producers are advised to use the Shewhart-type economic $\overline{X}$ and S charts depending on the convenience of using the chart.

Table 4.2. The Optimum Solution of “Economic Statistic $\overline{X}$ and S Charts” and “Shewhart Type Economic $\overline{X}$ and S Chart” with Producer Tolerance

Economic Statistical $\overline{X}$ and S Control Charts										
	d_c*	n*	h*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	25	7	8	6.468 (3.234)	-6.468 (3.234)	3.954 (1.977)	0.004 (0.002)	0.00188	0.2	1054.365
(2) δ=0 and σ=1	25	7	8	3.574 (3.574)	-3.574 (3.574)	1.851 (1.851)	0 (0)	0.00254	0.2	1051.746
(3) δ=1 and σ=2	25	7	8	7.38 (3.190)	-5.38 (3.190)	3.992 (1.996)	0.002 (0.001)	0.00197	0.2	1054.979
(4) δ=1 and σ=1	25	7	8	4.529 (3.529)	-2.529 (3.529)	1.866 (1.866)	0.003 (0.003)	0.00234	0.2	1051.899
Shewhart-Type Economic $\overline{X}$ and S Control Charts										
	d_c*	n*	h*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1076.541
(2) δ=0 and σ=1	25	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1073.910
(3) δ=1 and σ=2	25	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1077.121
(4) δ=1 and σ=1	25	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1074.055

4.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \text{EA}(d_p, n, h, k_1, k_2, k_3, \delta)\\
&\text{s.t.}\, 0.8 < d_p \le 25\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (δ_1, δ_2, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (1.5, 2, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \text{EA}(d_p, n, h, k_1, k_2, k_3, \delta, \sigma)\\
&\text{s.t.}\, 0.8 < d_p \le 25,\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad 0.5 \le \sigma \le 4,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 4.3 shows the optimal solutions of these objects and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 4.2, when μ is unknown, d_p^* and design parameters are the same, but the EA* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 4.2, when μ and σ are unknown, the width of the $\overline{X}$ and S charts is smaller, and EA* is smaller, too.

Table 4.3. The Optimum Solutions and In-Control Distribution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart Type Economic $\overline{X}$ and S Chart" with Producer Tolerance

Economic Statistical $\overline{X}$ and S Control Charts												
situation	δ*	σ*	d_p^*	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α^*	β^*	EA^*
(1) σ is known, δ is unknown (σ=2)	2.293E-13	--	25	7	8	6.334 (3.167)	-6.334 (3.167)	4.014 (2.00)	0.006 (0.003)	0.00230	0.2	1054.366
(2) δ and σ are unknown	1.002E-05	0.5	25	7	8	1.697 (3.393)	-1.697 (3.393)	0.957 (1.913)	0.0005 (0.001)	0.00192	0.2	1050.93
Shewhart-Type Economic $\overline{X}$ and S Control Charts												
situation	δ*	σ*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA^*
(1) σ is known, δ is unknown (σ=2)	9.858E-14	--	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1076.541
(2) δ and σ are unknown	1.001E-05	0.5	25	7	8	1.5 (3)	-1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1073.09

4.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 12 cost and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), A_p = (20,80,300), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1),a = (0.5,50,100), b = (0.1,1,5), and $(C_{s.r.},C_f)$ = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using an orthogonal array table $L_{27}(3^{13})$ (Table 4.4).

Table 4.4. Orthogonal Array $L_{27}(3^{13})$ for 27 Combinations of Input Parameters

No	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	C_{sr}	C_f
1	0	1	1	2	500	240	0.05	2	50	0.1	100	40
2	0	1	1	1.5	100	80	0.1	1	100	0.1	50	25
3	0	2	2.5	1.5	500	20	0.1	3	50	1	50	25
4	2	2	1	2	100	20	0.1	2	0.5	5	30	50
5	0	2.5	1.5	1	500	80	0.01	1	50	5	30	50
6	0	2	2.5	1	100	240	0.01	2	100	1	30	50
7	1	1	2.5	2	100	20	0.01	1	50	5	50	25
8	1	1	2.5	1	500	80	0.1	2	0.5	5	100	40
9	1	2.5	1	2	30	80	0.01	2	100	1	50	25
10	0	2	2.5	2	30	80	0.05	1	0.5	1	100	40
11	1	2	1.5	1	30	20	0.1	1	100	0.1	100	40
12	0	1	1	1	30	20	0.01	3	0.5	0.1	30	50
13	0	2.5	1.5	2	100	20	0.05	3	100	5	100	40
14	2	2.5	2.5	2	500	240	0.1	1	100	0.1	30	50
15	1	2	1.5	1.5	100	80	0.05	2	50	0.1	30	50
16	2	2.5	2.5	1.5	100	80	0.01	3	0.5	0.1	100	40
17	0	2.5	1.5	1.5	30	240	0.1	2	0.5	5	50	25
18	2	1	1.5	1	100	240	0.05	1	0.5	1	50	25
19	2	1	1.5	2	30	80	0.1	3	50	1	30	50
20	1	2	1.5	2	500	240	0.01	3	0.5	0.1	50	25
21	2	1	1.5	1.5	500	20	0.01	2	100	1	100	40
22	1	1	2.5	1.5	30	240	0.05	3	100	5	30	50
23	2	2	1	1	500	80	0.05	3	100	5	50	25
24	1	2.5	1	1.5	500	20	0.05	1	0.5	1	30	50
25	1	2.5	1	1	100	240	0.1	3	50	1	100	40
26	2	2	1	1.5	30	240	0.01	1	50	5	100	40
27	2	2.5	2.5	1	30	20	0.05	2	50	0.1	50	25

Table 4.5 shows the optimal solutions for these 27 combinations of input parameters.

Table 4.5. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with Producer Tolerance

No	d_p^*	k_p^*	n*	h*	k_1*	k_2*	k_3*	α^*	β^*	EA*
1	25	0.38	10	8	3.79	1.71	0.03	1.92E-03	0.20	9242.42
2	25	0.13	25	8	3.74	4.19	0.11	1.86E-04	0.20	1313.75
3	25	0.03	4	8	3.74	4.01	0.00	1.86E-04	0.20	6398.02
4	25	0.03	14	8	3.76	1.85	0.01	1.96E-04	0.20	1305.68
5	25	0.13	6	8	2.83	4.09	0.00	4.62E-03	0.20	18069.97
6	25	0.38	3	8	3.49	4.08	0.00	4.86E-04	0.20	3679.91
7	25	0.03	3	8	2.69	3.77	0.00	7.14E-03	0.20	3558.17
8	25	0.13	3	8	3.49	4.2	0.00	4.98E-04	0.20	6529.26
9	25	0.13	8	8	3.28	1.69	0.00	6.60E-03	0.20	1089.63
10	25	0.13	3	8	2.84	2.89	0.00	4.70E-03	0.20	573.55
11	25	0.03	10	8	3.9	3.73	0.01	9.55E-05	0.20	397.05
12	25	0.03	14	8	2.9	3.76	0.01	3.73E-03	0.20	1051.61
13	25	0.03	6	8	3.06	1.91	0.00	4.89E-03	0.20	1815.59
14	25	0.38	5	8	3.91	3.82	0.00	9.04E-05	0.20	9731.75
15	25	0.13	9	8	3.24	4.18	0.01	1.21E-03	0.20	1874.01
16	25	0.13	3	8	3.07	3.56	0.00	2.13E-03	0.20	3627.34
17	25	0.38	12	8	3.93	4.18	0.01	8.36E-05	0.20	430.81
18	25	0.38	6	8	2.83	4.17	0.00	4.62E-03	0.20	1902.74
19	25	0.13	14	8	3.93	4.18	0.02	8.52E-05	0.20	387.27
20	25	0.38	6	8	2.85	2.01	0.00	5.57E-03	0.20	18331.31
21	25	0.03	7	8	2.71	4.15	0.00	6.81E-03	0.20	17654.08
22	25	0.38	3	8	3.07	3.9	0.00	2.15E-03	0.20	558.59
23	25	0.13	12	8	2.62	4.1	0.03	8.73E-03	0.20	9271.83
24	25	0.03	13	8	2.71	1.65	0.01	7.88E-03	0.20	9315.34
25	25	0.38	23	8	3.95	3.29	0.08	7.68E-05	0.20	1425.03
26	25	0.38	14	8	2.92	1.61	0.05	4.95E-03	0.20	1126.16
27	25	0.03	3	8	3.49	4.2	0.00	4.87E-04	0.20	560.52

Table 4.6 shows the main effects of the optimal solutions and optimal values for three input parameter levels and it produces the following findings:

(i) δ_1 and λ are significant to average sample size *n**. When δ_1 increases, average *n** decreases. When λ increases, average *n** increases.

(ii) λ is significant to average k_1*. When λ increases, average k_1* increases.

(iii) δ_1 and δ_2 are significant to average k_2*. When δ_1 increases, average k_2* increases. When δ_2 increases, average k_2* decreases.

(iv) All input parameters are significant to average *EA**. When δ, σ, R, or A_p increase, average *EA** increase. When b increases, average *EA** decreases. When δ_1, increase, average *EA** increases first then decreases. δ_2, T_{sr}, or a increases, average *EA** decreases first then increases.

(v) All input parameters are not significant to average producer tolerance d_p*. Average d_p* equals to 25 for all levels of input parameters.

Table 4.6. Main Effect of the Optimal Solutions and Optimal Values under the Cost Model with Producer Tolerances

	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_p}$	1	25	25	25	25	25	25	25	25	25	25	25
	2	25	25	25	25	25	25	25	25	25	25	25
	3	25	25	25	25	25	25	25	25	25	25	25
	diff	0	0	0	0	0	0	0	0	0	0	0
$\bar{n}$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	9.22	9.44	14.78	8.89	9	8.22	7.11	9.44	8.22	9.44	9
	2	8.67	8.33	8.44	10	10.22	9.22	7.22	7.67	9.56	9	8.78
	3	8.67	8.78	3.33	7.67	7.33	9.11	12.22	9.44	8.78	8.11	8.78
	diff	0.56	1.11	**11.44**	2.33	2.89	1.00	**5.11**	1.78	1.33	1.33	0.22
$\bar{h}$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	8	8	8	8	8	8	8	8	8	8	8
	2	8	8	8	8	8	8	8	8	8	8	8
	3	8	8	8	8	8	8	8	8	8	8	8
	diff	0	0	0	0	0	0	0	0	0	0	0
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.37	3.24	3.3	3.28	3.36	3.22	2.97	3.24	3.15	3.43	3.32
	2	3.24	3.26	3.25	3.24	3.32	3.23	3.07	3.46	3.4	3.28	3.24
	3	3.25	3.36	3.31	3.35	3.18	3.42	3.82	3.15	3.31	3.15	3.3

	diff	0.13	0.12	0.06	0.11	0.18	0.2	**0.85**	0.31	0.24	0.28	0.07
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.42	3.78	2.65	3.96	3.35	3.22	3.19	3.41	3.14	3.46	3.5
	2	3.16	3.16	3.62	3.49	3.44	3.67	3.19	3.36	3.45	3.34	3.59
	3	3.51	3.15	3.82	2.65	3.3	3.2	3.72	3.32	3.51	3.29	3
	diff	0.36	0.63	**1.17**	**1.31**	0.14	0.48	0.53	0.09	0.37	0.17	0.59
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.02	0.02	0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
	2	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.02	0.01	0.02
	3	0.01	0.01	0	0.01	0.01	0.02	0.03	0.02	0.02	0.01	0.02
	diff	0	0.01	0.04	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.01
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0027	0.0029	0.0038	0.0026	0.0030	0.0029	0.0045	0.0031	0.0031	0.0018	0.0019
	2	0.0027	0.0033	0.0031	0.0028	0.0021	0.0034	0.0042	0.0021	0.0024	0.0033	0.0033
	3	0.0034	0.0027	0.0019	0.0035	0.0038	0.0025	0.0002	0.0036	0.0034	0.0037	0.0036
	diff	0.0007	0.0006	0.0019	0.0009	0.0017	0.0008	0.0043	0.0015	0.0010	0.0019	0.0016
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	2	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	diff	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	4730.63	4688.65	3904.61	4765.32	686.13	4672.9	7576.47	4762.95	4785.29	5125.53	5108.24
	2	4786.49	4773.06	6762.54	4699.79	2278.02	4748.51	3901.62	4707.37	4737.95	4713.95	4761.86
	3	5063.04	5118.44	3913.01	5115.04	11616.00	5158.75	3102.07	5109.83	5056.91	4740.67	4710.05
	diff	**332.42**	**429.79**	**2857.93**	**415.25**	**10929.87**	**485.85**	**4474.4**	**402.46**	**318.96**	**411.58**	**398.18**

In Table 4.7, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation "+", if the input parameter is significant and their relationship is linear and negative, we use notation "-", and if the input parameter is significant and their relationship is quadratic, we use notation "q"; otherwise, we use notation "N".

Table 4.7. The Significant Input Parameters of Each Design Parameter and EA under the Cost Model with Producer Tolerance

Optimal design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	A_p	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_p}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{n}$	N	N	-	N	N	N	+	N	N	N	N
$\vec{h}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{k}_1$	N	N	N	N	N	N	+	N	N	-	N
$\overline{k}_2$	N	N	+	-	N	N	N	N	N	N	N
$\overline{k}_3$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	-	q	q	-	-

5. DESIGN OF CONSUMER TOLERANCE, PRODUCER TOLERANCE, AND ECONOMIC STATISTICAL $\bar{X}$ AND S CHARTS

5.1 Consumer and Producer Loss Functions Are the Same but with Smaller Producer Tolerance

5.1.1 Derivation of Cost Models

This section continues to hold the assumptions of process distributions and economic statistical $\bar{X}$ and S charts and considers the consumer loss function and producer loss function simultaneously. The coefficients of the consumer loss function and of the producer loss function are assumed to be identical, and only A_c is known. This means that the producer knows only the cost of correcting a nonconforming product for a consumer after its shipment, but the cost of correcting a product before its shipment is unknown. Here, we want to determine the design parameters of $\bar{X}$ and S charts, consumer tolerance, and producer tolerance simultaneously.

The consumer loss function and producer loss function are

$$L_c(X)=\begin{cases} k_c(X-T)^2 & \text{if } |X-T|\le d_c \\ A_c & \text{if } |X-T|>d_c \end{cases},$$

$$L_p(X)=\begin{cases} k_p(X-T)^2 & \text{if } |X-T|\le d_p \\ A_p & \text{if } |X-T|>d_p \end{cases},$$

where $k_p = k_c$, Here, we assume $d_c > d_p > 0$.

Since $k_p = k_c$, such that $A_c/d_c^2 = A_p/d_p^2$. A_p can be calculated by $A_p = \frac{A_c}{d_c^2} d_p^2$.

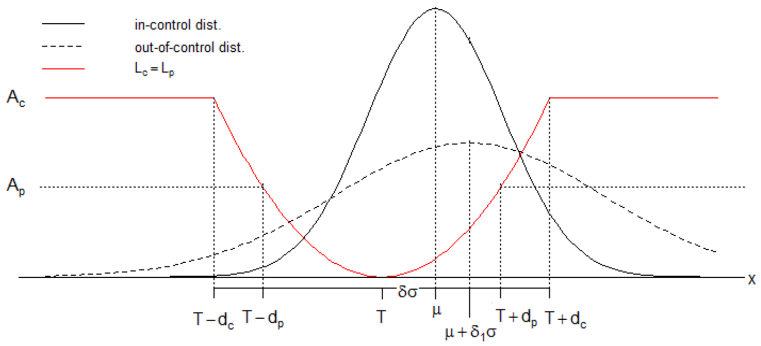

Figure 5.1.1. Consumer Loss Function, Producer Loss Function, In-Control and Out-Of-Control Distributions

If the producer implements a complete inspection plan in which all products are inspected before they ship to the consumer. When the process is in-control, the expected cost of a nonconforming product is the cost of rework plus the expected cost per unit when process is in-control.

$$L_I = IC + \left[1 - P(T - d_p \le X \le T + d_p)\right](A_p + L_I) + \int_{T-d_p}^{T-d_p} k_c (x-T)^2 f_X(x)dx \quad (5.1.1a)$$

Writing L_I as a function of d_c and d_p,

$$L_I = \frac{IC + A_p - A_p \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz + k_c\sigma^2 \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} (z+\delta)^2 \phi(z)dz}{\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz} \quad (5.1.1b)$$

When the process is in-control, the expected cost of a nonconforming product is the cost of rework plus the expected cost per unit when process is in-control.

$$\begin{aligned} L_O = IC + \left[1 - P(T - d_p \le X \le T - d_p)\right](A_p + L_I) \\ + \int_{T-d_p}^{T+d_p} k_c (x-T)^2 f_X(x)dx \end{aligned} \quad (5.1.2a)$$

Writing L_O as a function of d_c and d_p,

$$L_O = IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} \phi(z)dz\right](A_p + L_I) + k_c\sigma^2 \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} (\delta_2 z + \delta + \delta_1)^2 \phi(z)dz \quad (5.1.2b)$$

In the loss function with producer tolerance, the expected cost per unit time is

$$EA = \frac{R\frac{L_I}{\lambda} + RL_O\left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h} + \frac{1}{1-\beta}\right) + C_f \frac{\alpha}{\lambda h} + C_{s.r.}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + T_{s.r.}} \quad (5.1.3)$$

The design parameters can be determined by minimizing the cost function (5.1.3). A subroutine "DEoptim" in R program is used to solve the object. The optimization model is expressed as follows:

$$\begin{aligned} &\min \text{EA}(d_c, d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t.} \;\; d_{cL} \le d_c \le d_p \le d_{pU}, \\ &\qquad n_L \le n \le n_U, \\ &\qquad 0 < h \le h_U, \\ &\qquad 0 < k_1 \le k_{1U}, \\ &\qquad 0 < k_3 < k_2 \le k_{2U}, \\ &\qquad \alpha \le \alpha_U, \\ &\qquad \beta \le \beta_U. \end{aligned}$$

5.1.2 An Example and Numerical Analysis

5.1.2.1 Example

In this section, we give an example to show the application of the economic statistical control chart with consumer and producer tolerances. We compare the optimal solutions and the expected costs of three types of $\bar{X}$ and S control charts: (1) Shewhart-type economic $\bar{X}$ and S control charts with design *h*, d_c, and d_p, (2) economic statistical $\bar{X}$ and S control charts with a given *n*, and (3) economic statistical $\bar{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_c=100, R=30, λ=0.01, T_{sr}=3, a=0.5, b=0.1, C_{sr}=35, and C_f=50.

(1) Shewhart-type economic $\bar{X}$ and S control charts with design *h*, d_c, and d_p

To construct the Shewhart-type economic $\bar{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\bar{X}} = \alpha_S$ = 0.0027), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time of the optimum Shewhart-type economic $\bar{X}$ and S charts is

$$\begin{aligned} &\min EA(d_c, d_p, h) \\ &\text{s.t.}\ 0.8 < d_p \le d_c \le 25, \\ &\quad 0 < h \le 8. \end{aligned}$$

The EA^* is 1126.619, h^* is 8, d_c^* is 25, and d_p^* is 11.180. Under the d_p^*, the rate of nonconforming product is 0 and the optimal Shewhart-type economic $\bar{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\bar{X}} = 3 \\ LCL_{\bar{X}} = -3 \end{matrix} \quad \text{and} \quad \begin{matrix} UCL_S = 1.735 \\ LCL_S = 0.371 \end{matrix}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 5.1.2 shows that no points fall outside the limits of Shewhart-type $\bar{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 5.1.3 shows that all products fall into the consumer specification.

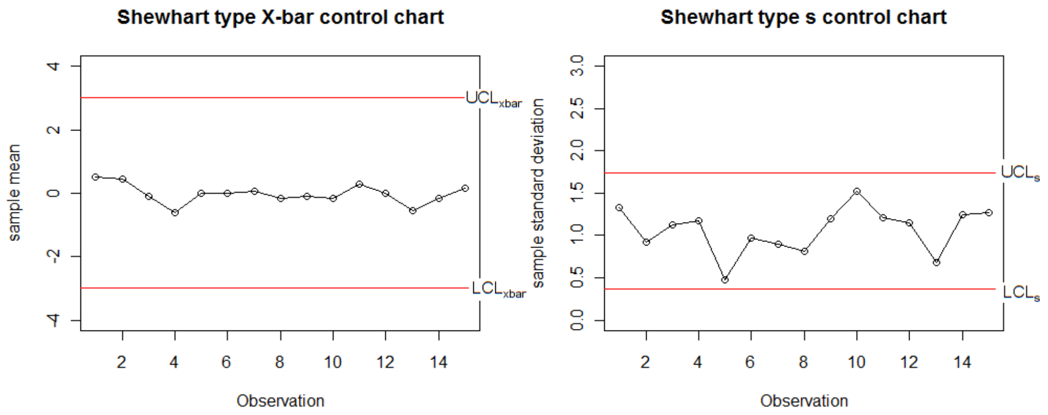

Figure 5.1.2. Shewhart-Type Economic $\overline{X}$ and S Control Charts with Consumer and Producer Tolerances

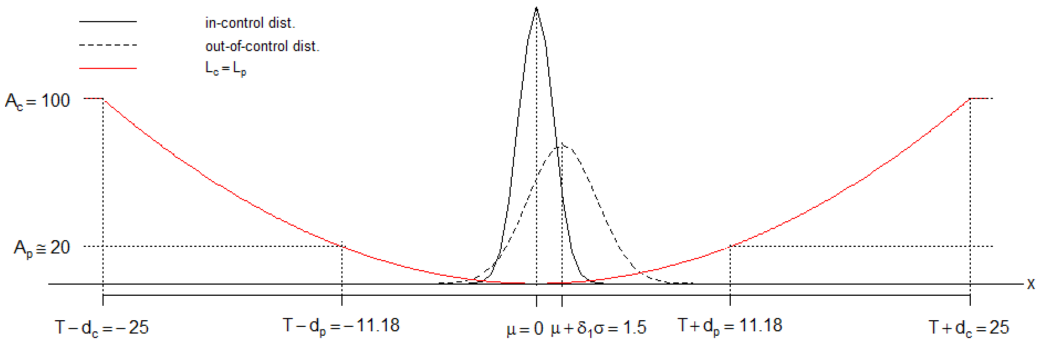

Figure 5.1.3. Optimal Consumer and Producer Loss Functions, In-Control and Out-Of-Control Distributions

(2) Economic statistical $\overline{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\overline{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\overline{X}$ and S charts is

$$\begin{aligned} &\min \ \mathrm{EA}(d_c, d_p, h, k_1, k_2, k_3) \\ &s.t. \ \ 0.8 < d_p \le d_c \le 25, \\ &\qquad 0 < h \le 8, \\ &\qquad 0 < k_1 \le 4, \\ &\qquad 0 < k_3 < k_2 \le 4.2, \\ &\qquad \alpha \le 0.01, \\ &\qquad \beta \le 0.2. \end{aligned}$$

The optimal design parameters are d_c* = 25, d_p*=11.180, h* = 8, k_1* = 3.061, k_2* = 3.933, k_3* = 0.006, α* = 0.002209, and β* = 0.2. The EA* is 1055.262. The optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 3.061 \quad \text{and} \quad UCL_S = 3.933$$
$$LCL_{\bar{X}} = -3.061 \quad \quad LCL_S = 0.006$$

Figure 5.1.4 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts. Because the optimal consumer and producer tolerance is the same as the previous, all in-control and out-of-control products are in the producer specification limits.

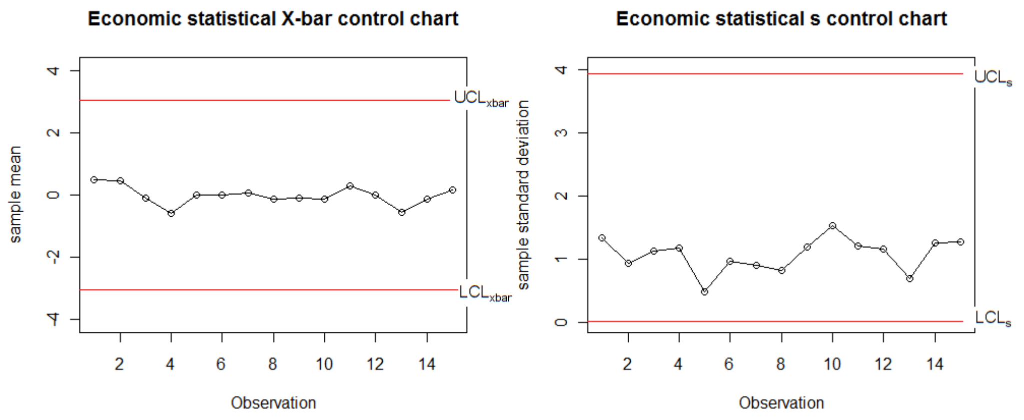

Figure 5.1.4 Optimal Economic Statistical $\bar{X}$ and S Control Chart with Consumer and Producer Tolerances and with a Given *n*

(3) Economic statistical $\bar{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$\begin{aligned} &\min \ \text{EA}(d_c, d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t. } 0.8 < d_p \le d_c \le 25, \\ &\quad 2 \le n \le 25, \\ &\quad 0 < h \le 8, \\ &\quad 0 < k_1 \le 4, \\ &\quad 0 < k_3 < k_2 \le 4.2, \\ &\quad \alpha \le 0.01, \\ &\quad \beta \le 0.2. \end{aligned}$$

The parameters are $d_c* = 25$, $d_p* = 19.298$,$n* = 7$, $h* = 8$, $k_1* = 3.328$ $k_2* = 1.938$, $k_3* = 0.0001$, $\alpha* = 0.000184$, and $\beta* = 0.2$. The $EA*$ is 1155.239. Under the d_p*, the rate of nonconforming product is 0 and the optimum economic statistical $\bar{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 3.328 \quad \text{and} \quad UCL_S = 1.938$$
$$LCL_{\bar{X}} = -3.328 \quad \quad LCL_S = 0.0001$$

Finally, we compare the optimal solutions and expected cost of these 3 types design charts (Table 5.1.1).

Comparing with "Shewhart-type" and "economic statistical chart with given *n*" leads to following findings:

(i) If producer can design the chart, k_1* and k_2* should increase and EA* will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart with a given *n*, EA* could save about 6%. And the false alarm rate of economic statistical chart with a given *n* will decrease, but its true alarm rate will decrease.

(iii) The optimal producer tolerance all equal to 25.

When comparing economic statistical chart with all design parameters and with given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, d_p* should be increase, *n** should decrease k_1* and k_2* should be decrease and EA* will reduce.

Because the expected cost per unit time cannot be saved a lot when we use economic statistical control chart with consumer tolerance, we advise that it is more convenience for using Shewhart-type economic control chart and let consumer tolerance equal to 25.

Table 5.1.1. Comparison of Three Types Design Charts under the Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(1) Shewhart-type economic $\overline{X}$ and S control charts	25	11.180	0	10	8	3	1.735	0.371	0.00539	0.06502	1126.619
	d_c*	d_p*	p_p	n	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(2) Economic statistical $\overline{X}$ and S control charts with a given *n*	25	11.180	0	10	8	3.061	3.933	0.006	0.002209	0.20000	1055.262
	d_c*	d_p*	p_p	n*	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	25	19.298	0	7	8	3.328	1.938	0.000	0.000184	0.20000	1055.239

5.1.2.2 The Effects of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are A_c = 100, T = 0, δ_1 = 1.5, δ_2 = 2, k = 4, R = 30, λ = 0.01, T_{sr} = 3, a = 0.5, b = 0.1, C_{sr} = 35, and C_f = 50.

The results of these objects are shown in Table 5.1.2. Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_c*, *n**, and *h** will not change, d_p* will be smaller, and the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 1.2%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_c*, d_p*, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 1.4%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_c*, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, *EA** will reduce about 0.2%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_c*, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, and *EA** will reduce about 0.1%.

Comparing with economic statistical control charts and Shewhart type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart type economic $\overline{X}$ and s charts'.

(vi) The α^* of Economic Statistic $\overline{X}$ and s chart is smaller, but its β^* is smaller, too.

According to the findings (i)-(iv), decreasing variance can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\overline{X}$ and S charts. In all situations, optimal consumer tolerance equals to 25 and all products are in the consumer specification limits.

According to findings (v)-(vi), the expected costs of the two charts are similar. Producers are advised to use the Shewhart-type economic $\overline{X}$ or S charts depending on the convenience of using the chart.

Table 5.1.2. The Optimum Solution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-Type Economic $\overline{X}$ and S Chart" with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

Economic Statistical $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	25	25	7	8	6.79 (3.395)	-6.79 (3.395)	3.824 (1.912)	0.006 (0.003)	0.00193	0.2	1068.346
(2) δ=0 and σ=1	25	19.298	7	8	3.328 (3.328)	-3.328 (3.328)	1.938 (1.938)	0 (0)	0.00184	0.2	1055.239
(3) δ=1 and σ=2	25	25	7	8	7.696 (3.348)	-5.696 (3.348)	3.86 (1.930)	0 (0)	0.00186	0.2	1071.416
(4) δ=1 and σ=1	25	25	7	8	4.348 (3.348)	-2.348 (3.348)	1.930 (1.930)	0 (0)	0.00186	0.2	1056.416
Shewhart-Type Economic $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	25	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1076.541
(2) δ=0 and σ=1	25	17.519	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1073.91
(3) δ=1 and σ=2	25	25	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1077.121
(4) δ=1 and σ=1	25	25	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1074.055

5.1.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{\mathrm{X}}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$
\begin{aligned}
&\min \mathrm{EA}\left(d_c, d_p, n, h, k_1, k_2, k_3, \delta\right)\\
&\text{s.t.}\, 0.8 < d_p \le d_c \le 25\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}
$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{\mathrm{X}}$ and S charts are unknown. Given (δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50)

$$
\begin{aligned}
&\min \mathrm{EA}\left(d_c, d_p, n, h, k_1, k_2, k_3, \delta, \sigma\right)\\
&\text{s.t.}\, 0.8 < d_p \le d_c \le 25,\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad 0.5 \le \sigma \le 4,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}
$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 5.1.3 shows the optimal solutions of these objects and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 5.1.2, when μ is unknown, d_c^*, d_p^* and design parameters are the same, but the EA* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 5.1.2, when μ and σ are unknown, d_p^* and the width of the $\overline{X}$ chart are smaller, the width of the S chart is larger, and the *EA** is smaller.

Table 5.1.3. The Optimum Solutions and In-Control Distribution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-Type Economic $\overline{X}$ and S Chart" with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

Economic Statistical $\overline{X}$ and S Control Charts												
situation	δ*	$σ^*$	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}^*$ (k_1*)	$LCL_{\overline{X}}^*$ (k_1*)	UCL_S^* (k_2*)	LCL_S^* (k_3*)	α*	β*	*EA**
(1) σ is known, δ is unknown (σ=2)	8.573E -14	--	25	25	7	8	6.334 (3.167)	-6.334 (3.167)	4.014 (2.00)	0.006 (0.003)	0.0023	0.2	1054.366
(2) δ and σ are unknown	9.791E -04	0.5	25	20.221	7	8	1.697 (3.393)	-1.697 (3.393)	0.957 (1.913)	0.001 (0.001)	0.00192	0.2	1050.93
Shewhart-Type Economic $\overline{X}$ and S Control Charts													
situation	δ*	$σ^*$	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	*EA**
(1) σ is known, δ is unknown (σ=2)	3.631E -14	--	25	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1076.541
(2) δ and σ are unknown	1.044E -05	0.5	25	24.971	7	8	1.5 (3)	-1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1073.09

5.1.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 12 cost and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), A_c = (100,200,300), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1),a = (0.5,50,100), b = (0.1,1,5), and ($C_{s.r.}$,C_f) = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using an orthogonal array table $L_{27}(3^{13})$ (use the same table as table 3.4). Table 5.1.4 shows the optimal solutions for these 27 combinations of input parameters.

Table 5.1.4. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

No	d_c^*	k_c^*	d_p^*	k_p^*	n*	h*	k1*	k2*	k3*	α^*	β^*	EA*
1	25.00	0.48	22.16	0.49	10	8.00	3.64	1.73	0.00	1.67E-03	0.20	9276.58
2	25.00	0.32	23.41	0.15	25	8.00	3.74	3.92	0.12	1.86E-04	0.20	1323.23
3	25.00	0.16	25.00	0.03	4	8.00	3.74	4.03	0.00	1.86E-04	0.20	6655.07
4	25.00	0.16	25.00	0.03	15	8.00	3.56	1.91	0.06	3.80E-04	0.20	1381.31
5	25.00	0.32	24.67	0.13	6	8.00	2.83	3.88	0.00	4.62E-03	0.20	18550.50
6	25.00	0.48	24.66	0.39	3	8.00	3.49	4.20	0.00	4.87E-04	0.20	3716.64
7	25.00	0.16	22.22	0.04	3	8.00	2.72	3.50	0.00	6.57E-03	0.20	3575.16
8	25.00	0.32	24.97	0.13	3	8.00	3.49	4.17	0.00	5.10E-04	0.20	6673.51
9	25.00	0.32	25.00	0.13	9	8.00	3.23	1.75	0.01	3.12E-03	0.20	1125.22
10	25.00	0.32	25.00	0.13	4	8.00	3.42	3.07	0.00	6.28E-04	0.20	600.85
11	25.00	0.16	19.52	0.05	10	8.00	3.90	4.11	0.01	9.55E-05	0.20	410.98
12	25.00	0.16	14.01	0.10	14	8.00	2.90	3.80	0.00	3.73E-03	0.20	1054.45
13	25.00	0.16	25.00	0.03	6	8.00	3.02	1.93	0.00	4.85E-03	0.20	1880.31
14	25.00	0.48	25.00	0.38	5	8.00	3.91	4.14	0.00	9.25E-05	0.20	10529.35
15	25.00	0.32	25.00	0.13	9	8.00	3.24	4.04	0.02	1.21E-03	0.20	1952.01
16	25.00	0.32	25.00	0.13	3	8.00	3.07	3.93	0.01	2.22E-03	0.20	3814.52
17	25.00	0.48	25.00	0.38	12	8.00	3.93	4.20	0.01	8.36E-05	0.20	441.85
18	25.00	0.48	18.07	0.73	6	8.00	2.83	4.17	0.00	4.62E-03	0.20	1907.64
19	25.00	0.32	22.76	0.15	14	8.00	3.93	4.19	0.06	8.52E-05	0.20	397.46
20	25.00	0.48	25.00	0.38	7	8.00	3.55	1.86	0.00	2.43E-03	0.20	18536.78
21	25.00	0.16	19.58	0.05	7	8.00	2.71	4.15	0.00	6.81E-03	0.20	17733.97
22	25.00	0.48	22.51	0.47	3	8.00	3.07	3.41	0.00	2.12E-03	0.20	562.95
23	25.00	0.32	24.99	0.13	12	8.00	2.62	3.43	0.02	8.73E-03	0.20	9705.89
24	25.00	0.16	25.00	0.03	14	8.00	2.88	1.62	0.01	5.07E-03	0.20	9648.06
25	25.00	0.48	24.40	0.40	23	8.00	3.95	2.48	0.10	7.68E-05	0.20	1464.25

26	25.00	0.48	24.60	0.40	13	8.00	2.75	1.62	0.01	7.49E-03	0.20	1138.75
27	25.00	0.16	25.00	0.03	3	8.00	3.49	4.10	0.00	4.86E-04	0.20	611.48

Table 5.1.5 shows the main effect of the optimal solutions and optimal values and it produces the following findings:

(1) σ and A_c are significant to average producer tolerance d_p. When σ increases, average d_p* increases. When A_c increases, average d_p* increases then decreases.

(2) δ_1 and λ are significant to average sample size *n*. When δ_1 increases, average *n** decreases. When λ increases, average *n** increases.

(3) λ is significant to average k_1. When λ increases, average k_1* increases.

(4) δ_1 and δ_2 are significant to average k_2. When δ_1 increases, average k_2* increases. When δ_2 increases, average k_2* decreases.

(5) All input parameters are significant to average *EA*. When δ, σ, R, or A_c increase, average *EA** increases. When b increases, average *EA** decreases. When δ_1, increases, average *EA** increases first then decreases. δ_2, $T_{s.r.}$, or a increase, average *EA** will decreases first then increases.

(6) All input parameters are not significant to average producer tolerance d_c. Average d_c* equals to 25 for all levels of input parameters.

Table 5.1.5. Main Effect of the Optimal Solutions and Optimal Values under the Cost Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	1	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	2	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	3	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	diff	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
$\overline{d_p}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	23.21	21.08	23.17	22.25	22.60	22.26	22.75	23.19	23.01	22.68	23.18
	2	23.74	24.31	22.73	23.90	23.64	24.53	23.64	24.04	23.98	23.27	23.74
	3	23.33	24.90	24.37	24.13	24.04	23.49	23.90	23.06	23.30	24.33	23.36
	diff	0.52	**3.82**	1.64	1.87	1.44	**2.27**	1.15	0.99	0.97	1.65	0.56
$\overline{n}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	9.33	9.44	15.00	8.89	9.11	8.44	7.22	9.56	8.67	9.56	9.22
	2	9.00	8.56	8.56	10.00	10.33	9.44	7.44	7.89	9.44	9.33	9.00
	3	8.67	9.00	3.44	8.11	7.56	9.11	12.33	9.56	8.89	8.11	8.78

	diff	0.67	0.89	**11.56**	1.89	2.78	1.00	**5.11**	1.67	0.78	1.44	0.44
$\vec{h}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	2	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	3	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.41	3.22	3.25	3.28	3.40	3.21	3.03	3.32	3.29	3.49	3.31
	2	3.34	3.36	3.33	3.24	3.29	3.28	3.13	3.42	3.37	3.35	3.32
	3	3.21	3.37	3.38	3.44	3.26	3.46	3.79	3.22	3.30	3.11	3.33
	diff	0.20	0.14	0.13	0.20	0.14	0.25	**0.77**	0.20	0.07	0.38	0.02
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.42	3.67	2.47	3.82	3.36	3.24	3.19	3.23	3.19	3.51	3.47
	2	2.99	3.14	3.61	3.44	3.34	3.60	3.06	3.36	3.29	3.30	3.44
	3	3.52	3.11	3.84	2.68	3.22	3.09	3.68	3.34	3.45	3.12	3.02
	diff	0.52	0.56	**1.36**	**1.14**	0.14	0.51	0.63	0.13	0.26	0.40	0.44
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.02	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.01	0.02	0.02
	2	0.02	0.01	0.01	0.02	0.03	0.03	0.01	0.01	0.02	0.02	0.02
	3	0.02	0.02	0.00	0.02	0.00	0.01	0.04	0.02	0.02	0.01	0.02
	diff	0.00	0.01	0.03	0.00	0.03	0.02	0.04	0.01	0.01	0.01	0.00
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0018	0.0029	0.0034	0.0026	0.0020	0.0031	0.0042	0.0027	0.0022	0.0013	0.0020
	2	0.0024	0.0024	0.0028	0.0028	0.0023	0.0024	0.0033	0.0016	0.0025	0.0023	0.0029
	3	0.0034	0.0023	0.0015	0.0022	0.0033	0.0021	0.0002	0.0033	0.0029	0.0039	0.0027
	diff	**0.0016**	**0.0006**	**0.0019**	**0.0006**	**0.0014**	**0.0010**	**0.0040**	**0.0016**	**0.0008**	**0.0026**	**0.0010**
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	2	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	diff	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	4833.27	4722.77	4013.08	4899.48	704.89	4772.31	7694.00	4896.85	4895.44	5278.82	5310.30
	2	4883.21	4899.81	6867.94	4807.82	2335.01	4904.80	4016.20	4768.06	4846.81	4805.46	4875.81
	3	5246.71	5340.62	4082.17	5255.89	11923.30	5286.09	3253.00	5298.28	5220.95	4878.91	4777.08
	diff	**413.43**	**617.84**	**2854.86**	**448.07**	**11218.41**	**513.78**	**4441.00**	**530.22**	**374.14**	**473.36**	**533.22**

In Table 5.1.6, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation “+”, if the input parameter is significant and their relationship is linear and negative, we use notation “-“, and if the input parameter is significant and their relationship is quadratic, we use notation “q“; otherwise, we use notation “N”.

Table 5.1.6. The Significant Input Parameters of Each Design Parameters and EA under the Cost Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Determined A_p

Optimal design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	A_c	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{d_p}$	N	+	N	N	N	q	N	N	N	N	N
$\overline{n}$	N	N	-	N	N	N	+	N	N	N	N
$\overline{h}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{k_1}$	N	N	N	N	N	N	+	N	N	N	N
$\overline{k_2}$	N	N	+	-	N	N	N	N	N	N	N
$\overline{k_3}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	-	q	q	-	q

5.2 Considering Different Consumer and Producer Loss Functions with Smaller Consumer Tolerance

5.2.1 Derivation of Cost Models

This section considers the difference between the consumer loss function and the producer loss function. Most producers only consider the producer loss. Without include the consumer loss, its cost is lower than the actual cost.

This section assumes that A_c and A_p are known, and that the consumer tolerance is smaller than or equal to the producer tolerance. This section determines the design parameters of $\overline{X}$ and S charts, the smaller consumer tolerance, and the larger producer tolerance simultaneously.

The consumer loss function and producer loss function are

$$L_c(X)=\begin{cases} k_c(X-T)^2 & if\ |X-T|\le d_c \\ A_c & if\ |X-T|>d_c \end{cases},$$

$$L_p(X)=\begin{cases} k_p(X-T)^2 & if\ |X-T|\le d_p \\ A_p & if\ |X-T|>d_p \end{cases},$$

where $d_p \geqq d_c \geqq 0$, $k_c > k_p \geqq 0$, and $A_c > A_p$.

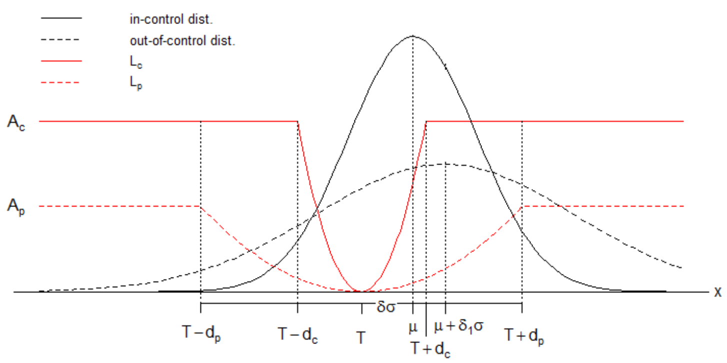

Figure 5.2.1. Consumer and Producer Loss Functions, In-Control and Out-Of-Control Distributions

When the product is within the consumer specification limits, only the consumer loss is considered because it is higher than the producer loss. When the product is out of the consumer specification limits and within the producer specification limits, the consumer loss is considered since $A_c > k_p(X-T)^2$. If the producer implements a complete inspection plan in which all products are inspected before they ship to the consumer, the expected cost per unit of nonconforming product of producer is rework cost plus the expected cost per unit when process is in-control.

$$L_I = IC + [1 - P(T - d_p \le X \le T + d_p)](A_p + L_I) + [P(T - d_p \le X \le T - d_c) + P(T + d_c \le X \le T + d_p)]A_c + \int_{T-d_c}^{T+d_c} k_c (x-T)^2 f_X(x)dx$$

(5.2.1a)

Writing L_I as a function of d_c and d_p,

$$L_I = \frac{IC + A_p - A_p \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz + A_c \left[\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta-\frac{d_c}{\sigma}} \phi(z)dz + \int_{-\delta+\frac{d_c}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz \right] + k_c\sigma^2 \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} (z+\delta)^2 \phi(z)dz}{\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz}$$

(5.2.1b)

When the process is out-of-control, the expected cost of a nonconforming product of producer is the cost of rework plus the expected cost per unit when process is in-control.

$$L_O = IC + [1 - P(T - d_p \le X \le T + d_p)](A_p + L_I) + [P(T - d_p \le X \le T - d_c) + P(T + d_c \le X \le T + d_p)]A_c + \int_{T-d_c}^{T+d_c} k_c (x-T)^2 f_{X^*}(x)dx \quad (5.2.2a)$$

Writing L_O as a function of d_c and d_p,

$$L_O = IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} \phi(z)dz \right](A_p + L_I) + A_c \left[\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta-\frac{d_c}{\sigma}} \phi(z)dz + \int_{-\delta+\frac{d_c}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz \right] + k_c\sigma^2 \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} (\delta_2 z + \delta + \delta_1)^2 \phi(z)dz \quad (5.2.2b)$$

With L_I and L_O, the expected cost per unit time is

$$EA = \frac{R\frac{L_I}{\lambda} + RL_O\left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h} + \frac{1}{1-\beta}\right) + C_f \frac{\alpha}{\lambda h} + C_{s.r.}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + T_{s.r.}}$$

(5.2.3)

The design parameters can be determined by minimizing the cost function (5.2.3). A subroutine "DEoptim" in R program is used to solve the object. The optimization model is expressed as follows:

$$\begin{aligned}
&\min \mathrm{EA}(d_c, d_p, n, h, k_1, k_2, k_3) \\
&\text{s.t. } d_{cL} \le d_c \le d_p \le d_{pU}, \\
&\quad n_L \le n \le n_U, \\
&\quad 0 < h \le h_U, \\
&\quad 0 < k_1 \le k_{1U}, \\
&\quad 0 < k_3 < k_2 \le k_{2\mathrm{U}}, \\
&\quad \alpha \le \alpha_{\mathrm{U}}, \\
&\quad \beta \le \beta_{\mathrm{U}}.
\end{aligned}$$

5.2.2 An Example and Numerical Analysis

5.2.2.1 Example

In this section, we give an example to show the application of the economic statistical $\overline{X}$ and S control charts considering different consumer and producer loss functions where $d_c \leqq d_p$. We compare the optimal solutions and the expected costs of three types of $\overline{X}$ and S control charts: (1) Shewhart-type economic $\overline{X}$ and S control charts with design h, d_c, and d_p, (2) economic statistical $\overline{X}$ and S control charts with a given n, and (3) economic statistical $\overline{X}$ and S control charts with all design parameters. A subroutine “DEoptim” in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_c=100, A_p=20, R=30, λ=0.01, T_{sr}=3, a=0.5, b=0.1, C_{sr}=35, and C_f=50.

(1) Shewhart-type economic $\overline{X}$ and S control charts with design h, d_c, and d_p

To construct the Shewhart-type economic $\overline{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\overline{X}} = \alpha_S = 0.0027$), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time of the optimal Shewhart-type economic $\overline{X}$ and S charts is

$$\begin{aligned} &\min EA(d_c, d_p, h) \\ &s.t.\ 0.8 < d_c \le d_p \le 25, \\ &\quad 0 < h \le 8. \end{aligned}$$

The EA^* is 1126.619, h^* is 8, d_c^* is 25, and d_p^* is 25. Under d_p^*, the rate of nonconforming product is 0. The optimal Shewhart-type economic $\overline{X}$ and S charts are constructed as follows.

$$\begin{aligned} UCL_{\overline{X}} &= 3 \\ LCL_{\overline{X}} &= -3 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 1.735 \\ LCL_S &= 0.371 \end{aligned}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 5.2.2 shows that no points fall outside the limits of Shewhart-type $\overline{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 5.2.3 shows that all products fall into the producer specification.

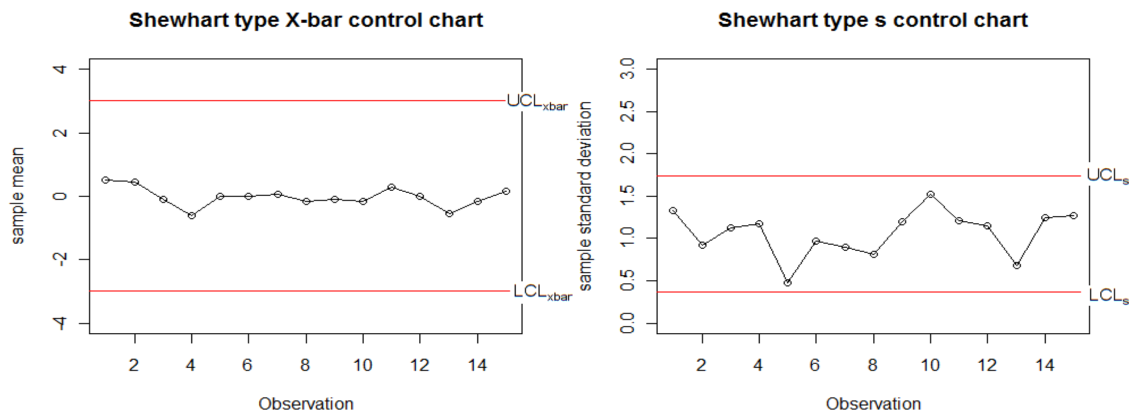

Figure 5.2.2. Shewhart-Type Economic $\overline{X}$ and S Control Charts with Consumer and Producer Tolerances

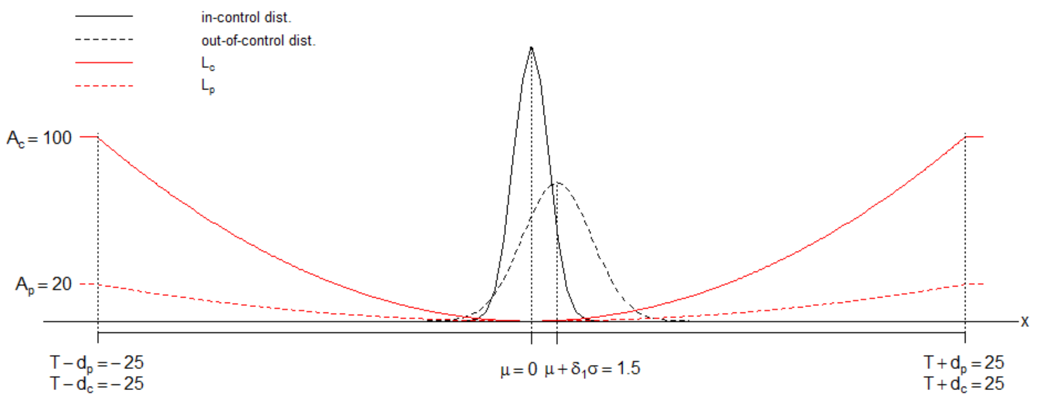

Figure 5.2.3. Optimal Consumer and Producer Loss Functions with In-Control and Out-Of-Control Distributions

(2) Economic statistical $\overline{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\overline{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\overline{X}$ and S charts is

$$\begin{aligned}
&\min \ \mathrm{EA}\left(d_c, d_p, h, k_1, k_2, k_3\right) \\
&s.t. \ \ 0.8 < d_c \le d_p \le 25, \\
&\qquad 0 < h \le 8, \\
&\qquad 0 < k_1 \le 4, \\
&\qquad 0 < k_3 < k_2 \le 4.2, \\
&\qquad \alpha \le 0.01, \\
&\qquad \beta \le 0.2.
\end{aligned}$$

The optimal design parameters are $d_c* = 25$, $d_p* = 25$, $h* = 8$, $k_1* = 3.061$, $k_2* = 4.041$, $k_3* = 0.005$, $\alpha* = 0.00221$, and $\beta* = 0.2$. Under the d_p*, the rate of nonconforming product is 0 and the $EA*$ is 1055.262. The optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$UCL_{\overline{X}} = 3.061 \quad \text{and} \quad UCL_S = 4.041$$
$$LCL_{\overline{X}} = -3.061 \qquad\qquad LCL_S = 0.005$$

Figure 5.2.4 shows the optimal economic statistical $\overline{X}$ and S control charts. No points fall outside the limits of the optimal charts. Because the optimal producer tolerance is the same as the previous, all in-control and out-of-control products are in the producer specification limits.

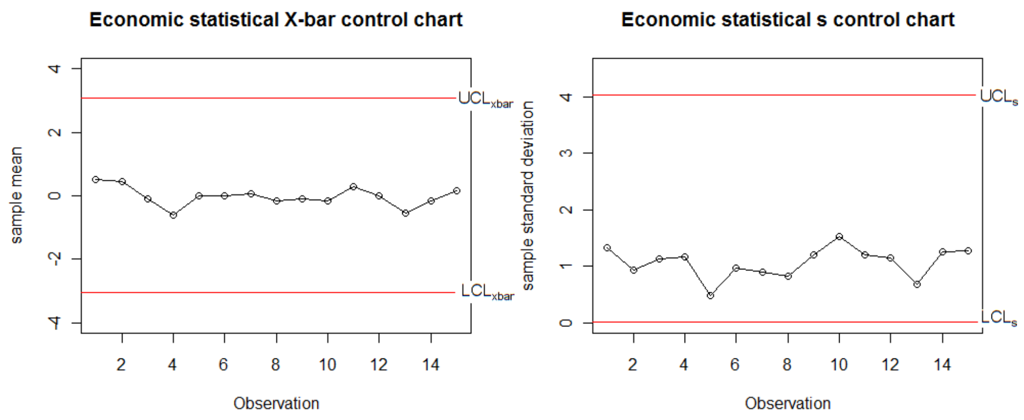

Figure 5.2.4. Optimal Economic Statistical $\overline{X}$ and S Control Charts with Consumer and Producer Tolerances and with a Given n

(3) Economic statistical $\overline{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimal economic statistical $\overline{X}$ and S charts is

$$\begin{aligned}
&\min \text{EA}(d_c, d_p, n, h, k_1, k_2, k_3) \\
&\text{s.t. } 0.8 < d_c \le d_p \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

The parameters are d_c* = 25, d_p* = 25, n* = 7, h* = 8, k_1* = 3.309, k_2* = 1.945, k_3* = 0.003, α* = 0.000184, and β* = 0.2. The EA* is 1155.239. Under the d_p*, the rate of nonconforming product is is 0, and the optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$UCL_{\overline{X}} = 3.309 \quad \text{and} \quad UCL_S = 1.945$$
$$LCL_{\overline{X}} = -3.309 \qquad\qquad LCL_S = 0.003$$

Finally, we compare the optimal solutions and expected cost of these 3 types design charts (Table 5.2.1).

Comparing with "Shewhart-type" and "economic statistical chart with given *n*" leads to following findings:

(i) If producer can design the chart, k_1* and k_2* should increase and *EA** will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart without design n, EA* could save about 6%. And the false alarm rate of economic statistical chart without design n will decrease, but its true alarm rate will decrease.

(iii) The optimal consumer and producer tolerances all equal to 25.

Comparing economic statistical chart with all design parameters and with a given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, d_p* and k_1* should increase, *n** and k_2* should decrease, and EA* will reduce.

To save the loss of the product between the consumer specification limits and producer specification limits, the optimal d_p will equal to optimal d_c. Because the expected cost per unit time cannot be saved a lot when we use economic statistical control chart with consumer tolerance, we advise that it is more convenience for using Shewhart-type control chart and let consumer and producer tolerances equal to 25.

Table 5.2.1. Comparison of Three Type Design Charts under the Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(1) Shewhart-type economic $\overline{X}$ and S control charts	25	25	0	10	8	3	1.735	0.371	0.00539	0.06502	1126.619
	d_c*	d_p*	p_p	n	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(2) Economic statistical $\overline{X}$ and S control charts with a given n	25	25	0	10	8	3.061	4.041	0.005	0.00221	0.20000	1055.262
	d_c*	d_p*	p_p	n*	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	25	25	0	7	8	3.309	1.945	0.003	0.001837	0.20000	1055.239

5.2.2.2 The Effect of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are $A_c = 100$, $A_p = 20$, T = 0, $\delta_1 = 1.5$, $\delta_2 = 2$, k = 4, R = 30, λ = 0.01, $T_{sr} = 3$, a = 0.5, b = 0.1, $C_{sr} = 35$, and $C_f = 50$.

The results of these objects are shown in Table 5.2.2. Comparing the optimal solutions of economic statistical $\bar{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_c*, d_p*, *n** and *h** will not change, and the width of $\bar{X}$ and S charts will be smaller, and *EA** will reduce about 0.2%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_c*, d_p*, *n** and *h** will not change, the width of $\bar{X}$ and S charts will be smaller, and *EA** will reduce about 0.2%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_c*, d_p*, *n**, *h** will not change, the width of $\bar{X}$ will increase and the width of S charts will decrease, *EA** will reduce about 0.1%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_c*, d_p*, *n**, *h** will not change, the width of $\bar{X}$ will increase and the width of S charts will decrease, and *EA** will reduce about 0.1%.

Comparing with economic statistical control charts and Shewhart type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\bar{X}$ and S charts are a little higher than Shewhart type economic $\bar{X}$ and s charts'.

(vi) The false alarm rate of Economic Statistic $\bar{X}$ and s chart is smaller, but its true alarm rate is smaller, too.

According to the findings (i)-(iv), we know that decreasing variance can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\bar{X}$ and S charts. In all situations, optimal consumer and producer tolerances equal to 25 and all products are in the producer specification limits According to findings (v)-(vi), the expected costs of the two charts are similar. Producers are advised to use the Shewhart-type economic $\overline{X}$ or S charts depending on the convenience of using the chart.

Table 5.2.2. The Optimum Solution of “Economic Statistic $\overline{X}$ and S Charts” and “Shewhart Type Economic $\overline{X}$ and S Chart” with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Economic Statistical $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	25	25	7	8	6.603 (3.302)	-6.603 (3.302)	3.897 (1.948	0.008 (0.004)	0.00184	0.2	1068.347
(2) δ=0 and σ=1	25	25	7	8	3.309 (3.309)	-3.309 (3.309)	1.945 (1.945)	0.003 (0.003)	0.00184	0.2	1055.239
(3) δ=1 and σ=2	25	25	7	8	8.236 (3.618)	-6.236 (3.618)	3.676 (1.838)	0.009 (0.005)	0.00278	0.2	1071.420
(4) δ=1 and σ=1	25	25	7	8	4.234 (3.234)	-2.234 (3.234)	1.977 (1.977)	0.001 (0.001)	0.00188	0.2	1056.006
Shewhart-Type Economic $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	25	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1090.573
(2) δ=0 and σ=1	25	25	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1077.418
(3) δ=1 and σ=2	25	25	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1093.475
(4) δ=1 and σ=1	25	25	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1078.143

5.2.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_c, A_p, R, k, λ, T_{sr}, a, b, C_{sr}, C_f) = (2, 1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \text{EA}\left(d_c, d_p, n, h, k_1, k_2, k_3, \delta\right) \\
&\text{s.t.}\, 0.8 < d_c \le d_p \le 25 \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (δ_1, δ_2, A_c, A_p, R, k, λ, T_{sr}, a, b, C_{sr}, C_f) = (1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \text{EA}\left(d_c, d_p, n, h, k_1, k_2, k_3, \delta, \sigma\right) \\
&\text{s.t.}\, 0.8 < d_c \le d_p \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad 0.5 \le \sigma \le 4, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 5.2.3 shows the optimal solutions of two situations and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 5.2.2, when μ is unknown, d_c^* and design parameters are the same, but EA* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 5.2.2, when μ and σ are unknown, the width of the $\overline{\mathrm{X}}$ chart is smaller, the width of the S chart is larger, and EA* is smaller.

Table 5.2.3. The Optimum Solutions and In-Control Distribution of "Economic Statistic $\overline{\mathrm{X}}$ and S Charts" and "Shewhart-Type Economic $\overline{\mathrm{X}}$ and S Chart" with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Economic Statistical $\overline{\mathrm{X}}$ and S Control Charts													
situation	δ*	σ^*	d_c^*	d_p^*	n^*	h^*	$UCL_{\bar{X}}^*$ (k_1*)	$LCL_{\bar{X}}^*$ (k_1*)	UCL_S^* (k_2*)	LCL_S^* (k_3*)	α*	β*	*EA**
(1) μ is unknown and σ is known μ-T=δσ (σ=2)	1.032E-09	--	25	25	7	8	6.369 (3.185)	-6.369 (3.185)	3.997 (1.998)	0 (0)	0.00198	0.2	1068.349
(2) μ and σ are unknown and μ-T=δσ	1.016E-05	0.5	25	25	7	8	1.641 (3.282)	-1.641 (3.282)	0.978 (1.956)	0.002 (0.005)	0.00184	0.2	1051.170
Shewhart-Type Economic $\overline{\mathrm{X}}$ and S Control Charts													
situation	δ*	σ^*	d_c^*	d_p^*	n^*	h^*	$UCL_{\bar{X}}$ (k_1)	$LCL_{\bar{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	*EA**
(1) μ is unknown and σ is known μ-T=δσ (σ=2)	2.768E-16	--	25	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1090.573
(2) μ and σ are unknown and μ-T=δσ	1.001E-05	0.5	25	25	7	8	1.5 (3)	-1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1073.316

5.2.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 13 cost and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), (A_c,A_p) = ((100,20),(200,80),(300,300)), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1),a = (0.5,50,100), b = (0.1,1,5), and ($C_{s.r.}$,C_f) = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using an orthogonal array table $L_{27}(3^{13})$ (Table 5.2.4).

Table 5.2.4. Orthogonal Array $L_{27}(3^{13})$ for 27 Combinations of Input Parameters

No	δ	σ	δ_1	δ_2	R	A_c	A_p	λ	T_{sr}	a	b	C_{sr}	C_f
1	0	1	1	2	500	300	240	0.05	2	50	0.1	100	40
2	0	1	1	1.5	100	200	80	0.1	1	100	0.1	50	25
3	0	2	2.5	1.5	500	100	20	0.1	3	50	1	50	25
4	2	2	1	2	100	100	20	0.1	2	0.5	5	30	50
5	0	2.5	1.5	1	500	200	80	0.01	1	50	5	30	50
6	0	2	2.5	1	100	300	240	0.01	2	100	1	30	50
7	1	1	2.5	2	100	100	20	0.01	1	50	5	50	25
8	1	1	2.5	1	500	200	80	0.1	2	0.5	5	100	40
9	1	2.5	1	2	30	200	80	0.01	2	100	1	50	25
10	0	2	2.5	2	30	200	80	0.05	1	0.5	1	100	40
11	1	2	1.5	1	30	100	20	0.1	1	100	0.1	100	40
12	0	1	1	1	30	100	20	0.01	3	0.5	0.1	30	50
13	0	2.5	1.5	2	100	100	20	0.05	3	100	5	100	40
14	2	2.5	2.5	2	500	300	240	0.1	1	100	0.1	30	50
15	1	2	1.5	1.5	100	200	80	0.05	2	50	0.1	30	50
16	2	2.5	2.5	1.5	100	200	80	0.01	3	0.5	0.1	100	40
17	0	2.5	1.5	1.5	30	300	240	0.1	2	0.5	5	50	25
18	2	1	1.5	1	100	300	240	0.05	1	0.5	1	50	25
19	2	1	1.5	2	30	200	80	0.1	3	50	1	30	50
20	1	2	1.5	2	500	300	240	0.01	3	0.5	0.1	50	25
21	2	1	1.5	1.5	500	100	20	0.01	2	100	1	100	40
22	1	1	2.5	1.5	30	300	240	0.05	3	100	5	30	50
23	2	2	1	1	500	200	80	0.05	3	100	5	50	25
24	1	2.5	1	1.5	500	100	20	0.05	1	0.5	1	30	50
25	1	2.5	1	1	100	300	240	0.1	3	50	1	100	40
26	2	2	1	1.5	30	300	240	0.01	1	50	5	100	40
27	2	2.5	2.5	1	30	100	20	0.05	2	50	0.1	50	25

Table 5.2.5 shows the optimal solutions for these 27 combinations of input parameters.

Table 5.2.5. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

No	d_c^*	k_c^*	d_p^*	k_p^*	$n*$	$h*$	$k1*$	$k2*$	$k3*$	α^*	β^*	$EA*$
1	25.00	0.48	25.00	0.38	10	8.00	3.32	1.78	0.01	1.68E-03	0.20	9276.58
2	25.00	0.32	25.00	0.13	25	8.00	3.74	4.03	0.02	1.86E-04	0.20	1323.23
3	25.00	0.16	25.00	0.03	4	8.00	3.74	3.96	0.00	1.86E-04	0.20	6655.07
4	25.00	0.16	25.00	0.03	14	8.00	3.55	1.88	0.02	4.04E-04	0.20	1381.31
5	25.00	0.32	25.00	0.13	6	8.00	2.83	4.08	0.00	4.62E-03	0.20	18550.50
6	25.00	0.48	25.00	0.38	3	8.00	3.49	4.20	0.00	4.86E-04	0.20	3716.64
7	25.00	0.16	25.00	0.03	3	8.00	2.79	3.07	0.01	5.32E-03	0.20	3575.16
8	25.00	0.32	25.00	0.13	3	8.00	3.49	4.20	0.00	5.02E-04	0.20	6673.51
9	25.00	0.32	25.00	0.13	8	8.00	3.20	1.70	0.01	6.32E-03	0.20	1125.15
10	25.00	0.32	25.00	0.13	4	8.00	3.33	3.88	0.00	8.63E-04	0.20	600.85
11	25.00	0.16	25.00	0.03	10	8.00	3.90	3.66	0.01	9.55E-05	0.20	410.98
12	25.00	0.16	25.00	0.03	14	8.00	2.90	3.37	0.01	3.73E-03	0.20	1054.45
13	25.00	0.16	25.00	0.03	6	8.00	3.04	1.92	0.00	4.85E-03	0.20	1880.30
14	25.00	0.48	25.00	0.38	5	8.00	3.93	3.53	0.00	8.59E-05	0.20	10520.12
15	25.00	0.32	25.00	0.13	9	8.00	3.24	4.19	0.01	1.21E-03	0.20	1952.01
16	25.00	0.32	25.00	0.13	3	8.00	3.07	3.78	0.01	2.18E-03	0.20	3814.46
17	25.00	0.48	25.00	0.38	12	8.00	3.93	4.20	0.02	8.36E-05	0.20	441.85
18	25.00	0.48	25.00	0.38	6	8.00	2.83	3.96	0.00	4.62E-03	0.20	1916.65
19	25.00	0.32	25.00	0.13	14	8.00	3.93	4.20	0.06	8.52E-05	0.20	397.53
20	25.00	0.48	25.00	0.38	7	8.00	3.10	2.04	0.00	2.30E-03	0.20	18536.78
21	25.00	0.16	25.00	0.03	7	8.00	2.71	4.15	0.00	6.81E-03	0.20	17733.97
22	25.00	0.48	25.00	0.38	3	8.00	3.07	3.97	0.00	2.15E-03	0.20	562.95
23	25.00	0.32	25.00	0.13	12	8.00	2.62	3.77	0.03	8.73E-03	0.20	9705.89
24	25.00	0.16	25.00	0.03	14	8.00	2.74	1.71	0.02	6.42E-03	0.20	9648.07
25	25.00	0.48	25.00	0.38	23	8.00	3.95	2.51	0.11	7.68E-05	0.20	1464.25
26	25.00	0.48	25.00	0.38	14	8.00	2.74	1.71	0.02	6.49E-03	0.20	1139.13
27	25.00	0.16	25.00	0.03	3	8.00	3.49	4.14	0.00	4.86E-04	0.20	611.48

Table 5.2.6 shows the main effect of the optimal solutions and optimal values and it produces the following findings:

(i) σ and A_c are significant to average producer tolerance d_p*. When σ increases, average d_p* increase. When A_c increases, average d_p* increase then decrease.

(ii) δ_1 and λ are significant to average sample size n*. When δ_1 increases, average n* decreases. When λ increases, average n* increases.

(iii) λ is significant to average k_1*. When λ increases, average k_1* increases.

(iv) δ_1 and δ_2 are significant to average k_2*. When δ_1 increases, average k_2* increases. When δ_2 increases, average k_2* decreases.

(v) All input parameters are significant to average EA*. When δ, σ, R, or A_c increase, average EA* increases. When b increases, average EA* decreases. Whenδ_1, increases, average EA* increase first then decreases. δ_2, $T_{s.r.}$, or a increase, average EA* decrease first then increases.

(vi) All input parameters are not significant to average consumer tolerance d_c*. Average d_c* equals to 25 for all levels of input parameters.

Table 5.2.6 Main Effect of The Optimal Solutions and Optimal Values under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	1	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	2	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	3	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_p}$	1	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	2	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	3	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{n}$	1	9.33	9.44	14.89	8.89	9.11	8.33	7.22	9.56	8.56	9.56	9.11
	2	8.89	8.56	8.56	10.11	10.22	9.33	7.44	7.67	9.56	9.22	8.89
	3	8.67	8.89	3.44	7.89	7.56	9.22	12.22	9.67	8.78	8.11	8.89
	diff	0.67	0.89	**11.44**	2.22	2.67	1.00	**5.00**	2.00	1.00	1.44	0.22

	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{h}$	1	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	2	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	3	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.37	3.20	3.20	3.28	3.39	3.21	2.98	3.27	3.22	3.41	3.30
	2	3.28	3.30	3.28	3.22	3.30	3.27	3.08	3.38	3.34	3.32	3.27
	3	3.21	3.35	3.38	3.35	3.16	3.37	3.80	3.20	3.30	3.12	3.28
	diff	0.16	0.16	0.18	0.13	0.22	0.17	**0.81**	0.17	0.12	0.29	0.03
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.49	3.64	2.50	3.77	3.43	3.10	3.12	3.28	3.22	3.39	3.46
	2	3.01	3.26	3.60	3.52	3.28	3.76	3.26	3.38	3.29	3.36	3.43
	3	3.46	3.06	3.86	2.67	3.25	3.10	3.58	3.29	3.44	3.20	3.07
	diff	0.48	0.57	**1.36**	**1.10**	0.18	0.66	0.45	0.10	0.21	0.19	0.39
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.01	0.01	0.03	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01
	2	0.02	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.02	0.02	0.01
	3	0.02	0.02	0.00	0.01	0.01	0.02	0.03	0.01	0.01	0.01	0.02
	diff	0.01	0.01	0.03	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0019	0.0028	0.0038	0.0026	0.0023	0.0031	0.0042	0.0027	0.0023	0.0013	0.0021
	2	0.0027	0.0023	0.0027	0.0029	0.0021	0.0027	0.0034	0.0020	0.0022	0.0029	0.0031
	3	0.0033	0.0028	0.0014	0.0024	0.0035	0.0020	0.0002	0.0032	0.0033	0.0037	0.0026
	diff	0.0015	0.0005	0.0024	0.0004	0.0013	0.0011	0.0041	0.0012	0.0011	0.0024	0.0010
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	2	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	diff	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	4833.27	4723.78	4013.12	4900.48	704.93	4772.31	7694.03	4896.85	4896.43	5277.79	5309.29
	2	4883.21	4899.85	6868.95	4807.86	2336.00	4904.79	4017.20	4768.06	4846.86	4806.46	4876.81
	3	5246.73	5339.58	4081.14	5254.87	11922.28	5286.10	3251.98	5298.30	5219.92	4878.96	4777.11
	diff	**413.45**	**615.79**	**2855.84**	**447.01**	**11217.35**	**513.79**	**4442.05**	**530.24**	**373.06**	**471.32**	**532.17**

In Table 5.2.7, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation “+”, if the input parameter is significant and their relationship is linear and negative, we use notation “-“, and if the input parameter is significant and their relationship is quadratic, we use notation “q“; otherwise, we use notation “N”.

Table 5.2.7. The Significant Input Parameters of Each Design Parameter and EA under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Optimal design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{d_p}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{n}$	N	N	-	N	N	N	+	N	N	N	N
$\overline{h}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{k_1}$	N	N	N	N	N	N	+	N	N	N	N
$\overline{k_2}$	N	N	+	-	N	N	N	N	N	N	N
$\overline{k_3}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	-	q	q	-	q

5.3 Considering Different Consumer and Producer Loss Functions with a Larger Consumer Tolerance

5.3.1 Smaller Coefficients of Consumer Loss Functions and Larger Consumer Tolerance

In this section, the consumer loss function and the producer loss function are different. Most producers in this situation consider only the producer loss. However, when a producer does not consider consumer loss, the calculated cost is lower than the actual cost. This section assumes that (1) A_c and A_p are known, (2) d_c is higher than the d_p, and (3) k_c is smaller k_p. This means that a producer loss of deviation from the target increases faster than the consumer loss of deviation from the target. This section determines the design parameters of $\overline{X}$ and S charts, consumer tolerance, and producer tolerance simultaneously when the consumer tolerance is larger than the producer tolerance.

The consumer loss function and producer loss function are

$$L_c(X)=\begin{cases} k_c(X-T)^2 & if\ |X-T|\le d_c \\ A_c & if\ |X-T|>d_c \end{cases},$$

$$L_p(X)=\begin{cases} k_p(X-T)^2 & if\ |X-T|\le d_p \\ A_p & if\ |X-T|>d_p \end{cases},$$

where T is the target value, $d_c > d_p$, and $k_c < k_p$. (See Figure 5.3.1.1)

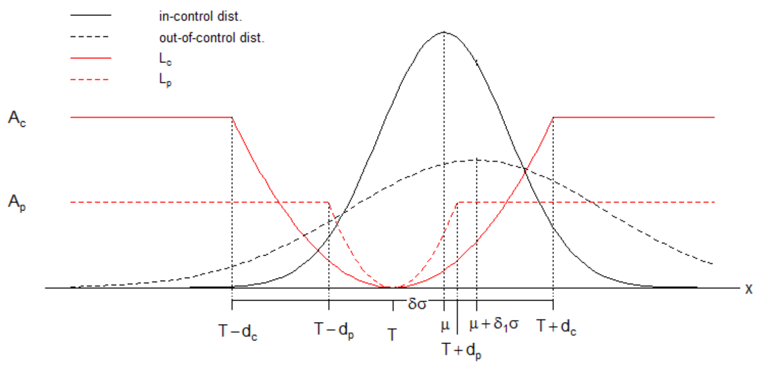

Figure 5.3.1.1. Consumer and Producer Loss Functions, In-Control and Out-Of-Control Distributions

When the product is within the producer specification limits, the producer loss is considered because it is higher than the consumer loss. When the product is out of the producer specification limits, the producer loss should be considered because the product will not ship to the consumer. The consumer loss function is not used in this situation. The expected cost per unit time considers only the producer loss function.

The expected cost per unit time is

$$EA = \frac{R\dfrac{L_I}{\lambda} + RL_O\left(\dfrac{h}{1-\beta} - \dfrac{h}{2} + \dfrac{\lambda h^2}{12}\right) + (a+bn)\left(\dfrac{1}{\lambda h} + \dfrac{1}{1-\beta}\right) + C_f\,\dfrac{\alpha}{\lambda h} + C_{s.r.}}{\dfrac{1}{\lambda} + h\left(\dfrac{1}{1-\beta} - \dfrac{1}{2} + \dfrac{\lambda h}{12}\right) + T_{s.r.}} \tag{5.3.1.1}$$

where $$L_I = \frac{IC + A_p - A_p\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}}\phi(z)dz + k_p\sigma^2\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}}(z+\delta)^2\phi(z)dz}{\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}}\phi(z)dz}$$

and

$$L_O = IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)}\phi(z)dz\right](A_p + L_I)$$
$$+ \frac{A_p}{d_p^2}\sigma^2\int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)}(\delta_2 z + \delta + \delta_1)^2\phi(z)dz$$

The remaining analysis, numerical analysis, and sensitivity analysis are identical to those in Section 4. If the producer knows that the consumer tolerance is larger than the producer tolerance, and the coefficient of the producer loss function is higher than consumer loss function, then the producer does not need to consider the consumer loss.

5.3.2 Larger Coefficient of Consumer Loss Function With Larger Consumer Tolerance

5.3.2.1 Derivation of Cost Models

This section assumes that (1) A_c and A_p are known, (2) d_c is higher than d_p, and (3) k_c is larger than k_p. This means that the producer loss of deviation from the target increases faster than the consumer loss of deviation from the target. This section determines the design parameters of $\overline{X}$ and S charts, consumer tolerance, and producer tolerance simultaneously.

The consumer loss function and the producer loss function are

$$L_c(X)=\begin{cases} k_c(X-T)^2 & if\ |X-T|\le d_c \\ A_c & if\ |X-T|>d_c \end{cases},$$

$$L_p(X)=\begin{cases} k_p(X-T)^2 & if\ |X-T|\le d_p \\ A_p & if\ |X-T|>d_p \end{cases},$$

where T is the target value, $d_c > d_p$, and $k_c > k_p$. (See Figure 5.3.1)

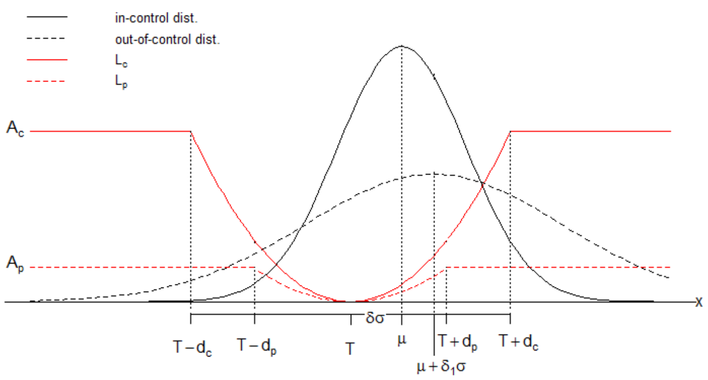

Figure 5.3.1. Consumer and Producer Loss Functions, In-Control and Out-Of-Control Distributions

When the product is within the producer specification limits, the consumer loss is considered because it is higher than the producer loss. If the producer implements a complete inspection plan in which all products are inspected before they ship to the consumer. If the product is out of the producer specifications and the expected cost per unit is rework cost plus the expected cost per unit when process is in-control.

$$L_I = IC + \left[1-P(T-d_p \le X \le T-d_p)\right](A_p + L_I) + \int_{T-d_p}^{T-d_p} k_c(x-T)^2 f_X(x)dx \tag{5.3.2.1a}$$

Writing L_I as a function of d_c and d_p,

$$L_I = \frac{IC + A_p - A_p \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz + k_c\sigma^2 \int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} (z+\delta)^2 \phi(z)dz}{\int_{-\delta-\frac{d_p}{\sigma}}^{-\delta+\frac{d_p}{\sigma}} \phi(z)dz} \tag{5.3.2.1b}$$

When the process is out-of-control, the expected cost of nonconforming product is rework cost plus the expected cost per unit when the process is in-control:

$$\begin{aligned} L_O &= IC + [1 - P(T - d_p \le X \le T - d_p)](A_p + L_I) \\ &+ \int_{T-d_p}^{T+d_p} k_c (x-T)^2 f_X(x)dx \end{aligned} \tag{5.3.2.2a}$$

Writing L_O as a function of d_c and d_p,

$$\begin{aligned} L_O &= IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} \phi(z)dz \right](A_p + L_I) \\ &+ k_c\sigma^2 \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} (\delta_2 z + \delta + \delta_1)^2 \phi(z)dz \end{aligned} \tag{5.3.2.2b}$$

With L_I and L_O, the expected cost per unit time is

$$EA = \frac{R\frac{L_I}{\lambda} + RL_O\left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h} + \frac{1}{1-\beta}\right) + C_f\frac{\alpha}{\lambda h} + C_{s.r.}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + T_{s.r.}} \tag{5.3.2.3}$$

The design parameters can be determined by minimizing the cost function (5.3.2.3). A subroutine "DEoptim" in R program is used to solve the object. The optimization model is expressed as follows:

$$\begin{aligned} &\min \mathrm{EA}(d_c, d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t. } d_{pL} \le d_p \le d_c \le d_{cU}, \\ &\quad n_L \le n \le n_U, \\ &\quad 0 < h \le h_U, \\ &\quad 0 < k_1 \le k_{1U}, \\ &\quad 0 < k_3 < k_2 \le k_{2\mathrm{U}}, \\ &\quad \alpha \le \alpha_U, \\ &\quad \beta \le \beta_U, \\ &\quad k_p = A_p/d_p^2 < k_c = A_c/d_c^2. \end{aligned}$$

5.3.2.2 An Example and Numerical Analysis

5.3.2.2.1 Example

In this section, we give an example to show the application of the economic statistical $\bar{X}$ and S control chart considering $d_c > d_p$. We compare the optimal solutions and the expected costs of three types of control charts: (1) Shewhart-type economic $\bar{X}$ and S control charts with design h, d_c, and d_p, (2) economic statistical $\bar{X}$ and S control charts with a given n, and (3) economic statistical $\bar{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_c=100, A_p=20, R=30, λ=0.01, $T_{s.r.}$=3, a=0.5, b=0.1, $C_{s.r.}$=35, and C_f=50.

(1) Shewhart-type economic $\bar{X}$ and S control charts with design h, d_c, and d_p

To construct the Shewhart-type economic $\bar{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\bar{X}} = \alpha_S$ = 0.0027), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time the optimal Shewhart-type economic $\bar{X}$ and S charts is

$$\begin{aligned}&\min EA(d_c, d_p, h)\\ &\text{s.t.}\ 0.8 < d_p \le d_c \le 25,\\ &\quad 0 < h \le 8\\ &\quad k_p = A_p/d_p^2 < k_c = A_c/d_c^2.\end{aligned}$$

The EA^* is 1126.619, h^* is 8, d_c^* is 25, and d_p^* is 17.308. Under the d_p^*, the rate of nonconforming product is 0, and the optimal Shewhart-type economic $\bar{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\bar{X}} = 3 \\ LCL_{\bar{X}} = -3 \end{matrix} \quad \text{and} \quad \begin{matrix} UCL_S = 1.735 \\ LCL_S = 0.371 \end{matrix}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 5.3.2.2 shows that no points fall outside the limits of Shewhart-type $\bar{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 5.3.2.3 shows that all products fall into the produce specification.

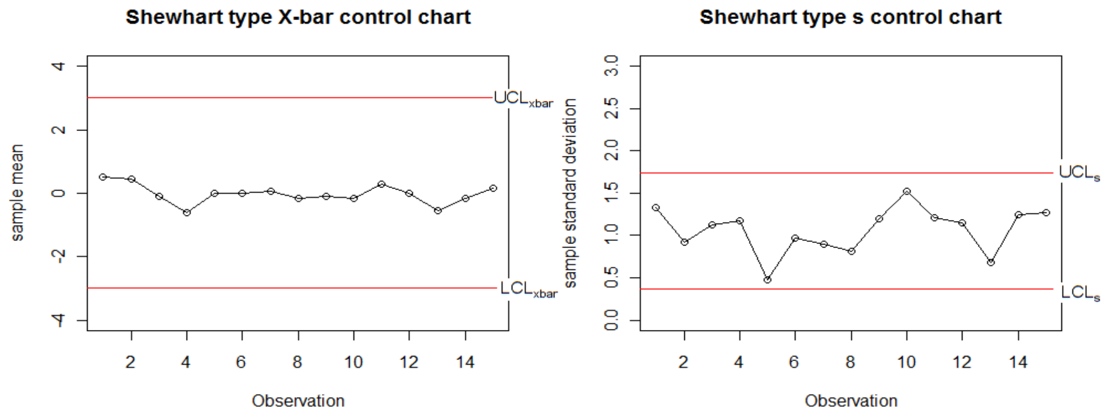

Figure 5.3.2.2. Shewhart-Type Economic $\bar{X}$ and S Control Charts with Consumer and Producer Tolerances

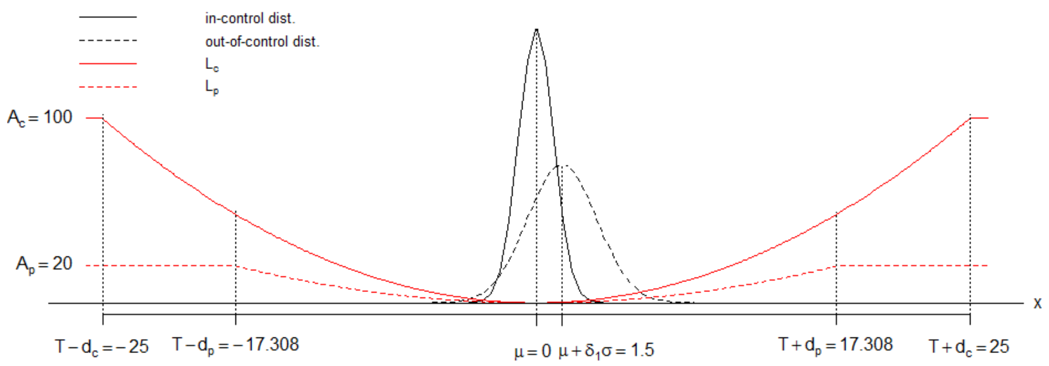

Figure 5.3.2.3. Optimal Consumer and Producer Loss Functions with In-Control and Out-Of-Control Distributions

(2) Economic statistical $\bar{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\bar{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$
\begin{aligned}
&\min \mathrm{EA}(d_c, d_p, h, k_1, k_2, k_3) \\
&s.t.\ \ 0.8 < d_p \le d_c \le 25, \\
&\qquad 0 < h \le 8, \\
&\qquad 0 < k_1 \le 4, \\
&\qquad 0 < k_3 < k_2 \le 4.2, \\
&\qquad \alpha \le 0.01, \\
&\qquad \beta \le 0.2 \\
&\qquad k_p = A_p/d_p^2 < k_c = A_c/d_c^2 .
\end{aligned}
$$

The optimal design parameters are $d_c* = 25$, $d_p* = 12.157$, $h* = 8$, $k_1* = 3.061$, $k_2* = 3.856$, $k_3* = 0.008$, $\alpha* = 0.00221$, and $\beta* = 0.2$. The $EA*$ is 1055.262. The optimal economic statistical $\bar{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 3.061 \quad \text{and} \quad UCL_S = 3.856$$
$$LCL_{\bar{X}} = -3.061 \qquad\quad LCL_S = 0.008$$

Figure 5.3.2.4 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts. Figure 5.3.2.5 shows that under this optimal consumer and producer tolerances, all in-control and out-of-control products are in the producer specification limits.

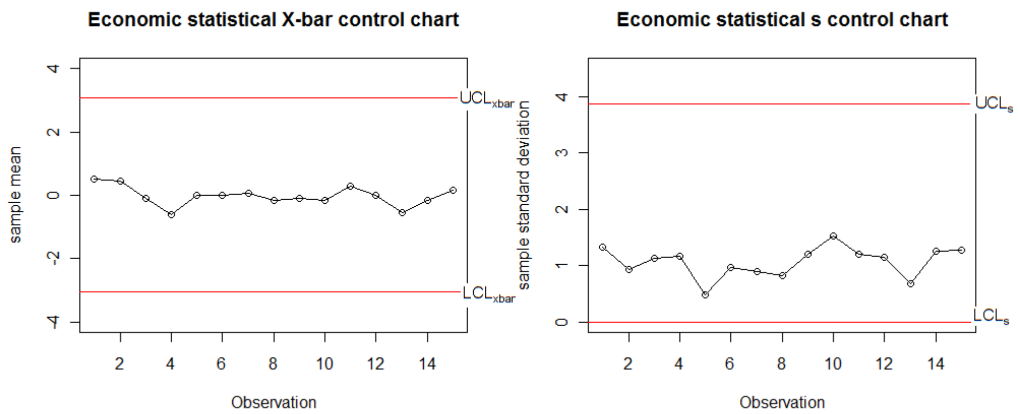

Figure 5.3.2.4. Optimal Economic Statistical $\bar{X}$ and S Control Chart with Consumer and Producer Tolerances and with a Given *n*

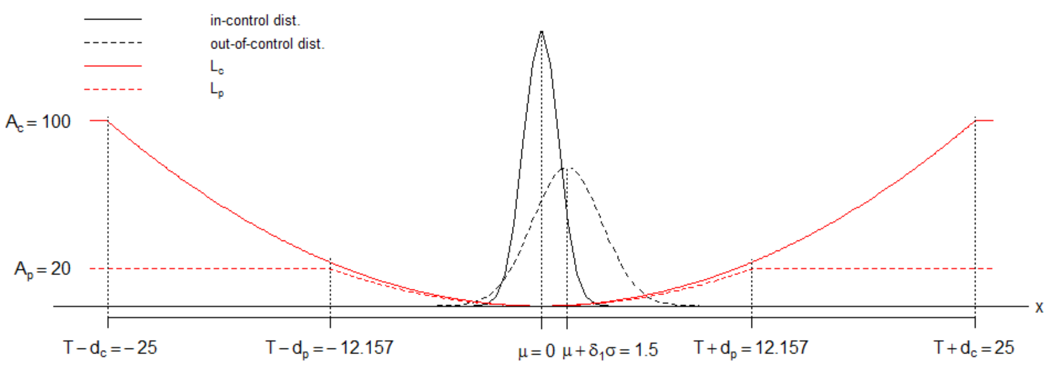

Figure 5.3.2.5. Optimal Consumer and Producer Loss Functions with In-Control and Out-Of-Control Distributions

(3) Economic statistical $\bar{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time the optimum economic statistical $\bar{X}$ and S charts is

$$
\begin{aligned}
&\min \ \mathrm{EA}\left(d_c, d_p, n, h, k_1, k_2, k_3\right) \\
&\text{s.t. } 0.8 < d_c \le d_p \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2 \\
&\quad k_p = A_p / d_p^2 < k_c = A_c / d_c^2 .
\end{aligned}
$$

The parameters are $d_c^* = 25$, $d_p^* = 24.737$, $n^* = 7$, $h^* = 8$, $k_1^* = 3.051$, $k_2^* = 2.046$, $k_3^* = 0.001$, $\alpha^* = 0.00255$, and $\beta^* = 0.2$. The EA^* is 1055.242. Under the d_p^*, the rate of nonconforming product of producer is 0, and the optimal economic statistical $\overline{X}$ and S charts are constructed as follows.

$$
\begin{aligned} UCL_{\overline{X}} &= 3.051 \\ LCL_{\overline{X}} &= -3.051 \end{aligned} \quad \text{and} \quad \begin{aligned} UCL_S &= 2.046 \\ LCL_S &= 0.001 \end{aligned}
$$

Finally, we compare the optimal solutions and expected cost of these 3 types design charts (Table 5.3.2.1).

Comparing with “Shewhart-type” and “economic statistical chart with given *n*” leads to following findings:

(i) If producer can design the chart, d_p^* should decrease, k_1^* and k_2^* should increase, and EA* will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart with given n, EA* could save about 6%. And the false alarm rate of economic statistical chart with a given n will decrease, but it’s true alarm rate will decrease.

(iii) The optimal consumer tolerances all equals to 25.

Comparing economic statistical chart with design *n* and with given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, d_p^* should be increase, n^* should decrease from 10 to 7, and k_2^* should decrease and EA* will reduce.

Because the expected cost per unit time cannot be saved a lot when we use economic statistical control chart with consumer tolerance, we advise that it is more convenience for using Shewhart type control chart.

Table 5.3.2.1. Comparison of Three Types Design Charts under the Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(1) Shewhart-type economic $\overline{X}$ and S control charts	25	17.038	0	10	8	3	1.735	0.371	0.00539	0.06502	1126.619
	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(2) Economic statistical $\overline{X}$ and S control charts with a given n	25	12.157	0	10	8	3.061	3.856	0.008	0.00221	0.20000	1055.262
	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	25	24.737	0	7	8	3.051	2.064	0.002	0.00255	0.20000	1055.242

5.3.2.2.2 The Effects of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are $A_c = 100$, $A_p = 20$, $T = 0$, $\delta_1 = 1.5$, $\delta_2 = 2$, $k = 4$, $R = 30$, $\lambda = 0.01$, $T_{s.r.} = 3$, $a = 0.5$, $b = 0.1$, $C_{s.r.} = 35$, and $C_f = 50$.

The results of these objects are shown in Table 5.3.2.2. Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_c*, n* and h* will not change, d_p* will increase, and the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 0.24%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_c*, n* and h* will not change, d_p* will decrease, and the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 0.29%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_c*, n*, and h* will not change, d_p* will increase, the width of $\overline{X}$ will increase and the width of S charts will decrease, *EA** will reduce about 0.1%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_c*, n*, and h* will not change, d_p* will decrease a little, the width of $\overline{X}$ will increase and the width of S charts will decrease, and *EA** will reduce about 0.1%.

Comparing with economic statistical control charts and Shewhart type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart type economic $\overline{X}$ and s charts'.

(vi) The α* of Economic Statistic $\overline{X}$ and s chart is smaller, but its β* is smaller, too.

According to the findings (i)-(iv), decreasing variance can reduce costs more than improving the mean can. If a producer decrease the variance, the producer should reduce the width of the $\overline{X}$ and S charts. In all situations, optimal consumer tolerance equals to 25s. According to findings (v)-(vi), the expected costs of the two charts are similar. Producers are advised to use the Shewhart-type economic $\overline{X}$ or S charts depending on the convenience of using the chart.

Table 5.3.2.2. The Optimum Solution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart-Type Economic $\overline{X}$ and S Chart" with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Economic statistical $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	25	22.213	7	8	6.238 (3.119)	-6.238 (3.119)	4.059 (2.030)	0.005 (0.003)	0.00220	0.2	1068.347
(2) δ=0 and σ=1	25	24.737	7	8	3.051 (3.051)	-3.051 (3.051)	2.064 (2.064)	0.002 (0.002)	0.00255	0.2	1055.242
(3) δ=1 and σ=2	25	22.355	7	8	7.191 (3.095)	-5.191 (3.095)	4.083 (2.041)	0.002 (0.001)	0.00231	0.2	1071.418
(4) δ=1 and σ=1	25	15.377	7	8	4.323 (3.323)	-2.323 (3.323)	1.940 (1.940)	0.001 (0.001)	0.00184	0.2	1056.006
Shewhart-Type Economic $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	25	18.670	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1090.573
(2) δ=0 and σ=1	25	23.322	7	8	3 (3)	-3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1077.418
(3) δ=1 and σ=2	25	12.487	7	8	7 (3)	-5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1093.475
(4) δ=1 and σ=1	25	15.198	7	8	4 (3)	-2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1078.143

5.3.2.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(d_c, d_p, n, h, k_1, k_2, k_3, \delta) \\
&s.t.\ 0.8 < d_p \le d_c \le 25 \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2 \\
&\quad k_p = A_p/d_p^2 < k_c = A_c/d_c^2.
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(d_c, d_p, n, h, k_1, k_2, k_3, \delta, \sigma) \\
&s.t.\ 0.8 < d_p \le d_c \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad 0 < \delta \le 2, \\
&\quad 0.5 \le \sigma \le 4, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2 \\
&\quad k_p = A_p/d_p^2 < k_c = A_c/d_c^2.
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 5.3.2.3 shows the optimal solutions of these objects and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 5.3.2.2, when μ is unknown, d_c^* and design parameters are the same, but EA* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 5.3.2.2, when μ and σ are unknown, the width of the $\bar{X}$ chart is smaller, and the width of the S chart is larger, and EA* is smaller.

Table 5.3.2.3. The Optimum Solutions and In-Control Distribution of "Economic Statistic $\bar{X}$ and S Charts" and "Shewhart-Type Economic $\bar{X}$ and S Chart" with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Economic Statistical $\bar{X}$ and S Control Charts													
situation	δ*	σ*	d_c^*	d_p^*	n^*	h^*	$UCL^*_{\bar{X}}$ (k_1*)	$LCL^*_{\bar{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) σ is known, δ is unknown (σ=2)	2.368E-11	--	25	25	7	8	6.234 (3.117)	6.234 (3.117)	4.061 (2.030)	0.009 (0.005)	0.00221	0.2	1068.347
(2) δ and σ are unknown	1.006E-05	0.5	25	25	7	8	1.674 (3.349)	1.674 (3.349)	0.965 (1.930)	0.002 (0.004)	0.00186	0.2	1051.170
Shewhart-Type Economic $\bar{X}$ and S Control Charts													
situation	δ*	σ*	d_c^*	d_p^*	n^*	h^*	$UCL_{\bar{X}}$ (k_1)	$LCL_{\bar{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) σ is known, δ is unknown (σ=2)	1.013E-16	--	25	25	7	8	6 (3)	6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1090.573
(2) δ and σ are unknown	1.001E-05	0.5	25	25	7	8	1.5 (3)	1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1073.316

5.3.2.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 12 costs and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), (A_c,A_p) = ((100,20),(200,80),(300,300)), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1),a = (0.5,50,100), b = (0.1,1,5), and ($C_{s.r.}$,C_f) = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using a same orthogonal array table $L_{27}(3^{13})$ in Table 5.2.4. Table 5.3.2.4 shows the optimal solutions for these 27 combinations of input parameters.

Table 5.3.2.4. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

No	d_c^*	k_c^*	d_p^*	k_p^*	$n*$	$h*$	$k1*$	$k2*$	$k3*$	α^*	β^*	$EA*$
1	25.00	0.48	24.12	0.41	11	8.00	3.90	1.74	0.00	9.17E-04	0.20	9276.58
2	25.00	0.32	22.23	0.16	25	8.00	3.74	3.69	0.17	1.86E-04	0.20	1323.23
3	25.00	0.16	17.07	0.07	4	8.00	3.74	3.56	0.02	1.95E-04	0.20	6655.07
4	25.00	0.16	11.41	0.15	14	8.00	3.67	1.86	0.03	2.65E-04	0.20	1381.31
5	25.00	0.32	17.75	0.25	6	8.00	2.83	3.97	0.00	4.62E-03	0.20	18550.50
6	25.00	0.48	22.73	0.46	3	8.00	3.49	3.50	0.00	5.02E-04	0.20	3716.64
7	25.00	0.16	22.28	0.04	3	8.00	2.83	2.92	0.03	5.55E-03	0.20	3575.16
8	25.00	0.32	22.62	0.16	3	8.00	3.49	3.14	0.01	5.77E-04	0.20	6673.51
9	25.00	0.32	20.64	0.19	9	8.00	3.13	1.77	0.00	3.30E-03	0.20	1125.22
10	25.00	0.32	21.62	0.17	4	8.00	3.33	3.92	0.00	8.68E-04	0.20	600.85
11	25.00	0.16	17.27	0.07	10	8.00	3.90	3.75	0.01	9.55E-05	0.20	410.98
12	25.00	0.16	17.45	0.07	14	8.00	2.90	3.20	0.01	3.73E-03	0.20	1054.45
13	25.00	0.16	12.15	0.14	6	8.00	2.96	1.95	0.00	4.92E-03	0.20	1880.30
14	25.00	0.48	22.67	0.47	5	8.00	3.91	3.91	0.00	9.12E-05	0.20	10520.12
15	25.00	0.32	17.56	0.26	9	8.00	3.24	4.17	0.00	1.21E-03	0.20	1952.01
16	25.00	0.32	19.73	0.21	3	8.00	3.07	3.78	0.00	2.17E-03	0.20	3814.46
17	25.00	0.48	23.38	0.44	12	8.00	3.93	4.04	0.04	8.36E-05	0.20	441.85
18	25.00	0.48	23.08	0.45	6	8.00	2.83	4.17	0.00	4.62E-03	0.20	1916.65
19	25.00	0.32	21.56	0.17	14	8.00	3.93	4.20	0.05	8.52E-05	0.20	397.53
20	25.00	0.48	22.55	0.47	8	8.00	3.06	2.23	0.00	2.26E-03	0.20	18536.79
21	25.00	0.16	19.22	0.05	7	8.00	2.71	3.78	0.04	6.81E-03	0.20	17733.97
22	25.00	0.48	24.41	0.40	4	8.00	3.74	3.87	0.00	1.86E-04	0.20	563.03
23	25.00	0.32	19.80	0.20	12	8.00	2.62	3.62	0.00	8.73E-03	0.20	9705.89
24	25.00	0.16	14.70	0.09	13	8.00	2.82	1.59	0.05	7.25E-03	0.20	9648.05

25	25.00	0.48	22.68	0.47	23	8.00	3.95	3.59	0.05	7.68E-05	0.20	1464.25
26	25.00	0.48	23.73	0.43	15	8.00	2.61	4.02	0.02	9.04E-03	0.20	1139.51
27	25.00	0.16	16.25	0.08	3	8.00	3.49	4.20	0.00	4.86E-04	0.20	611.48

Table 5.3.2.5 shows the main effects of the optimal solutions and optimal values for three input parameter levels and it produces the following findings:

(i) σ and (A_c,A_p) are significant to average producer tolerance d_p*. When σ increase, average d_p* increases then decreases. When (A_c,A_p) increase, average d_p* increases then increases.

(ii) δ_1 and λ are significant to average sample size *n**. When δ_1 increases, average *n** decreases. When λ increases, average *n** increases.

(iii) λ is significant to average k_1*. When λ increases, average k_1* increases.

(iv) δ_1 and δ_2 are significant to average k_2*. When δ_1 increases, average k_2* increases. When δ_2 increases, average k_2* decreases.

(v) All input parameters are significant to average *EA**. When δ, σ, R, or A_c increase, average *EA** increases. When b increases, average *EA** decreases. Whenδ_1, increases, average *EA** increase first then decreases. δ_2, $T_{s.r.}$, or a increase, average *EA** decreases first then increases.

(vi) All input parameters are not significant to average consumer tolerance d_c*. Average d_c* equals to 25 for all levels of input parameters.

Table 5.3.2.5. Main Effect of The Optimal Solutions and Optimal Values under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	1	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	2	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	3	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\overline{d_p}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	19.83	21.88	19.64	19.96	20.70	16.42	20.67	19.71	19.61	19.98	18.92
	2	20.52	19.31	19.39	20.22	19.32	20.39	19.30	19.77	20.33	20.37	20.81
	3	19.72	18.88	21.04	19.89	20.06	23.26	20.10	20.59	20.12	19.73	20.35
	diff	0.81	**3.00**	1.65	0.34	1.38	**6.84**	1.37	0.88	0.72	0.64	1.89

	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\bar{n}$	1	9.44	9.67	15.11	8.89	9.44	8.22	7.56	9.78	8.56	9.78	9.11
	2	9.11	8.78	8.67	10.22	10.22	9.44	7.56	7.89	9.78	9.22	9.11
	3	8.78	8.89	3.56	8.22	7.67	9.67	12.22	9.67	9.00	8.33	9.11
	diff	0.67	0.89	**11.56**	2.00	2.56	1.44	**4.67**	1.89	1.22	1.44	0.00
$\vec{h}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	2	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	3	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.43	3.34	3.26	3.28	3.44	3.22	2.96	3.33	3.23	3.47	3.39
	2	3.35	3.29	3.27	3.29	3.31	3.26	3.21	3.45	3.39	3.32	3.26
	3	3.20	3.34	3.45	3.41	3.23	3.49	3.81	3.20	3.36	3.19	3.33
	diff	0.22	0.05	0.19	0.13	0.21	0.27	**0.85**	0.25	0.16	0.28	0.13
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.28	3.41	2.79	3.68	3.66	2.98	3.24	3.33	3.10	3.41	3.36
	2	3.00	3.40	3.58	3.61	3.29	3.58	3.25	3.13	3.60	3.34	3.36
	3	3.73	3.20	3.64	2.72	3.06	3.45	3.53	3.55	3.32	3.27	3.30
	diff	0.72	0.21	**0.86**	**0.96**	0.60	0.60	0.29	0.42	0.49	0.14	0.07
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.03	0.03	0.04	0.01	0.02	0.02	0.01	0.02	0.02	0.02	0.02
	2	0.02	0.01	0.02	0.04	0.03	0.03	0.01	0.01	0.02	0.02	0.03
	3	0.02	0.02	0.01	0.01	0.01	0.01	0.04	0.03	0.02	0.01	0.02
	diff	0.01	0.02	0.03	0.03	0.02	0.01	0.03	0.02	0.01	0.01	0.01
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0018	0.0025	0.0037	0.0026	0.0020	0.0033	0.0042	0.0025	0.0024	0.0012	0.0020
	2	0.0023	0.0026	0.0027	0.0030	0.0022	0.0024	0.0032	0.0016	0.0025	0.0026	0.0028
	3	0.0036	0.0026	0.0012	0.0020	0.0035	0.0020	0.0002	0.0036	0.0028	0.0038	0.0028
	diff	0.0018	0.0001	0.0025	0.0010	0.0015	0.0013	0.0040	0.0020	0.0003	0.0025	0.0008
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	2	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	diff	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)

	1	4833.27	4723.79	4013.17	4900.48	704.99	4772.31	7694.08	4896.86	4896.43	5277.79	5309.29
	2	4883.22	4899.89	6868.95	4807.91	2336.00	4904.80	4017.20	4768.06	4846.90	4806.47	4876.81
	3	5246.77	5339.58	4081.15	5254.87	11922.27	5286.16	3251.98	5298.34	5219.93	4879.01	4777.16
	diff	**413.49**	**615.79**	**2855.79**	**446.96**	**11217.29**	**513.85**	**4442.10**	**530.27**	**373.03**	**471.32**	**532.14**

In Table 2.6, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation "+", if the input parameter is significant and their relationship is linear and negative, we use notation "-", and if the input parameter is significant and their relationship is quadratic, we use notation "q"; otherwise, we use notation "N".

Table 5.3.2.6. The Significant Input Parameters of Each Design Parameter and EA under the Cost Model with $k_c > k_p$, $d_c \leqq d_p$, and Specified A_c > Specified A_p

Design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{d_p}$	N	-	N	N	N	+	N	N	N	N	N
$\overline{n}$	N	N	-	N	N	N	+	N	N	N	N
$\overline{h}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{k_1}$	N	N	N	N	N	N	+	N	N	N	N
$\overline{k_2}$	N	N	+	-	N	N	N	N	N	N	N
$\overline{k_3}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	-	q	q	-	-

5.3.3 Equal Consumer Loss Function and Producer Loss Function Coefficients but With Larger Consumer Tolerance

5.3.3.1 Derivation of Cost Models

This section assumes that (1) A_c and A_p are known, (2) the consumer tolerance is higher than the producer tolerance, and (3) the consumer loss function coefficient is equal to that of the producer loss function. This section determines the design parameters of $\overline{X}$ and S charts and the producer tolerance simultaneously. After the producer tolerance is determined, the consumer tolerance can be calculated.

The consumer' loss function and producer loss function are

$$L_c(X)=\begin{cases} k_c(X-T)^2 & \text{if } |X-T|\le d_c \\ A_c & \text{if } |X-T|> d_c \end{cases},$$

$$L_p(X)=\begin{cases} k_p(X-T)^2 & \text{if } |X-T|\le d_p \\ A_p & \text{if } |X-T|> d_p \end{cases},$$

where $d_c > d_p$, and $k_c = k_p$. The value of d_c can be calculated such that $A_c/d_c^2 = A_p/d_p^2$.

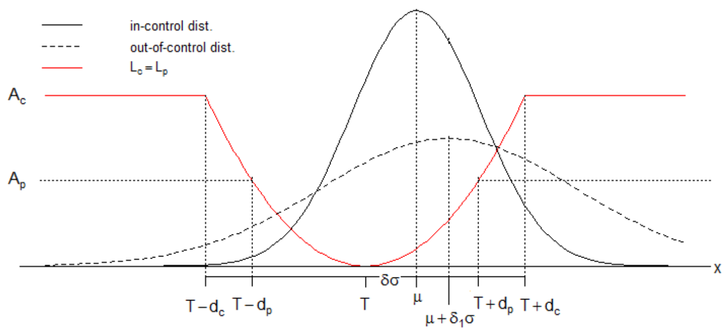

Figure 5.3.3.1. Consumer and Producer Loss Functions, In-Control and Out-Of-Control Distributions

If the producer implements a complete inspection plan in which all products are inspected before they ship to the consumer, then the expected cost of nonconforming product is rework cost plus the expected cost per unit when the process is in-control.

$$L_I = IC + [1-P(T-d_p \le X \le T+d_p)](A_p + L_I) + \int_{T-d_p}^{T-d_p} k_c(x-T)^2 f_X(x)dx \tag{5.3.3.1a}$$

Writing L_I as a function of d_p,

$$L_I = \frac{IC + A_p - A_p \int_{-\delta - \frac{d_p}{\sigma}}^{-\delta + \frac{d_p}{\sigma}} \phi(z)dz + k_c \sigma^2 \int_{-\delta - \frac{d_p}{\sigma}}^{-\delta + \frac{d_p}{\sigma}} (z+\delta)^2 \phi(z)dz}{\int_{-\delta - \frac{d_p}{\sigma}}^{-\delta + \frac{d_p}{\sigma}} \phi(z)dz} \tag{5.3.3.1b}$$

When the process is out-of-control, the expected cost of nonconforming product is rework cost plus the expected cost per unit when process is in-control.

$$\begin{aligned} L_O &= IC + [1 - P(T - d_p \le X \le T - d_p)](A_p + L_I) \\ &+ \int_{T-d_p}^{T+d_p} k_c (x-T)^2 f_{X^*}(x)dx \end{aligned} \tag{5.3.3.2a}$$

Writing L_O as a function of d_p we have,

$$\begin{aligned} L_O &= IC + \left[1 - \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} \phi(z)dz \right](A_p + L_I) \\ &+ k_c \sigma^2 \int_{\frac{1}{\delta_2}\left(-\delta-\delta_1-\frac{d_p}{\sigma}\right)}^{\frac{1}{\delta_2}\left(-\delta-\delta_1+\frac{d_p}{\sigma}\right)} (\delta_2 z + \delta + \delta_1)^2 \phi(z)dz \end{aligned} \tag{5.3.3.2b}$$

The expected cost per unit time is

$$EA = \frac{R\frac{L_I}{\lambda} + RL_O\left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right) + (a+bn)\left(\frac{1}{\lambda h} + \frac{1}{1-\beta}\right) + C_f \frac{\alpha}{\lambda h} + C_{s.r.}}{\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + T_{s.r.}} \tag{5.3.3.3}$$

The design parameters can be determined by minimizing the cost function (5.3.3.3). A subroutine "DEoptim" in R program is used to solve the object. The optimization model is expressed as follows:

$$\begin{aligned} &\min \mathrm{EA}(d_p, n, h, k_1, k_2, k_3) \\ &\text{s.t. } d_{pL} \le d_p \le d_{pU}, \\ &\quad n_L \le n \le n_U, \\ &\quad 0 < h \le h_U, \\ &\quad 0 < k_1 \le k_{1U}, \\ &\quad 0 < k_3 < k_2 \le k_{2U}, \\ &\quad \alpha \le \alpha_U, \\ &\quad \beta \le \beta_U. \end{aligned}$$

5.3.3.2 An Example and Numerical Analysis

5.3.3.2.1 Example

In this section, we give an example to show the application of the economic statistical $\bar{X}$ and S control chart considering $k_c = k_p$ and $d_c > d_p$. We compare the optimal solutions and the expected costs of three types of $\bar{X}$ and S control charts: (1) Shewhart-type economic $\bar{X}$ and S control charts with design h and d_p, (2) economic statistical $\bar{X}$ and S control charts with a given n, and (3) economic statistical $\bar{X}$ and S control charts with all design parameters. A subroutine "DEoptim" in R program is used to determine the optimal solutions in the optimization models.

The data which we use in this section is the same as 2.2.1, and the input parameters are set by δ_1=1.5, δ_2=2, A_c = 100,A_p=20, R=30, λ=0.01, $T_{s.r.}$=3, a=0.5, b=0.1, $C_{s.r.}$=35, and C_f=50.

(1) Shewhart-type economic $\bar{X}$ and S control charts with design h and d_p

To construct the Shewhart-type economic $\bar{X}$ and S charts when n = 10 and α = 0.00539 ($\alpha_{\bar{X}} = \alpha_S$ = 0.0027), we calculated that k_1 = 3, k_2 = 1.735, k_3 = 0.371, and β = 0.06502. The expected cost per unit time of the optimal Shewhart-type economic $\bar{X}$ and S charts is

$$\begin{aligned} &\min EA(d_p, h) \\ &\text{s.t.} 0.8 < d_p \le 25, \\ &0 < h \le 8. \end{aligned}$$

The EA^* is 1123.083, h^* is 8, and d_p^* is 25. Under the d_p^*, the rate of nonconforming product is 0, and the optimal Shewhart-type economic $\bar{X}$ and S charts are constructed as follows.

$$\begin{matrix} UCL_{\bar{X}} = 3 & \text{and} & UCL_S = 1.735 \\ LCL_{\bar{X}} = -3 & & LCL_S = 0.371 \end{matrix}$$

Plotting the data in Shewhart-type control charts shows whether they are in-control. Figure 5.3.3.2 shows that no points fall outside the limits of Shewhart-type $\bar{X}$ and S control charts, thus indicating that these charts can be used to monitor the future process. Figure 5.3.3.3 shows that all products fall into the producer specification.

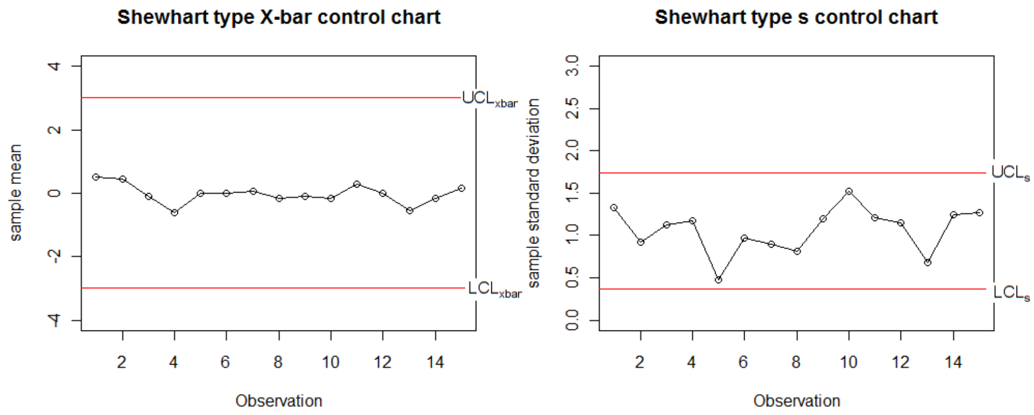

Figure 5.3.3.2. Shewhart-Type Economic $\bar{X}$ and S Control Chart with Consumer and Producer Tolerances

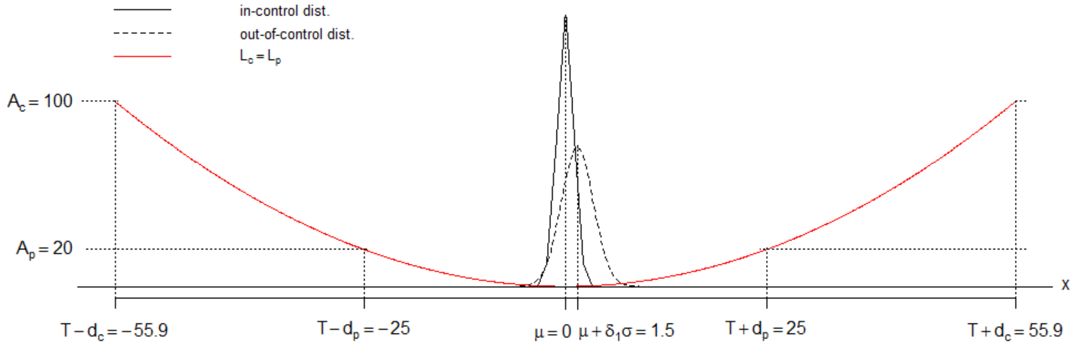

Figure 5.3.3.3. Optimal Consumer and Producer Loss Functions with In-Control and Out-Of-Control Distributions

(2) Economic statistical $\bar{X}$ and S control charts with a given n

The design parameters are determined with a given n by minimizing the cost function to construct the economic statistical $\bar{X}$ and S control charts. The expected cost per unit time of the optimal economic statistical $\bar{X}$ and S charts is

$$
\begin{aligned}
&\min \ \mathrm{EA}(d_p, h, k_1, k_2, k_3) \\
&s.t. \ \ 0.8 < d_p \le 25, \\
&\quad\ \ 0 < h \le 8, \\
&\quad\ \ 0 < k_1 \le 4, \\
&\quad\ \ 0 < k_3 < k_2 \le 4.2, \\
&\quad\ \ \alpha \le 0.01, \\
&\quad\ \ \beta \le 0.2.
\end{aligned}
$$

The optimal design parameters are $d_p^* = 25$, $h^* = 8$, $k_1^* = 3.075$, $k_2^* = 3.100$, $k_3^* = 0.018$, $\alpha^* = 0.00210$, and $\beta^* = 0.2$. The EA^* is 1051.767. The optimal economic statistical $\bar{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 3.075 \quad \text{and} \quad UCL_S = 3.100$$
$$LCL_{\bar{X}} = -3.075 \qquad\qquad LCL_S = 0.018$$

Figure 5.3.3.4 shows the optimal economic statistical $\bar{X}$ and S control charts. No points fall outside the limits of the optimal charts. Because the optimal producer tolerance is the same as the previous, all in-control and out-of-control products are in the producer specification limits.

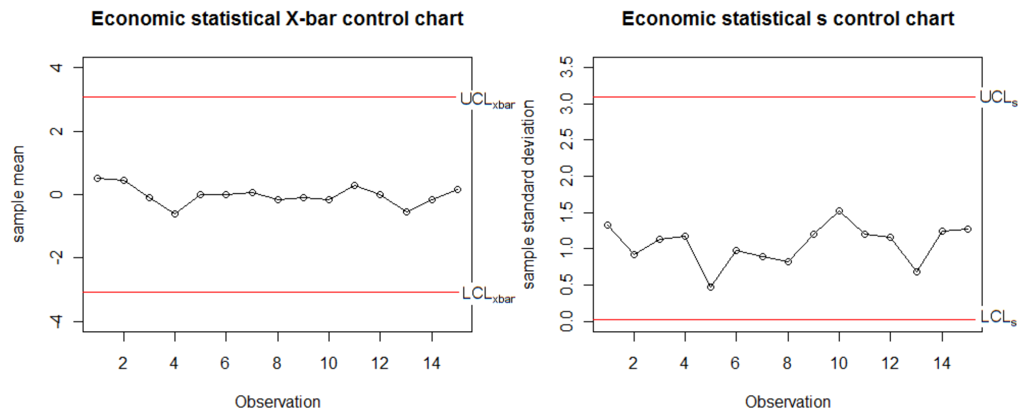

Figure 5.3.3.4. Optimal Economic Statistical $\bar{X}$ and S Control Charts with Consumer and Producer Tolerances and with a Given *n*

(3) Economic statistical $\bar{X}$ and S control charts with all design parameters

Assuming that all design parameters can be determined by minimizing the cost function, the expected cost per unit time of the optimum economic statistical $\bar{X}$ and S charts is

$$\begin{aligned}
&\min \text{EA}(d_p, n, h, k_1, k_2, k_3) \\
&\text{s.t. } 0.8 < d_p \le 25, \\
&\quad 2 \le n \le 25, \\
&\quad 0 < h \le 8, \\
&\quad 0 < k_1 \le 4, \\
&\quad 0 < k_3 < k_2 \le 4.2, \\
&\quad \alpha \le 0.01, \\
&\quad \beta \le 0.2.
\end{aligned}$$

The parameters are $d_p* = 25$, $n* = 7$, $h* = 8$, $k_1* = 3.265$, $k_2* = 1.963$, $k_3* = 0.002$, $\alpha* = 0.000184$, and $\beta* = 0.2$. The $EA*$ is 1051.743. Under the d_p*, the rate of nonconforming product is 0, and the optimal economic statistical $\bar{X}$ and S charts are constructed as follows.

$$UCL_{\bar{X}} = 3.265 \quad \text{and} \quad UCL_S = 1.963$$
$$LCL_{\bar{X}} = -3.265 \qquad\qquad LCL_S = 0.002$$

Finally, we compare the optimal solutions and expected cost of these 3 types design charts (Table 5.3.3.1).

Comparing with “Shewhart-type” and “economic statistical chart with a given *n*” leads to following findings:

(i) If producer can design the chart, k_1* and k_2* should increase and EA* will reduce.

(ii) Using Economic statistical $\overline{X}$ and s chart without design n, EA* could save about 6%. And the false alarm rate of economic statistical chart without design n will decrease, but it’s true alarm rate will decrease.

(iii) The optimal producer tolerance all equal to 25 and consumer tolerance equaling to 55.0917 can be calculated.

Comparing economic statistical chart with design *n* and with given *n* leads to following findings:

(iv) If producer can decide all design parameter of control chart, d_p* should be increase, *n** should decrease from 10 to 7, k_1* should increase and k_2* should decrease and EA* will reduce.

Because the expected cost per unit time cannot be saved a lot when we use economic statistical control chart with consumer tolerance, we advise that it is more convenience for using Shewhart type control chart and let consumer and producer tolerances equal to 25.

Table 5.3.3.1. Comparison of Three Types Design Charts under the Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Specified A_p

	d_c*	d_p*	p_p	n	h*	k_1	k_2	k_3	α	β	EA*
(1) Shewhart-type economic $\overline{X}$ and S control charts	55.902	25	0	10	8	3	1.735	0.371	0.00539	0.06502	1123.083
	d_c*	d_p*	p_p	n	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(2) Economic statistical $\overline{X}$ and S control charts with a given *n*	55.902	25	0	10	8	3.075	3.100	0.018	0.00210	0.20000	1051.767
	d_c*	d_p*	p_p	n*	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(3) Economic statistical $\overline{X}$ and S control charts with all design parameters	55.902	25	0	7	8	3.265	1.963	0.002	0.00185	0.20000	1051.743

5.3.3.2.2 The Effects of Optimal Design Parameters under Different Combination δ and σ for a Given In-control Distribution

This section sets the process mean and variance in different combinations to show the manner in which the process mean and variance affect the design parameters and the expected cost. Furthermore, it compares these optimal economic statistical control charts with Shewhart-type economic control charts, which fix the false alarm rate of each chart under 0.0027. Other input parameters of the cost function are A_c = 100, A_p = 20, T = 0, δ_1 = 1.5, δ_2 = 2, k = 4, R = 30, λ = 0.01, $T_{s.r.}$ = 3, a = 0.5, b = 0.1, $C_{s.r.}$ = 35, and C_f = 50.

The results of these objects are shown in Table 5.3.3.2. Comparing the optimal solutions of economic statistical $\overline{X}$ and S charts under different combinations of process mean and variance leads to the following findings:

(i) Under δ equals to 0, when σ decreases from 2 to 1, d_p*, *n** and *h** will not change, and the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 1.3%.

(ii) Under δ equals to 1, when σ decreases from 2 to 1, d_p*, *n** and *h** will not change, the width of $\overline{X}$ and S charts will be smaller, and *EA** will reduce about 1.7%.

(iii) Under σ equals to 1, when σ increases from 0 to 1, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, *EA** will reduce about 0.3%.

(iv) Under σ equals to 2, when σ increases from 0 to 1, d_p*, *n**, *h** will not change, the width of $\overline{X}$ will increase and the width of S charts will decrease, and *EA** will reduce about 0.1%.

Comparing with economic statistical control charts and Shewhart type economic control charts base on same combination of process mean and variance leads to following findings:

(v) *EA** of economic statistical $\overline{X}$ and S charts are a little higher than Shewhart type economic $\overline{X}$ and s charts'.

(vi) The α* of Economic Statistic $\overline{X}$ and s chart is smaller, but its β* is smaller, too.

According to the findings (i)-(iv), decreasing variance can reduce costs more than improving the mean can. If a producer improves the variance, the producer should reduce the width of the $\overline{X}$ and S charts. In all situations, optimal producer tolerance equals to 25 and all products are in the producer specification limits. According to findings (v)-(vi), the expected costs of the two charts are similar.

Producers are advised to use the Shewhart-type economic $\overline{X}$ and S charts depending on the convenience of using the chart.

Table 5.3.3.2. The Optimum Solution of "Economic Statistic $\overline{X}$ and S Charts" and "Shewhart Type Economic $\overline{X}$ and S Chart" with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Specified A_p

Economic Statistical $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL^*_{\overline{X}}$ (k_1*)	$LCL^*_{\overline{X}}$ (k_1*)	UCL^*_S (k_2*)	LCL^*_S (k_3*)	α*	β*	EA*
(1) δ=0 and σ=2	55.902	25	7	8	6.885 (3.443)	6.885 (3.443)	3.790 (1.895)	0.003 (0.001)	0.00204	0.2	1054.366
(2) δ=0 and σ=1	55.902	25	7	8	3.265 (3.265)	3.265 (3.265)	1.963 (1.963)	0.002 (0.002)	0.00185	0.2	1051.743
(3) δ=1 and σ=2	55.902	25	7	8	7.577 (3.288)	5.577 (3.288)	3.907 (1.954)	0.003 (0.002)	0.00184	0.2	1054.979
(4) δ=1 and σ=1	55.902	25	7	8	4.431 (3.431)	2.431 (3.431)	2.018 (2.018)	0 (0)	0.00078	0.2	1051.900
Shewhart-Type Economic $\overline{X}$ and S Control Charts											
	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA*
(1) δ=0 and σ=2	55.902	25	7	8	6 (3)	6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1076.541
(2) δ=0 and σ=1	55.902	25	7	8	3 (3)	3 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1073.910
(3) δ=1 and σ=2	55.902	25	7	8	7 (3)	5 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1077.121
(4) δ=1 and σ=1	55.902	25	7	8	4 (3)	2 (3)	1.903 (1.903)	0.266 (0.266)	0.00539	0.16024	1074.055

5.3.3.2.3 Determine Optimal in Control Distribution with Minimum Expected Cost Per Unit Time.

This section determines the optimal solutions for 2 situations.

Situation (1): σ is known, δ is unknown, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (σ, δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (2, 1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(d_p, n, h, k_1, k_2, k_3, \delta)\\
&s.t.\ 0.8 < d_p \le 25\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}$$

Situation (2): δ, σ, and the optimal design parameters of $\overline{X}$ and S charts are unknown. Given (δ_1, δ_2, A_c, A_p, R, k, λ, $T_{s.r.}$, a, b, C_{sr}, C_f) = (1.5, 2, 100, 20, 30, 4, 0.01, 3, 0.5, 0.1, 35, 50).

$$\begin{aligned}
&\min \mathrm{EA}(d_p, n, h, k_1, k_2, k_3, \delta, \sigma)\\
&s.t.\ 0.8 < d_p \le 25,\\
&\quad 2 \le n \le 25,\\
&\quad 0 < h \le 8,\\
&\quad 0 < k_1 \le 4,\\
&\quad 0 < k_3 < k_2 \le 4.2,\\
&\quad 0 < \delta \le 2,\\
&\quad 0.5 \le \sigma \le 4,\\
&\quad \alpha \le 0.01,\\
&\quad \beta \le 0.2.
\end{aligned}$$

To determine the optimal solutions in above models, we use a subroutine "DEoptim" in R program.

Table 5.3.3.3 shows the optimal solutions of these objects and leads to the following findings:

(i) In situation (1), the δ* is approximately 0. This means that if a producer can design a process mean, it should choose a mean as close to the target as possible.

(ii) In situation (2), δ* is approximately 0 and σ* is 0.5. This means that if a producer can design the mean and variance, μ* should be as close to the target as possible, and σ* should be small.

(iii) Compare situation (1) to (1) in Table 5.3.3.2, when μ is unknown, d_p^* and design parameters are the same, but EA^* is smaller for μ = T.

(iv) Compare situation (2) to (1) in Table 5.3.3.2, when μ and σ are unknown, the width of the $\overline{X}$ chart is smaller, and the width of the S chart is larger, and EA^* is smaller.

Table 5.3.3.3. The Optimum Solutions and In-Control Distribution of "Economic Statistic $\overline{X}$ Aand S Charts" and "Shewhart-Type Economic $\overline{X}$ and S Chart" with $k_p = k_c$, $d_c > d_p$, and Specified $A_c >$ Specified A_p

Economic statistical $\overline{X}$ and S control charts												
situation	δ*	σ*	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}^*$ (k_1*)	$LCL_{\overline{X}}^*$ (k_1*)	UCL_S^* (k_2*)	LCL_S^* (k_3*)	α^*	β^*	EA^*
(1) σ is known, δ is unknown (σ=2)	1.295E-12	--	55.902	25	7	8	6.577 (3.289)	-6.577 (3.289)	3.907 (1.954)	0.003 (0.001)	0.00184	0.2	1054.365
(2) δ and σ are unknown	1.014E-05	0.5	55.902	25	7	8	1.705 (3.411)	-1.705 (3.411)	0.953 (1.907)	0.002 (0.004)	0.00196	0.2	1050.933
Shewhart type Economic $\overline{X}$ and S control charts												
situation	δ*	σ*	d_c^*	d_p^*	n^*	h^*	$UCL_{\overline{X}}$ (k_1)	$LCL_{\overline{X}}$ (k_1)	UCL_S (k_2)	LCL_S (k_3)	α	β	EA^*
(1) σ is known, δ is unknown (σ=2)	4.964E-14	--	55.902	25	7	8	6 (3)	-6 (3)	3.806 (1.903)	0.532 (0.266)	0.00539	0.16024	1090.573
(2) δ and σ are unknown	1.001E-05	0.5	55.902	25	7	8	1.5 (3)	-1.5 (3)	0.9515 (1.903)	0.133 (0.266)	0.00539	0.16024	1073.316

5.3.3.3 Sensitivity Analysis

The economic cost model without tolerance requires the user to specify 12 cost and process parameters. Consider the levels of these parameters to be: δ = (1,1.5,2), σ = (1,2,2.5), δ_1 = (1,1.5,2.5), δ_2 = (1,1.5,2), R = (30,100,500), (A_c,A_p) = ((100,20),(200,80),(300,300)), λ = (0.01,0.05,0.1), $T_{s.r.}$ = (3,2,1),a = (0.5,50,100), b = (0.1,1,5), and ($C_{s.r.}$,C_f) = ((35,50),(50,25),(100,40)). We adopt 27 combinations of these parameters by using a same orthogonal array table $L_{27}(3^{13})$ (Table 5.2.4). Table 5.3.3.4 shows the optimal solutions for these 27 combinations of input parameters and

Table 5.3.3.4. The Optimal Solutions for 27 Combinations of Input Parmaters under the Cost Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Specified A_p

No	d_c^*	k_c^*	d_p^*	k_p^*	n^*	h^*	k_1^*	k_2^*	k_3^*	α^*	β^*	EA^*
1	27.95	0.38	25.00	0.38	10	8.00	3.41	1.77	0.02	1.58E-03	0.20	9242.42
2	39.53	0.13	25.00	0.13	25	8.00	3.74	4.04	0.02	1.86E-04	0.20	1313.75
3	55.90	0.03	25.00	0.03	4	8.00	3.74	4.07	0.00	1.86E-04	0.20	6398.02
4	55.90	0.03	25.00	0.03	17	8.00	3.73	1.93	0.03	1.89E-04	0.20	1305.68
5	39.53	0.13	25.00	0.13	6	8.00	2.83	3.99	0.00	4.62E-03	0.20	18069.97
6	27.95	0.38	25.00	0.38	3	8.00	3.49	4.12	0.00	4.86E-04	0.20	3679.91
7	55.90	0.03	25.00	0.03	3	8.00	2.82	2.96	0.00	4.93E-03	0.20	3558.16
8	39.53	0.13	25.00	0.13	3	8.00	3.49	4.01	0.00	4.86E-04	0.20	6529.26
9	39.53	0.13	25.00	0.13	8	8.00	3.25	1.70	0.00	6.47E-03	0.20	1089.63
10	39.53	0.13	25.00	0.13	3	8.00	2.73	3.44	0.01	6.50E-03	0.20	573.55
11	55.90	0.03	25.00	0.03	10	8.00	3.90	4.03	0.01	9.55E-05	0.20	397.05
12	55.90	0.03	25.00	0.03	14	8.00	2.90	3.16	0.04	3.73E-03	0.20	1051.61
13	55.90	0.03	25.00	0.03	6	8.00	3.10	1.89	0.00	5.00E-03	0.20	1815.59
14	27.95	0.38	25.00	0.38	5	8.00	3.91	4.16	0.00	9.25E-05	0.20	9731.75
15	39.53	0.13	25.00	0.13	9	8.00	3.24	4.19	0.01	1.21E-03	0.20	1874.01
16	39.53	0.13	25.00	0.13	3	8.00	3.07	3.59	0.00	2.13E-03	0.20	3627.34
17	27.95	0.38	25.00	0.38	12	8.00	3.93	4.16	0.02	8.36E-05	0.20	430.81
18	27.95	0.38	25.00	0.38	6	8.00	2.83	4.17	0.00	4.62E-03	0.20	1902.74
19	39.53	0.13	25.00	0.13	14	8.00	3.93	4.13	0.01	8.52E-05	0.20	387.27
20	27.95	0.38	25.00	0.38	7	8.00	3.24	1.97	0.00	1.87E-03	0.20	18331.31
21	55.90	0.03	25.00	0.03	7	8.00	2.71	4.18	0.00	6.81E-03	0.20	17654.08
22	27.95	0.38	25.00	0.38	3	8.00	3.07	3.74	0.00	2.14E-03	0.20	558.59
23	39.53	0.13	25.00	0.13	12	8.00	2.62	3.30	0.02	8.73E-03	0.20	9271.83

24	55.90	0.03	25.00	0.03	13	8.00	2.96	1.53	0.00	8.08E-03	0.20	9315.35
25	27.95	0.38	25.00	0.38	23	8.00	3.95	3.90	0.08	7.68E-05	0.20	1425.03
26	27.95	0.38	25.00	0.38	13	8.00	2.98	1.53	0.00	8.32E-03	0.20	1125.80
27	55.90	0.03	25.00	0.03	3	8.00	3.49	4.20	0.00	4.95E-04	0.20	560.52

Table 5.3.3.5 shows the main effects of the optimal solutions and optimal values for three input parameter levels and it produces the following findings:

(i) δ_1 and λ are significant to average sample size n^*. When δ_1 increases, average n^* decreases. When λ increases, average n^* increases.

(ii) λ is significant to average k_1^*. When λ increases, average k_1^* increases.

(iii) δ_1 and δ_2 are significant to average k_2^*. When δ_1 increases, average k_2^* increases. When δ_2 increases, average k_2^* decreases.

(iv) All input parameters are significant to average EA^*. When δ, σ, R, or (A_c,A_p) increase, average EA^* increase. When δ_1, increase, average EA^* increases first then decreases. δ_2, $T_{s.r.}$, a, or b increases, average EA^* decrease first then increases.

(5) All input parameters are not significant to average producer tolerance d_p^*. Average d_p^* equals to 25 for all levels of input parameters. Since d_c^* is calculated by d_p, A_c, and A_p, the combination of (A_c,A_p) is significant to d_c^*.

Table 5.3.3.5. Main Effect of the Optimal Solutions and Optimal Values under the Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Specified A_p

	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	1	41.13	41.13	41.13	41.13	41.13	55.90	41.13	41.13	41.13	41.13	41.13
	2	41.13	41.13	41.13	41.13	41.13	39.53	41.13	41.13	41.13	41.13	41.13
	3	41.13	41.13	41.13	41.13	41.13	27.95	41.13	41.13	41.13	41.13	41.13
	diff	0.00	0.00	0.00	0.00	0.00	27.95	0.00	0.00	0.00	0.00	0.00
$\overline{d_p}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	2	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	3	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\overline{n}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	9.22	9.44	15.00	8.89	8.89	8.56	7.11	9.56	8.67	9.56	9.33
	2	8.78	8.67	8.56	9.89	10.56	9.22	7.22	8.00	9.44	9.00	8.89
	3	8.89	8.78	3.33	8.11	7.44	9.11	12.56	9.33	8.78	8.33	8.67

	diff	0.44	0.78	**11.67**	1.78	3.11	0.67	**5.44**	1.56	0.78	1.22	0.67
$\vec{h}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	2	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	3	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
	diff	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\bar{k}_1$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.32	3.21	3.28	3.28	3.35	3.26	3.03	3.29	3.21	3.43	3.34
	2	3.33	3.30	3.30	3.27	3.33	3.21	3.05	3.41	3.38	3.29	3.30
	3	3.25	3.39	3.31	3.35	3.21	3.42	3.81	3.19	3.31	3.18	3.26
	diff	0.07	0.18	0.03	0.08	0.14	0.21	**0.78**	0.23	0.17	0.26	0.08
$\bar{k}_2$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	3.40	3.57	2.54	3.87	3.34	3.11	3.02	3.30	3.11	3.46	3.44
	2	3.11	3.18	3.64	3.45	3.42	3.60	3.14	3.36	3.41	3.47	3.40
	3	3.47	3.24	3.81	2.66	3.22	3.28	3.83	3.32	3.46	3.06	3.15
	diff	0.35	0.40	**1.27**	**1.21**	0.20	0.49	0.80	0.06	0.35	0.41	0.29
$\bar{k}_3$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.01	0.01	0.02	0.02	0.01	0.01	0.00	0.02	0.01	0.01	0.01
	2	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01
	3	0.01	0.01	0.00	0.01	0.01	0.01	0.02	0.00	0.00	0.01	0.01
	diff	0.00	0.00	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
$\bar{\alpha}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.0025	0.0027	0.0042	0.0026	0.0031	0.0033	0.0044	0.0027	0.0031	0.0013	0.0023
	2	0.0028	0.0031	0.0027	0.0032	0.0021	0.0034	0.0043	0.0020	0.0024	0.0037	0.0031
	3	0.0035	0.0030	0.0019	0.0030	0.0036	0.0021	0.0002	0.0042	0.0033	0.0038	0.0034
	diff	0.0010	0.0003	0.0022	0.0006	0.0015	0.0012	0.0042	0.0022	0.0009	0.0026	0.0012
$\bar{\beta}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	2	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	diff	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\overline{EA}$	Level	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
	1	4730.63	4688.65	3904.57	4765.32	686.09	4672.90	7576.42	4762.95	4785.29	5125.53	5108.24
	2	4786.49	4773.02	6762.54	4699.75	2278.02	4748.51	3901.62	4707.37	4737.91	4713.95	4761.86
	3	5063.00	5118.44	3913.01	5115.04	11616.00	5158.71	3102.07	5109.79	5056.91	4740.63	4710.01
	diff	**332.38**	**429.79**	**2857.97**	**415.29**	**10929.91**	**485.81**	**4474.35**	**402.42**	**319.00**	**411.57**	**398.22**

In Table 5.3.3.6, if the input parameter is significant to optimal design parameter and their relationship is linear and positive, we use notation "+", if the input parameter is significant and their relationship is linear and negative, we use notation "-", and if the input parameter is significant and their relationship is quadratic, we use notation "q"; otherwise, we use notation "N".

Table 5.3.3.6. The Significant Input Parameters of Each Design Parameter and EA under the Model with $k_p = k_c$, $d_c > d_p$, and Specified A_c > Specified A_p

Optimal design parameters and EA	Input parameters										
	δ	σ	δ_1	δ_2	R	(A_c,A_p)	λ	T_{sr}	a	b	(C_{sr},C_f)
$\overline{d_c}$	N	N	N	N	N	-	N	N	N	N	N
$\overline{d_p}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{n}$	N	N	-	N	N	N	+	N	N	N	N
$\overline{h}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{k_1}$	N	N	N	N	N	N	+	N	N	N	N
$\overline{k_2}$	N	N	+	-	N	N	N	N	N	N	N
$\overline{k_3}$	N	N	N	N	N	N	N	N	N	N	N
$\overline{EA}$	+	+	q	q	+	+	-	q	q	q	q

6. EXAMPLES AND SENSITIVITY ANALYSIS COMPARISON FOR ALL TYPES OF LOSS FUNCTIONS

6.1 Examples Comparison

Previous sections mentioned the cost models considering different types of loss functions. This section shows a comparison of the optimal solutions and optimal expected cost per unit time of these cost models. All optimal solutions of economic statistical $\overline{X}$ and S charts with a given n are shown in Table 6.1 and leads to following findings:

(i) In (1), the EA^* is smaller than those in (3) to (8) since it does not include the rework cost and consumer loss. Furthermore, The width of $\overline{X}$ chart is larger and the width of S chart is smaller.

(ii) In (2), the EA^* is the smallest in 8 models since it only consider the loss of consumer. Furthermore，The width of $\overline{X}$ chart is larger and the width of S chart is smaller.

(iv) In (4), when we only know the A_c and $d_c > d_p$, the $d_p{*}$ is smaller than those in (3) to (8) since we want to avoid nonconforming product for consumer and reduce the rework cost.

(iii) In (5), when $k_c > k_p$ and $d_c \leqq d_p$, $d_c{*}$ should be equal to $d_p{*}$, since we want to reduce the rate of nonconforming product for consumer and reduce the loss of consumer.

(iv) In (6), when $k_c < k_p$ and $d_c > d_p$, the optimal design of control charts is the same as those in (3) since we don't have to consider consumer loss in this situation.

(v) In (7), when $k_c > k_p$ and $d_c > d_p$, $d_c{*}$ is smaller since $L_c > L_p$ and we want to reduce the rate of nonconforming product for consumer.

(vi) In (8), $d_c{*}$ is higher than other models because we only determine $d_p{*}$, and $d_c{*}$ is calculated by $k_p = k_c$.

(vii) If $k_c > k_p$, producer are advised to use (7) and let $d_p{*}$ be smaller. If $k_c \leqq k_p$, producer are advised to use (3) for convenience and EA^* is smaller.

Table 6.2. Compare all economic statistical design of $\overline{X}$ and S charts with given n of the example

model	d_c*	d_p*	h*	k_1*	k_2*	k_3*	α*	β*	EA*
(1) Without tolerance	None	None	8	3.581	2.194	0.017	0.00034	0.2	109.572
(2) Only with d_c	25	None	8	4.000	2.001	0.005	0.00010	0.2	4.712
(3) Only with d_p	None	25	8	3.061	3.881	0.000	0.00221	0.2	1051.767
(4) $k_p = k_c$, $d_c > d_p$, and $A_c > A_p$ where A_c is specified and A_p is determined	25	11.18	8	3.061	3.933	0.006	0.00221	0.2	1055.262
(5) $k_c > k_p$, $d_c \leqq d_p$, and $A_c > A_p$ where A_c and A_p are specified	25	25	8	3.061	4.042	0.005	0.00221	0.2	1055.262
(6) $k_c < k_p$, $d_c > d_p$ and $A_c > A_p$ where A_c and A_p are specified	> 25	25	8	3.061	3.881	0.000	0.00221	0.2	1051.767
(7) $k_c > k_p$, $d_c > d_p$ and $A_c > A_p$ where A_c and A_p are specified	25	12.157	8	3.061	3.856	0.008	0.00221	0.2	1055.262
(8) $k_p = k_c$, $d_c > d_p$, and $A_c > A_p$ where A_c and A_p are specified	55.902	25	8	3.075	3.100	0.018	0.00210	0.2	1051.767

6.2 Sensitivity Analysis Comparison

Figures 6.1-6.10 show a comparison of the variations of each optimal parameter design under 8 models using the following main effect plot. The x-axis represents the levels of each parameter, and the y-axis represents the value of the average optimal design of the parameter under each level. The comparison of the models produces the following results:

(i) In Figure 6.1, all input parameters are not significant to average d_c* in all models.

(ii) In Figure 6.2, only in model (4) and (7), σ and (A_c,A_p) are significant to average d_p*. The average d_p* is smaller in model (7). The input parameters are not significant to d_p *in other models.

(iii) In Figure 6.3, δ_1 is significant to average *n** in 8 models, λ is significant to average *n** in 7 models except model (1), and a and b are only significant to average *n** in model (1). The average *n** is higher in model (1).

(iv) In Figure 6.4, all input parameters are not significant to average *h** in 8 models. However, average *h** in model (1) is smaller than in other 7 models.

(v) In Figure 6.5, λ is significant to average k_1* in 7 models except model (1). However, average k_1* is smaller in model (1).

(vi) In Figure 6.6, δ_1 is significant to average k_2 in 7 models except model (1), and δ_2 is significant to average k_2* in all models. However, average k_2* in mode (1) is smaller.

(vii) In Figure 6.7 and 6.8, all input parameters are not significant to average k_3* and average α* in all models.

(viii) In Figure 6.9, all input parameters are not significant to average β* in 7 models except model (1), and a is significant to average β* in model (1). However, average β* is smaller in model (1).

(ix) In Figure 6.10, all input parameters are significant to average *EA** in model (1) but not significant in model (3). Only R and λ are significant to average *EA** in rest 6 models.

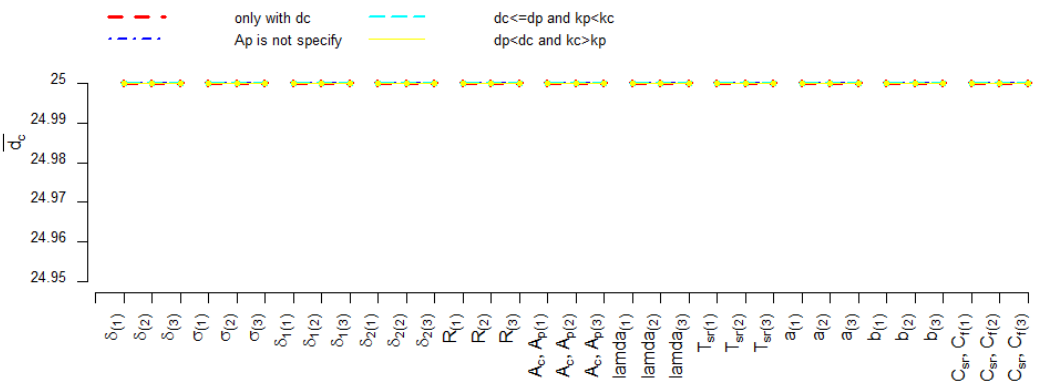

Figure 6.1. Main Effect Plot of Consumer Tolerance with Different Models

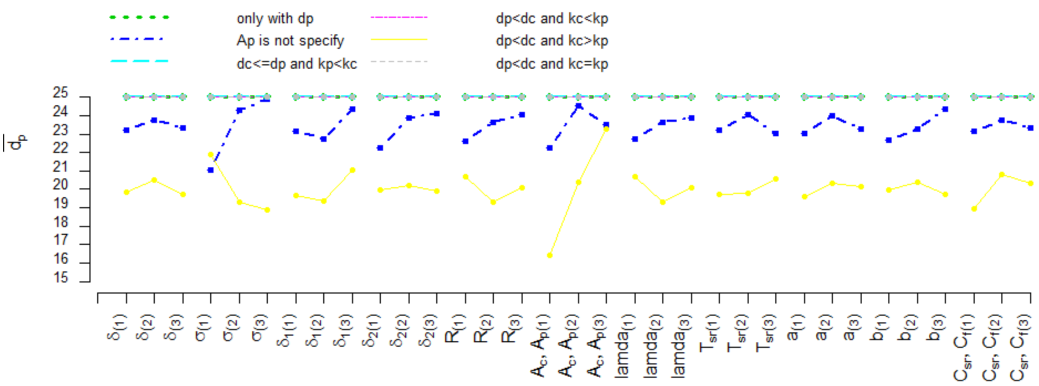

Figure 6.2. Main Effect Plot of Producer Tolerance with Different Models

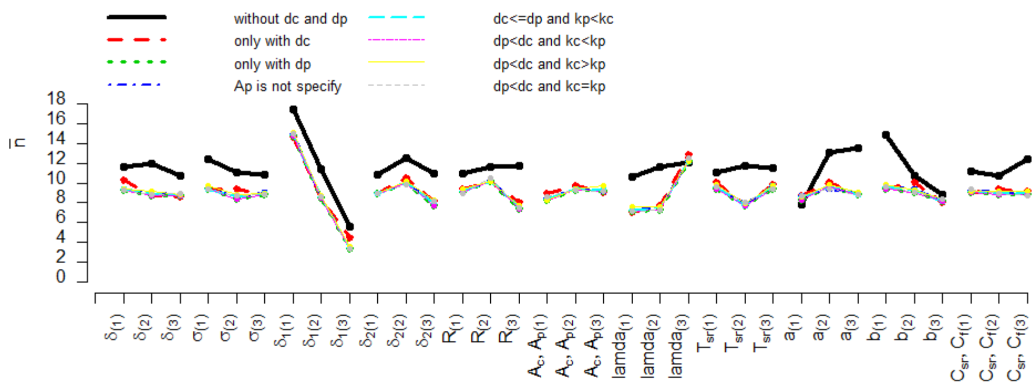

Figure 6.3. Main Effect Plot of *n* Tolerance with Different Models

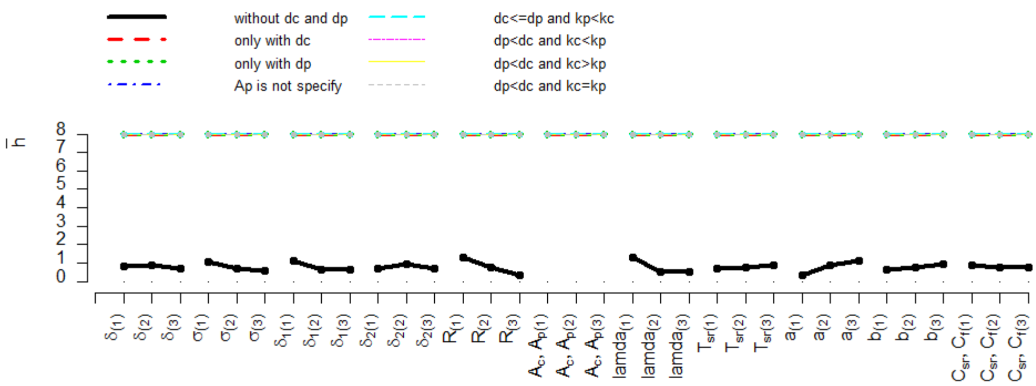

Figure 6.4. Main Effect Plot of *h* with Different Models

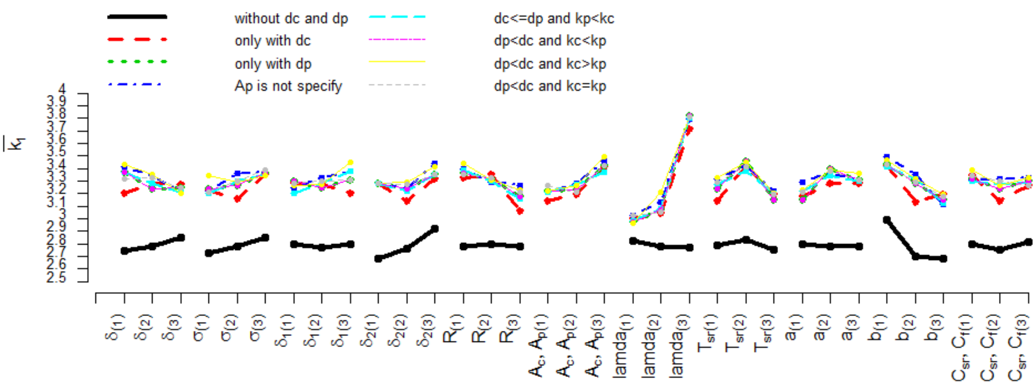

Figure 6.5. Main Effect Plot of k_1 with Different Models

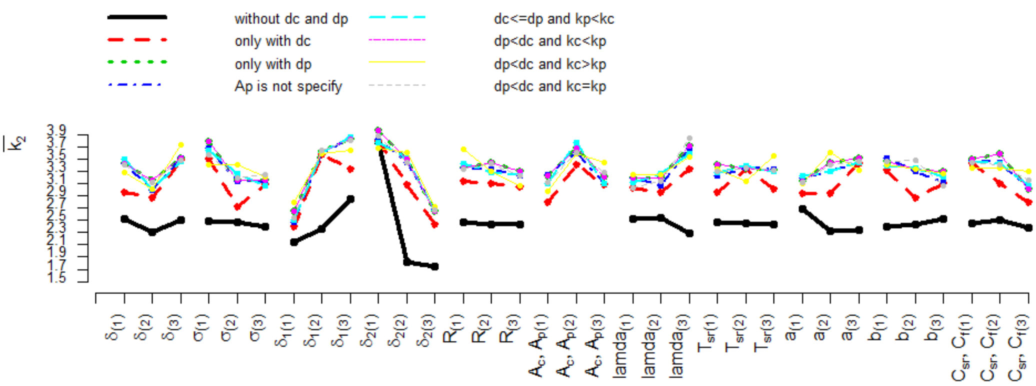

Figure 6.6. Main Effect Plot of k_2 with Different Models

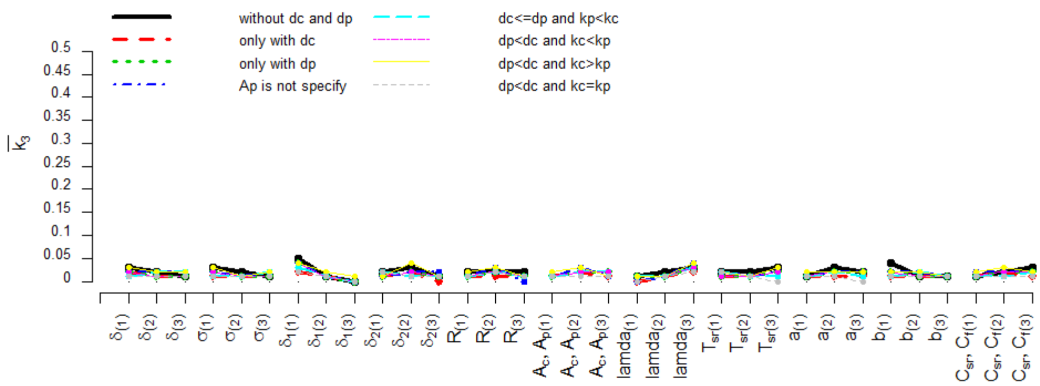

Figure 6.7. Main Effect Plot of k_3 with Different Models

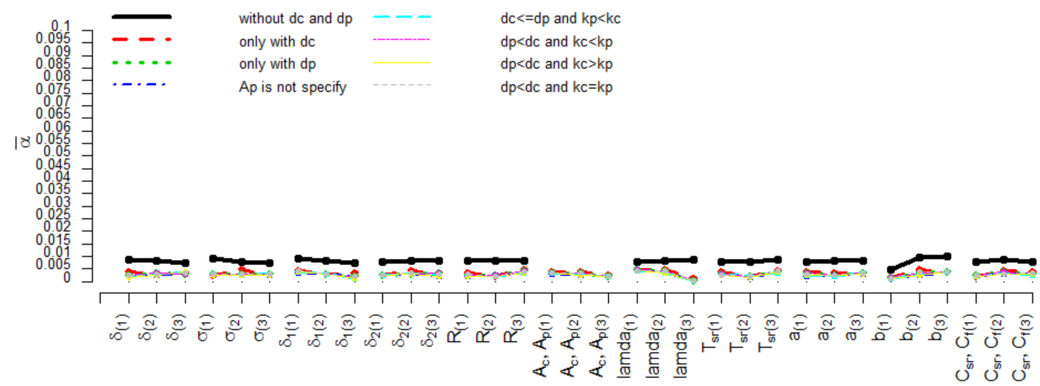

Figure 6.8. Main Effect Plot of False Alarm Rate with Different Models

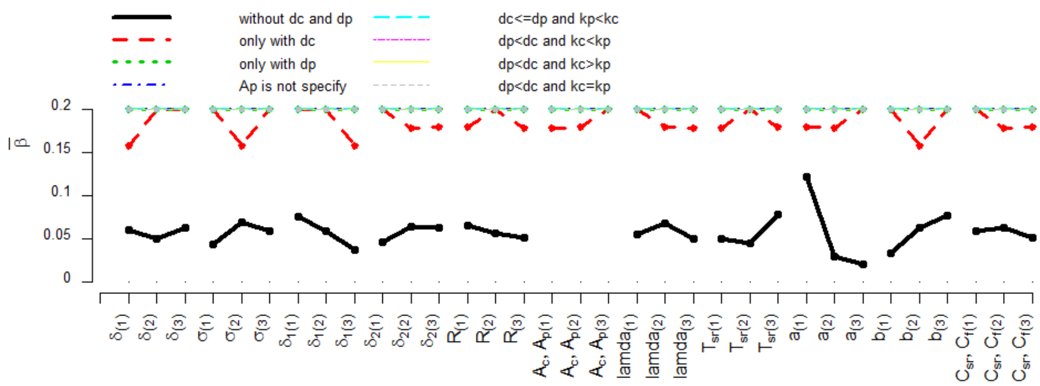

Figure 6.9. Main Effect Plot of True Alarm Rate with Different Models

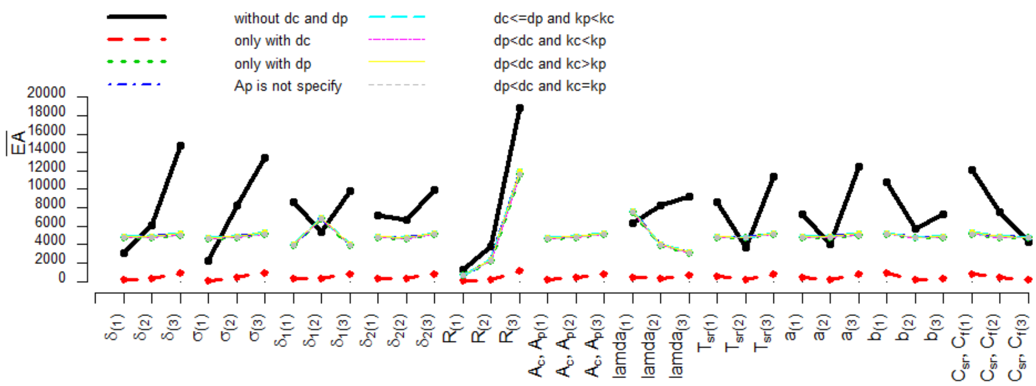

Figure 6.10. Main Effect Plot of *EA* with Different Models

7. CONCLUSION

To monitor the process and reduce the rate of nonconforming products with a complete inspection plan, this study proposes the design of economic $\bar{X}$ and S charts, consumer tolerance and producer tolerance to minimize expected cost per unit time. These models include the concept of tolerance and loss function in the economic control charts. This study assumes that consumer loss function and producer loss function are not necessarily the same. To conclude all situations of consumer and producer loss functions, eight cost models are in this study. These eight cost models are derived by using renewal processes and renewal reward processes approach, and the differential evolution algorithm is applied to find the optimal solutions of the eight proposed cost models.

The example of each proposed cost model shows the construction and application of control charts and specifications. The numerical analysis shows that it is more convenient to use Shewhart-type economic $\bar{X}$ and S charts because their expected costs are almost same, and decreasing the variance can significant reduce costs. Sensitivity analysis of each proposed cost model shows the significant input parameters to optimal design parameter and their relationship. The comparisons of examples and sensitivity analysis produced the following results:

(1) The expected cost per unit time is lower than the actual cost per unit time when the cost model only considering consumer loss or producer loss. However, if the producer loss is higher than the consumer loss, the producer is advised to use the cost model only with producer tolerance.

(2) In the cost models considering producer tolerance and consumer tolerance, δ_1 and λ are significant to average n^*, all input parameters are not significant to average h^*, λ is significant to average k_1^*, and δ_1 and δ_2 is significant to average k_2^*. All input parameters are not significant to average d_c^*, σ and (A_c, A_p) are significant to average d_p^* in model "$k_p = k_c$, $d_c > d_p$, and specified A_c > determined A_p" and "$k_c > k_p$, $d_c > d_p$ and specified A_c > specified A_p". All input parameters are significant to average EA^*.

(3) The design parameters of economic statistical $\bar{X}$ and S control charts are not sensitive to the cost models considering the consumer tolerance and the producer tolerance.

(4) f the producer tolerance is smaller than the consumer tolerance, and the producer loss is smaller than the consumer loss, the optimal producer tolerance should be small.

In the future, the study can be extended to a cost model containing the EWMA and EWMS charts and tolerances of customers and producers. Furthermore, the economic model may contain multiple assignable causes.

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