行政院國家科學委員會專題研究計畫 成果報告

在預算限制下網路頻寬的最佳化過程（II）
研究成果報告(精簡版)

計畫類別：個別型
計畫編號：NSC 97-2221-E-004-005-
執行期間：97年08月01日至98年07月31日
執行單位：國立政治大學應用數學學系

計畫主持人：陸行

計畫參與人員：學士級-專任助理人員：謝智宇
碩士班研究生-兼任助理人員：黃瀚陞
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博士班研究生-兼任助理人員：王嘉宏

報告附件：出席國際會議研究心得報告及發表論文

處理方式：本計畫可公開查詢

中華民國 98年09月19日

行政院國家科學委員會補助專題研究計畫成果報告
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※　在預算限制下網路頻寬的最佳化過程(II)　※
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計畫類別：□v個別型計畫　　□整合型計畫
計畫編號：NSC 97-2221-E-004-005
執行期間：　97年8月1日至　98年7月31日

計畫主持人：陸行

共同主持人：

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位：國立政治大學應用數學系

中　華　民　國　98　年　8　月　1　日

行政院國家科學委員會專題研究計畫成果報告
在預算限制下網路頻寬的最佳化過程 (II)
Network Bandwidth Optimization Under Budget Constraints

計畫編號: **NSC 97-2221-E-004-005**
執行期限: 97年08月01日至98年07月31日

Abstract. We present a bandwidth allocation scheme offering optimal solutions for the network optimization problem. The bandwidth allocation policy in class-based networks can be defined by utility functions. This scheme is formulated as a mixed-integer nonlinear programming model, preparing a database identifying suitable end-to-end paths upon each connection request. A fast branch and bound algorithm is proposed for solving the optimization problem.

1 Introduction

In recent years, we have witnessed considerable accomplishments in the design and deployment of broadband communication networks. Network capabilities grow at a remarkable rate. At the same time, a phenomenal growth in data traffic and a wide range of new requirements of emerging applications call for new mechanisms for the control and management of communication networks. The idea of a single shared physical network that will support multiple heterogeneous applications with different traffic characteristics and different Quality of Service (QoS) requirements, is widely regarded as the way to meet the telecommunication challenges of the future [3]. QoS has always been the major issue for telecom providers [13]. Packet-switched networks have been proposed to offer the QoS guarantees in integrated-services networks because individual packets may exhibit a significant variation in network service quality.

We deal with the problem of dimensioning bandwidth for elastic data applications in packet-switched communication networks. Each network user is allowed to request more than one type of service, and users' satisfaction is summarized by means of their utility functions. We focus on allocating resources and finding a routing scheme on All-IP communication networks. An approach is presented for the bandwidth allocation problem and QoS routing in All-IP networks. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the sum of the users' satisfaction. These operational processes are involved in the efficient set-up and usage of a network. Three main components of these processes are designed for which links to develop to meet certain connectivity requirements, determining how much capacity to put on the links to serve all traffic demands, and choosing which paths to use for the various traffic streams to meet demand without violating capacity restrictions on links.

2 Network Management Schemes

Consider a directed network topology $G = (\mathbb{V}, \mathbb{E})$, where $\mathbb{V}$ and $\mathbb{E}$ denote the set of nodes and the set of links in the network respectively. The maximal possible link capacity is U_e on each link $e \in \mathbb{E}$. Suppose, for each link e, there is a mean delay ℓ_e related to the link's speed, propagation delay, and maximal transfer unit. Suppose there is the link cost κ_e for using one unit bandwidth. There are m different QoS classes of connections in the network. Let $\mathbb{I} = \{1, \ldots, m\}$ be an index set consists of m different QoS classes. The specific QoS requirements, for each class i, include minimal bandwidth requirement b_i and maximal end-to-end delay constraint D_i. We denote the total number of connections, for each class i, by K_i. Let $\mathbb{J}_i$, for each class i, be an index set which consists of K_i connections, that is, $\mathbb{J}_i = \{1, \ldots, K_i\}$. All connections are delivered between the same source o and destination d in this (core) network. Every connection in class i is allocated the same bandwidth θ_i and has the same QoS requirement.

A connection j in each class i should be routed through some path $p_{i,j}$ between o and d. When a connection j in class i is routed along a path $p_{i,j} = \{e \in \mathbb{E} |\ \chi_{i,j}(e) = 1\}$, the end-to-end delay $D(p_{i,j})$ is computed from the following formula (Atov et al. [2], Johari and Tan [5], etc.):

$$D(p_{i,j}) = \frac{n(p_{i,j}) \cdot \sigma_i}{\theta_i} + \sum_{e \in p_{i,j}} \ell_e, \tag{1}$$

where $n(p_{i,j})$ is the number of links along path $p_{i,j}$ and σ_i is the mean packet size for each class i, $i \in \mathbb{I}$. A path $p_{i,j}$ between o and d is feasible, for a connection j of class i, if $D(p_{i,j}) \le D_i$.

Under a limited available budget B, we plan to allocate the bandwidth in order to provide each class with maximal possible QoS and determine the optimal end-to-end path under guaranteed service. Decision variable θ_i represents the bandwidth allocated to each connection in class i, and binary variable $\chi_{i,j}(e)$ determines whether the link e is chosen for connection j in class i. Bandwidth sharing in a network is frequently evaluated in terms of a utility function [4], [6], [7], etc. The utility of a connection (user) in class i, $f_i(\theta_i)$, is assumed to be an increasing concave function of its bandwidth θ_i. The utility function $f_i(\theta_i)$ can be formulated as

$$f_i(\theta_i) = \log \theta_i \tag{2}$$

as introduced by Kelly et al. [6]. Our goal is to maximize the total utility of all competing classes. The utility maximization model is formulated as follows [10]:

$$\text{Max} \quad \sum_{i \in \mathbb{I}} w_i \cdot f_i(\theta_i) \tag{3}$$

$$\text{s. t.} \quad \sum_{e \in \mathbb{E}} \sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}_i} \kappa_e \theta_i \chi_{i,j}(e) \le B \tag{4}$$

$$\sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}_i} \theta_i \chi_{i,j}(e) \le U_e \tag{5}$$

$$\sum_{e \in \mathbb{E}} \sigma_i \chi_{i,j}(e) + \sum_{e \in \mathbb{E}} \ell_e \theta_i \chi_{i,j}(e) \le D_i \cdot \theta_i \tag{6}$$

$$\theta_i \ge b_i \tag{7}$$

$$\sum_{e \in \mathbb{E}_o} \chi_{i,j}(e) = 1,\ \forall\ j \in \mathbb{J}_i \tag{8}$$

$$\sum_{e \in \mathbb{E}_\nu^{in}} \chi_{i,j}(e) = \sum_{e \in \mathbb{E}_\nu^{out}} \chi_{i,j}(e) \tag{9}$$

$$\sum_{e \in \mathbb{E}_d} \chi_{i,j}(e) = 1 \tag{10}$$

$$\theta_i \ge 0 \tag{11}$$

$$\chi_{i,j}(e) = 0 \text{ or } 1,\ \forall\ e \in \mathbb{E},\ \forall\ j \in \mathbb{J}_i \tag{12}$$

where $w_i \in (0, 1)$ is the weight assigned to each class i and $\sum_{i \in \mathbb{I}} w_i = 1$. Since pages are limited, proofs of the following results are skipped and will be provided for requests.

The budget constraint (4) is due to the limited budget on network planning. The constraint (5) means that the aggregate bandwidth of all connections at any link does not exceed the capacity. We have the end-to-end delay constraint (6) since every connection has the maximal end-to-end delay constraint. Constraint (7) shows that every connection in the same class has the same bandwidth requirement. Constraints (8), (9), and (10) express the node conservation relations indicating that flow in equals flow out for every connection j in class i. Constraints (8)-(10) are standard flow conservation constraints. Continuous decision variables and binary variables must be nonnegative in constraints (11)-(12).

Theorem 1 *The network management scheme is NP-hard.*

3 A Fast Branch and Bound Algorithm

The relaxation model of the network management scheme is formulated as follows:

$$
\begin{aligned}
\text{Max} \quad & \sum_{i\in\mathbb{I}} w_i \cdot f_i(\theta_i) \\
\text{s. t.} \quad & \text{constraints } (4)-(11) \\
& 0 \le \chi_{i,j}(e) \le 1,\ \forall\ e\in\mathbb{E},\ \forall\ j\in\mathbb{J}_i,\ i\in\mathbb{I} \qquad (13)
\end{aligned}
$$

where $w_i \in (0,1)$ is the weight assigned to each class i and $\sum_{i\in\mathbb{I}} w_i = 1$. The relaxation constraints (13) are obtained by dropping the integer constraint on (12) in the network management scheme. Let $\digamma = \{\theta_i, \chi_{i,j}(e) | j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$ be the set of all feasible solutions to Relaxation Model. The fast branch and bound algorithm branches by fixing the fractional decision variable $0<\chi_{i,j}(e)<1$. Branch and bound searches stop when every solution in $\digamma$ has been branched or terminated. The incumbent solution at any stage in a search of a discrete model is the best feasible solution known so far. We denote the incumbent solution $\tilde{\mathbf{X}} = \{\tilde{\theta}_i, \tilde{\chi}_{i,j}(e) | \forall j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$ and its objective function value $\tilde{f} = \sum_{i\in\mathbb{I}} w_i f_i(\tilde{\theta}_i)$. Moreover, we denote the Lagrangian dual value of the incumbent solution $\tilde{\mathbf{X}}$ by $\tilde{f}_L$. If all the solutions have been either branched or fathom, then the final incumbent solution is the optimum. The following is the fast branch and bound algorithm, which is implemented to solve the network management scheme.

Subprogram 1: (Path Search Algorithm)

Step 1. Compute the incidence matrix of given network topology $G=(\mathbb{V},\mathbb{E})$. Proceed to Step 2.

Step 2. Find all candidates of end-to-end paths from the incidence matrix. Proceed to Step 3.

Step 3. Set up the set of all end-to-end paths $\mathbb{P} = \{\chi(e) | \sum_{e\in\mathbb{E}_o}\chi(e) = 1, \sum_{e\in\mathbb{E}_d}\chi(e) = 1, \sum_{e\in\mathbb{E}_\nu^{in}}\chi(e) = \sum_{e\in\mathbb{E}_\nu^{out}}\chi(e), \chi_{i,j}(e) = 0 \text{ or } 1,\ \forall\ e\in\mathbb{E}\}$, and calculate the cardinal number $|\mathbb{P}|$. Then the procedure stops, and go to Subprogram 2.

From the output of Subprogram 1, $\mathbb{P}$, the network management scheme can be simplified to Model 2:

$$
\begin{aligned}
\text{Max} \quad & \sum_{i\in\mathbb{I}} w_i \cdot f_i(\theta_i) \\
\text{s. t.} \quad & \text{constraints } (4)-(7), (11) \\
& p_{i,j} = \{\chi_{i,j}(e) | \forall\ e\in\mathbb{E}\} \in \mathbb{P}, \forall\ j\in\mathbb{J}_i,\ \forall\ i\in\mathbb{I}
\end{aligned}
$$

Subprogram 2:

Step 1. (Relaxation.) Solve the relaxation of the network management scheme, Relaxation Model. Let $f^* = \sum_{i\in\mathbb{I}} w_i f_i^*$ be the optimal value of Relaxation Model. Proceed to Step 2.

Step 2. (Initialization.) Set $t=0$ and $\varepsilon>0$. Put the initial solution $\tilde{\mathbf{X}} = \{\theta_i^0, \chi_{i,j}^0(e) | \theta_i^0 = b_i, \chi_{i,j}^0(e) = \chi(e),\ \forall j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$, and $\tilde{f} = \sum_{i\in\mathbb{I}} w_i f_i(\theta_i^0)$. Proceed to Step 3.

Step 3. (Branching.) Set $t \leftarrow t+1$, and select one solution $\mathbf{X}^t = \{\theta_i^t, \chi_{i,j}^t(e) | j \in\in \mathbb{E}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\} \in \digamma$. Choose a connection (a pair of indices) $(i,j)^t \in \mathbb{I}\times\mathbb{J}_i$ whose $\chi_{i,j}^t(e)$, $e\in\mathbb{E}$ is a fractional part of the solution $\mathbf{X}^t$ node, then create $|\mathbb{P}|$ new active nodes and select one different candidate $p_{i,j}^t = \{\chi(e) | \forall\ e\in\mathbb{E}\} \in \mathbb{P}$ for each new active node. Add them into $\digamma$ and update $\mathbb{I}\times\mathbb{J}_i \leftarrow \mathbb{I}\times\mathbb{J}_i \setminus (i,j)^t$. Proceed to Step 4.

Step 4. (Termination by Bound.) If $\sum_{i\in\mathbb{I}} w_i f_i(\theta_i^t) < \tilde{f}$, then set $\digamma \leftarrow \digamma \setminus \{\mathbf{X}^t\}$ and go to Step 6. Otherwise, proceed to Step 5.

Step 5. (Termination by Solving.) If $\sum_{i\in\mathbb{I}} w_i f_i(\theta_i^t) \ge \tilde{f}$ and $\{\chi_{i,j}^t(e) | j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$ are integer solutions, then update $\tilde{f} \leftarrow \sum_{i\in\mathbb{I}} w_i f_i(\theta_i^t)$, $\tilde{\mathbf{X}} \leftarrow \mathbf{X}^t$, $\digamma \leftarrow \digamma \setminus \{\mathbf{X}^t\}$, and proceed to Step 6. Otherwise, go to Step 3.

Step 6. (Optimal Criteria.) If $\digamma = \emptyset$ or $|\tilde{f} - f^*| < \varepsilon$, then the procedure stops. The incumbent solution $\tilde{\mathbf{X}}$ is called the ε-optimal solution. Otherwise, go to Step 3.

Theorem 2 *Model 2 is equivalent to the original model.*

For positive integer n, let $\tilde{f}_n$ be the objective function value of the n-th incumbent solution in the above fast branch and bound algorithm. Then, for this utility maximization model, $\{\tilde{f}_n\}$ is an increasing sequence.

Theorem 3 *The sequence $\{\tilde{f}_n\}$ of objective function values of incumbent solutions is increasing.*

Theorem 4 *The sequence of objective function values of nodes for each consecutive branch in the fast branch and bound algorithm is decreasing.*

Theorem 5 *The complexity of the fast branch and bound algorithm in the worst case is $O(|\mathbb{P}|^{\sum_{i\in\mathbb{I}} K_i})$.*

The complexity of this algorithm is much better than that of original utility maximization model, which is $O(2^{|\mathbb{E}|\sum_{i\in\mathbb{I}} K_i})$.

4 Conclusions

We present an approach for the bandwidth allocation and QoS routing in All-IP networks. Solving the network management scheme by the fast branch and bound algorithm, we can find the optimal bandwidth allocation on the network under a limited available budget. Our approach is executed in advance and its purpose is to precompute solutions as a database for later usage, which selects one of the solutions by performing a few additional computations when connections arrive.

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保存年限：5年

行政院國家科學委員會補助國內專家學者出席學術會議報告

填表日期：　　年　　月　　日

報告人姓名	陸　行	服務機構及職稱	政大應數系教授
時間 會議 地點	0980601--0980605 中國湖南長沙市	本會核定補助文號	
會議名稱	（中文）第一屆全球工程及科學最佳化會議 （英文）The first World Congress on Global Optimization in Engineering & Science (WCGO2009)		
發表論文題目	(中文) (英文) A Fast Branch and Bound Based Algorithm for Bandwidth Allocation and QoS Routing on Class-based IP Networks		

一、內容摘要

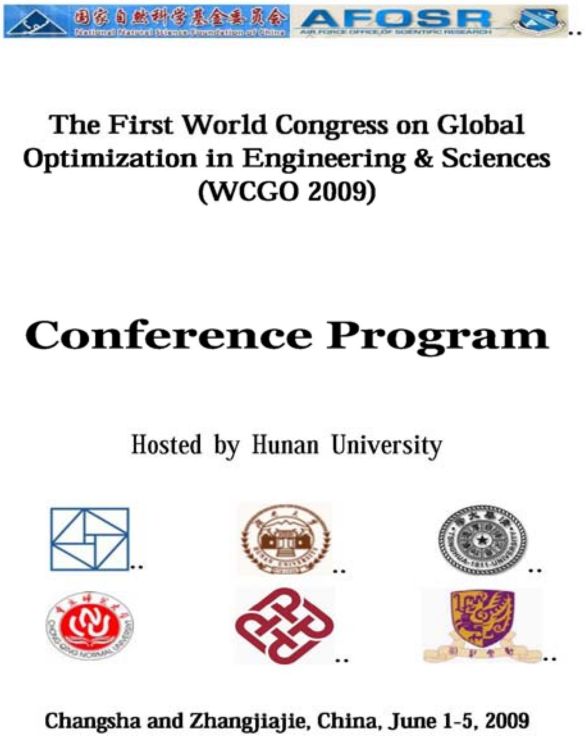
此研討會是由中國科學院主辦，而且本人忝爲會議委員會委員，具有連絡台灣學者的責任。本研討會爲作業研究和最優化學界在世界之重要領導人第一次籌辦的學術研討會，對於討論新興研究議題和日後發展具有深刻影響。 本人會議論文的主題網路最佳化之演算機制，是本人長期研究的主要課題之一。希望藉由這個機會結合國際研究相關議題的研究者交換研究心得，共創展新契機。今年在湖南大學舉行，估計參加的人數將超過 200 人。 目前報名的學者專家分別來自美國、日本、韓國、俄羅斯、中國、越南、泰國、印度和新加坡。若能於此會議建立彼此交流管道，讓本校、或國內學者與國際學者交換研究經驗，實有助於日後教學和研究的國際化與產業經濟發展。

研討會議程（中、英文） 如附件一。

二、重要結論或研究成果（中英文論文、期刊、光碟、出版或獲校外研究經費補助）
如附件 二。

三、建議
大陸地區發展作業研究突飛猛進。值得我們學習。

四、相關聯結(活動網頁、與本學術活動有關聯結…)
http://madis1.iss.ac.cn/wcgo2009/



国家自然科学基金委员会 National Natural Science Foundation of China

AFOSR AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

The First World Congress on Global
Optimization in Engineering & Sciences
(WCGO 2009)

Conference Program

Hosted by Hunan University

Changsha and Zhangjiajie, China, June 1-5, 2009

備註:本執行成果報告應於活動執行完畢後一個月內提出。

附件 一

WCGO2009 Schedule

June 1-5, 2009

Hunan University, Changsha, China

Organized by
Hunan University, China
Academy of Mathematics and Systems Science of Chinese Academy of Sciences

Organizing Committee

Honorary Co-Chairs:	Pardalos(Univeristy of Florida, USA),(Hunan University,	
General Co-Chairs:	hu-Cherng	Carolina State University,
	houyang	Chinese Academy of Sciences, China
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	Prof. Chris Floudas	Princeton University, USA
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	R.Enkhbat (National University of Mongolia)	
	Masao Fukushima (Japan)	
	Duan Li, (Chinese University, HK)	
	Hsing Paul Luh (Taiwan)	
	Zhaosong Lu (Operations Research of Simon	

	Fraser University)
	Rubem Mondaini (Federal University of Rio de Janeiro)
	Dumitru Motreanu (motreanu@univ-perp.fr)
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	Jianming Shi (Department of Computer Science and Systems Engineering Muroran Institute of Technology, Muroran)
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	Yongkang Sui (ysui@bjut.edu.cn)
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	Kok Lay Teo (Curtin University, Australia)
	Michel Thera (France)
	Soon-yi Wu (Nat'l Cheng Kung University, Taiwan)
	Wenxun Xing (Tsinghua University, China)
	Qiang Xu (Department of Chemical Engineering Lamar University,Beaumont)
	Xinmin Yang (China)
	Yaxiang Yuan (China)
	Wuyi Yue (Konan University, Japan)
	Jun Zhang (University of Michigan)
	Xiangsun Zhang (Chinese Academy of Sciences, China)
	Julius Zilinskas (Institute of Mathematics and Informatics, Lithuania)

Local Organizing Committee

Chair:	(Hunan University)
Conference Coordinators:	(Hunan University)
	(Hunan University)
Publication Coordinator :	(Hunan University)
Members:	Yan Gao (Shanghai University of Science and Tech)
	Guoshan Liu (Chinese Ren Ming University)
	Lizhi Liao (Hong Kong Baptize University)
	Dingou Pu (Tongji University)
	Xiao-Ling Sun (Fudan University)
	Zhengbo Wang (Tsing Hua University)
Secretary:	(Hunan University)

Conference Program (Overview)

Date	Time	Themes
June 1, 2009	8:30-9:00	Welcome
	9:45-10:15	Photo taking
	10:25-11:30	Keynote Addresses
	12:00	Lunch
	14:00-15 :30	Keynote Addresses
	15:30-15:45	Coffee Break
	15:45-18 :00	Keynote Addresses
	18:30	Reception
June 2, 2009	8:30-10:00	Keynote Addresses
	10:00-10:15	Coffee Break
	10:15-11:30	Keynote Addresses

	12:00	Lunch
	14:00-15:30	Paper Presentation
	15:30-15:45	Coffee Break
	16:00-18:00	Paper Presentation
	18:15	Dinner
June 3, 2009	8:30-10:00	Paper Presentation
	10:00-10:15	Coffee Break
	10:15-11:30	Paper Presentation
	12:00	Lunch
	14:00 -15 :30	Paper Presentation
	15:30-15:45	Coffee Break
	16:00-18:00	Paper Presentation
	18:15	Dinner, Closing Address
June 4-5, 2009	Tour: Zhangjiajie National Forest Park	

附件 二

A Fast Branch and Bound Based Algorithm for Bandwidth Allocation and QoS Routing on Class-based IP Networks

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Abstract We present a bandwidth allocation scheme offering optimal solutions for the network optimization problem. The bandwidth allocation policy in class-based networks can be defined by utility functions. This scheme is formulated as a mixed-integer nonlinear programming model, preparing a database identifying suitable end-to-end paths upon each connection request. A fast branch and bound algorithm is proposed for solving the optimization problem.

Keywords bandwidth allocation; QoS routing; branch and bound algorithm

1 Introduction

We deal with the problem of dimensioning bandwidth for elastic data applications in packet-switched communication networks. Each network user is allowed to request more than one type of service, and users' satisfaction is summarized by means of their utility functions. We focus on allocating resources and finding a routing scheme on All-IP communication networks. An approach is presented for the bandwidth allocation problem and QoS routing in All-IP networks, offering multiple services to users. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the sum of the users' satisfaction. These operational processes are involved in the efficient set-up and usage of a network. Three main components of these processes are designed for which links to develop to meet certain connectivity requirements, determining how much capacity to put on the links to serve all traffic demands, and choosing which paths to use for the various traffic streams to meet demand without violating capacity restrictions on links.

2 Network Management Schemes

Consider a directed network topology $G = (\mathbb{V}, \mathbb{E})$, where $\mathbb{V}$ and $\mathbb{E}$ denote the set of nodes and the set of links in the network respectively. The maximal possible link capacity is U_e on each link $e \in \mathbb{E}$. Suppose, for each link e, there is a mean delay ℓ_e related to the link's speed, propagation delay, and maximal transfer unit. Suppose there is the link cost κ_e for using one unit bandwidth.

There are m different QoS classes of connections in the network. Let $\mathbb{I} = \{1, \ldots, m\}$ be an index set consists of m different QoS classes. A small example with m = 3 from [9] is shown in Fig. 1. The specific QoS requirements, for each class i, include minimal bandwidth requirement b_i and maximal end-to-end delay constraint D_i. We denote the total number of connections, for each class i, by K_i. Let $\mathbb{J}_i$, for each class i, be an index set which consists of K_i connections, that is, $\mathbb{J}_i = \{1, \ldots, K_i\}$. All connections are delivered between the same source o and destination d in this (core) network. Every connection, in the same class i, is allocated the same bandwidth θ_i and has the same QoS requirement.

Under a limited available budget B, we plan to allocate the bandwidth in order to provide each class with maximal possible QoS and determine the optimal end-to-end path under guaranteed service. Decision variable θ_i represents the bandwidth allocated to each connection in class i, and binary variable $\chi_{i,j}(e)$ determines whether the link e is chosen for connection j in class i.

Bandwidth sharing in a network is frequently evaluated in terms of a utility function [4], [6], [7], etc. The utility of a connection (user) in class i, $f_i(\theta_i)$, is assumed to be an increasing concave function of its bandwidth θ_i. The utility function $f_i(\theta_i)$ can be formulated as

$$f_i(\theta_i) = \log \theta_i \tag{1}$$

as introduced by Kelly et al. [6]. Our goal is to maximize the total utility of all competing classes. The utility maximization model is formulated as follows [10]:

$$\text{Max} \quad \sum_{i \in \mathbb{I}} w_i \cdot f_i(\theta_i) \tag{2}$$

$$\text{s. t.} \quad \sum_{e \in \mathbb{E}} \sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}_i} \kappa_e \theta_i \chi_{i,j}(e) \leq B \tag{3}$$

$$\sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}_i} \theta_i \chi_{i,j}(e) \leq U_e, \ \forall\, e \in \mathbb{E} \tag{4}$$

$$\textstyle\sum_{e \in \mathbb{E}} \sigma_i \chi_{i,j}(e) + \sum_{e \in \mathbb{E}} \ell_e \theta_i \chi_{i,j}(e) \leq D_i \cdot \theta_i, \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{5}$$

$$\theta_i \geq b_i, \ \forall\, i \in \mathbb{I} \tag{6}$$

$$\sum_{e \in \mathbb{E}_o} \chi_{i,j}(e) = 1, \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{7}$$

$$\sum_{e \in \mathbb{E}_v^{in}} \chi_{i,j}(e) = \sum_{e \in \mathbb{E}_v^{out}} \chi_{i,j}(e), \ \forall\, v \in \mathbb{V}', \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{8}$$

$$\sum_{e \in \mathbb{E}_d} \chi_{i,j}(e) = 1, \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{9}$$

$$\theta_i \geq 0, \ \forall\, i \in \mathbb{I} \tag{10}$$

$$\chi_{i,j}(e) = 0 \text{ or } 1, \ \forall\, e \in \mathbb{E}, \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{11}$$

where $w_i \in (0,1)$ is the weight assigned to each class i and $\sum_{i \in \mathbb{I}} w_i = 1$. Since pages are limited, proofs of the following results are skipped and will be provided for requests.

Theorem 1. *The network management scheme is NP-hard.*

3 A Fast Branch and Bound Algorithm

The relaxation model of the network management scheme is formulated as follows:

$$
\begin{aligned}
&\text{Max} \qquad && \sum_{i\in\mathbb{I}} w_i \cdot f_i(\theta_i) \\
&\text{s. t.} && \text{constraints } (3)-(10) \\
& && 0 \le \chi_{i,j}(e) \le 1,\ \forall\, e\in\mathbb{E},\ \forall\, j\in\mathbb{J}_i,\ i\in\mathbb{I}
\end{aligned}
\tag{12}
$$

where $w_i \in (0,1)$ is the weight assigned to each class i and $\sum_{i\in\mathbb{I}} w_i = 1$. The relaxation constraints (12) are obtained by dropping the integer constraint on (11) in the network management scheme.

Let $\mathcal{F} = \{\theta_i, \chi_{i,j}(e) | j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$ be the set of all feasible solutions to Relaxation Model. The fast branch and bound algorithm branches by fixing the fractional decision variable $0 < \chi_{i,j}(e) < 1$. Branch and bound searches stop when every solution in $\mathcal{F}$ has been branched or terminated. The incumbent solution at any stage in a search of a discrete model is the best feasible solution known so far. We denote the incumbent solution $\tilde{\mathbf{X}} = \{\tilde{\theta}_i, \tilde{\chi}_{i,j}(e) | \forall j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$ and its objective function value $\tilde{f} = \sum_{i\in\mathbb{I}} w_i f_i(\tilde{\theta}_i)$. Moreover, we denote the Lagrangian dual value of the incumbent solution $\tilde{\mathbf{X}}$ by $\tilde{f}_L$. If all the solutions have been either branched or fathom, then the final incumbent solution is the optimum. The following is the fast branch and bound algorithm, which is implemented to solve the network management scheme.

Subprogram 1: (Path Search Algorithm)

Step 1. Compute the incidence matrix of given network topology $G = (\mathbb{V}, \mathbb{E})$. Proceed to Step 2.

Step 2. Find all candidates of end-to-end paths from the incidence matrix. Proceed to Step 3.

Step 3. Set up the set of all end-to-end paths $\mathbb{P} = \{\chi(e) | \sum_{e\in\mathbb{E}_o} \chi(e) = 1, \sum_{e\in\mathbb{E}_d} \chi(e) = 1, \sum_{e\in\mathbb{E}_v^{in}} \chi(e) = \sum_{e\in\mathbb{E}_v^{out}} \chi(e), \chi_{i,j}(e) = 0 \text{ or } 1,\ \forall\, e\in\mathbb{E}\}$, and calculate the cardinal number $|\mathbb{P}|$. Then the procedure stops, and go to Subprogram 2.

From the output of Subprogram 1, $\mathbb{P}$, the network management scheme can be simplified to Model 2:

$$
\begin{aligned}
&\text{Max} \quad && \sum_{i\in\mathbb{I}} w_i \cdot f_i(\theta_i) \\
&\text{s. t.} && \text{constraints } (3)-(6) \\
& && p_{i,j} = \{\chi_{i,j}(e) | \forall\, e\in\mathbb{E}\} \in \mathbb{P}, \forall\, j\in\mathbb{J}_i,\ \forall\, i\in\mathbb{I} \\
& && \theta_i \ge 0,\ \forall\, i\in\mathbb{I}
\end{aligned}
$$

Subprogram 2:

Step 1. (Relaxation.) Solve the relaxation of the network management scheme, Relaxation Model. Let $f^* = \sum_{i\in\mathbb{I}} w_i f_i^*$ be the optimal value of Relaxation Model. Proceed to Step 2.

Step 2. (Initialization.) Set $t = 0$ and $\varepsilon > 0$. Put the initial solution $\tilde{\mathbf{X}} = \{\theta_i^0, \chi_{i,j}^0(e) | \theta_i^0 = b_i, \chi_{i,j}^0(e) = \chi(e),\ \forall j\in\mathbb{J}_i,\ i\in\mathbb{I},\ e\in\mathbb{E}\}$, and $\tilde{f} = \sum_{i\in\mathbb{I}} w_i f_i(\theta_i^0)$. Proceed to Step 3.

Step 3. (Branching.) Set $t \leftarrow t+1$, and select one solution $\mathbf{X}^t = \{\theta_i^t, \chi_{i,j}^t(e) | j \in\in \mathbb{E}_i,\ i \in \mathbb{I},\ e \in \mathbb{E}\} \in \mathcal{F}$. Choose a connection (a pair of indices) $(i,j)^t \in \mathbb{I} \times \mathbb{J}_i$ whose $\chi_{i,j}^t(e)$, $e \in \mathbb{E}$ is a fractional part of the solution $\mathbf{X}^t$ node, then create $|\mathbb{P}|$ new active nodes and select one different candidate $p_{i,j}^t = \{\chi(e) | \forall\ e \in \mathbb{E}\} \in \mathbb{P}$ for each new active node. Add them into $\mathcal{F}$ and update $\mathbb{I} \times \mathbb{J}_i \leftarrow \mathbb{I} \times \mathbb{J}_i \setminus (i,j)^t$. Proceed to Step 4.

Step 4. (Termination by Bound.) If $\sum_{i \in \mathbb{I}} w_i f_i(\theta_i^t) < \tilde{f}$, then set $\mathcal{F} \leftarrow \mathcal{F} \setminus \{\mathbf{X}^t\}$ and go to Step 6. Otherwise, proceed to Step 5.

Step 5. (Termination by Solving.) If $\sum_{i \in \mathbb{I}} w_i f_i(\theta_i^t) \geq \tilde{f}$ and $\{\chi_{i,j}^t(e) | j \in \mathbb{J}_i,\ i \in \mathbb{I},\ e \in \mathbb{E}\}$ are integer solutions, then update $\tilde{f} \leftarrow \sum_{i \in \mathbb{I}} w_i f_i(\theta_i^t)$, $\tilde{\mathbf{X}} \leftarrow \mathbf{X}^t$, $\mathcal{F} \leftarrow \mathcal{F} \setminus \{\mathbf{X}^t\}$, and proceed to Step 6. Otherwise, go to Step 3.

Step 6. (Optimal Criteria.) If $\mathcal{F} = \emptyset$ or $|\tilde{f} - f^*| < \varepsilon$ or $|\tilde{f} - \tilde{f}_L| < \varepsilon$, then the procedure stops. The incumbent solution $\tilde{\mathbf{X}}$ is called the ε-optimal solution. Otherwise, go to Step 3.

Theorem 2. *Model 2 is equivalent to the original model.*

For positive integer n, let $\tilde{f}_n$ be the objective function value of the n-th incumbent solution in the above fast branch and bound algorithm. Then, for this utility maximization model, $\{\tilde{f}_n\}$ is an increasing sequence.

Theorem 3. *The sequence $\{\tilde{f}_n\}$ of objective function values of incumbent solutions is increasing.*

Theorem 4. *The sequence of objective function values of nodes for each consecutive branch in the fast branch and bound algorithm is decreasing.*

Theorem 5. *The complexity of the fast branch and bound algorithm in the worst case is $O(|\mathbb{P}|^{\sum_{i \in \mathbb{I}} K_i})$.*

The complexity of this algorithm is much better than that of original utility maximization model, which is $O(2^{|\mathbb{E}| \sum_{i \in \mathbb{I}} K_i})$.

4 Numerical Results

Consider a sample network (as Fig. 1 shows) taken from [9], where $\mathbb{V} = \{$node o, node 1, …, node $d\}$ and $\mathbb{E} = \{e_k,\ k = 1, 2, \ldots, 20\}$ denote the set of nodes and the set of links in the network respectively. Each connection is delivered from source node o to destination node d. There are three different QoS classes, where class 1 has the highest priority and class 3 has the lowest priority. Every connection in class i, for $i = 1, 2, 3$, has the same bandwidth requirement $b_1 = 160$ (Mbps), $b_2 = 80$ (Mbps), $b_3 = 25$ (Mbps) mean packet size $\sigma_1 = 35$ (Mb), $\sigma_2 = 16.6$ (Mb), $\sigma_3 = 12.5$ (Mb), and maximal end-to-end delay constraint $D_1 = 0.89$ (millisecond), $D_2 = 1.02$ (millisecond), $D_3 = 2.34$ (millisecond).

Under the total available budget $B = \$2,000,000$, we plan to allocate the bandwidths in order to provide each class with maximal utility (1). Table 1 provides a database (an optimal solution) for given parameters $(K_1, K_2, K_3) = (80, 120, 150)$ and $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$. In Table 1, the column of Optimal Paths p's shows the paths selected in the network management scheme. The path flow $\theta_{i,p}$ is the aggregate bandwidth of connections through path p in class i. The number of connections (in each class) on some paths is also determined. By the computation of (5), we list the maximal ene-to-end delay $D(p)$ along the path p for each class. We conclude numerical results of several test problems in Table 2. Optimal objective value, bandwidth allocation and CPU time are listed in Table 2 to compare those results solved by GAMS and by the fast branch and bound algorithm. We find that it takes less time by the fast branch and bound algorithm than by GAMS when the network (size) is large. In the case of $(|\mathbb{V}|,|\mathbb{E}|) = (20,43)$, the (optimal) solution obtained by the fast branch and bound algorithm is better than that by GAMS.

5 Conclusions

We present an approach for the bandwidth allocation and QoS routing in All-IP networks. Solving the network management scheme by the fast branch and bound algorithm, we can find the optimal bandwidth allocation on the network under a limited available budget. Our approach is executed in advance and its purpose is to precompute solutions as a database for later usage, which selects one of the solutions by performing a few additional computations when connections arrive.

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Table 1: A database as $(K_1, K_2, K_3) = (80, 120, 150)$ and $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$

Class i	Optimal Path p	Path Flow $\theta_{i,p}$ (Mbps)	No. of Connections	No. of Links $n(p)$	Delay $D(p)$ (millisecond)
1	$e_1 - e_3 - e_{11} - e_{17}$	2713	9	4	0.556
	$e_2 - e_5 - e_{11} - e_{17}$	2412	8	4	0.558
	$e_2 - e_6 - e_8 - e_{11} - e_{17}$	13264	44	5	0.723
	$e_2 - e_6 - e_{10} - e_{13} - e_{15} - e_{19}$	2713	9	6	0.886
	$e_2 - e_6 - e_{10} - e_{13} - e_{16} - e_{20}$	3014	10	6	0.886
2	$e_1 - e_3 - e_{11} - e_{17}$	2822	17	4	0.492
	$e_2 - e_5 - e_{11} - e_{17}$	3652	22	4	0.494
	$e_2 - e_6 - e_8 - e_{11} - e_{17}$	9794	59	5	0.642
	$e_2 - e_6 - e_{10} - e_{13} - e_{15} - e_{19}$	1826	11	6	0.789
	$e_2 - e_6 - e_{10} - e_{13} - e_{16} - e_{20}$	1826	11	6	0.789
3	$e_1 - e_3 - e_{11} - e_{17}$	1904	34	4	0.985
	$e_2 - e_5 - e_{11} - e_{17}$	1960	35	4	0.987
	$e_2 - e_6 - e_8 - e_{11} - e_{17}$	1624	29	5	1.258
	$e_2 - e_6 - e_{10} - e_{13} - e_{15} - e_{19}$	1568	28	6	1.528
	$e_2 - e_6 - e_{10} - e_{13} - e_{16} - e_{20}$	1344	24	6	1.528

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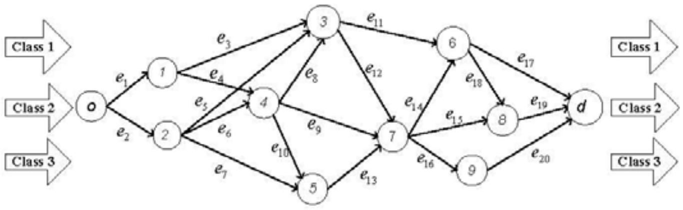
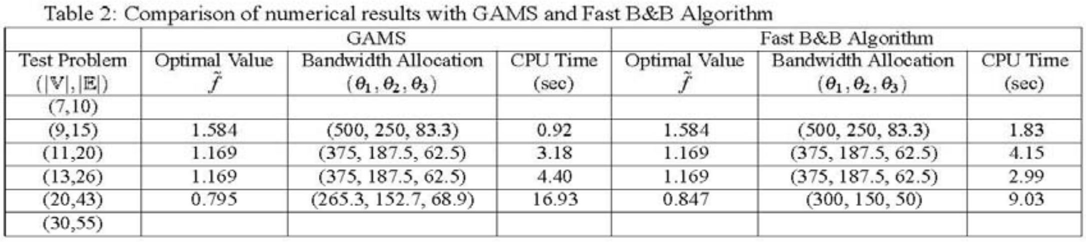


Figure 1: A network architecture

Table 2: Comparison of numerical results with GAMS and Fast B&B Algorithm



	GAMS			Fast B&B Algorithm		
Test Problem $(\lvert\mathbb{V}\rvert, \lvert\mathbb{E}\rvert)$	Optimal Value $\tilde{f}$	Bandwidth Allocation $(\theta_1, \theta_2, \theta_3)$	CPU Time (sec)	Optimal Value $\tilde{f}$	Bandwidth Allocation $(\theta_1, \theta_2, \theta_3)$	CPU Time (sec)
(7,10)						
(9,15)	1.584	(500, 250, 83.3)	0.92	1.584	(500, 250, 83.3)	1.83
(11,20)	1.169	(375, 187.5, 62.5)	3.18	1.169	(375, 187.5, 62.5)	4.15
(13,26)	1.169	(375, 187.5, 62.5)	4.40	1.169	(375, 187.5, 62.5)	2.99
(20,43)	0.795	(265.3, 152.7, 68.9)	16.93	0.847	(300, 150, 50)	9.03
(30,55)						