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非線性阻尼型雙曲方程之研究 (On Nonlinear Damped Hyperbolic Equations)

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共同主持人：
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中英文摘要

On Nonlinear Damped Hyperbolic Equations

Abstract : We show the existence of global solutions of the initial boundary value problem for Rayleigh equations. The existence proof is based on fixed point theorem, Sobolev - Poincaré inequality, and Gagliardo-Nirenberg inequality.

Keywords : Rayleigh equation, global solution

非線性阻尼型雙曲方程之研究

中文摘要 ： 我們探討 Rayleigh 方程的邊界值問題，利用定點理論及一些不等式諸如 Sobolev –Poincaré 和 Gagliardo-Nirenberg 不等式，我們證明了大域解的存在性。

關鍵詞： Rayleigh 方程，大域解

On nonlinear damped hyperbolic equations

1. Introduction

We shall consider the existence of global solutions of the following initial boundary valueproblem (*) for the Rayleigh equation

$$\begin{cases} u_{tt} - u_{xx} = \alpha u_t - \beta (u_t)^3 \text{ in } Q, \\ u(x,t)|_{\partial\Omega} = 0, \ t \in (0,T), \\ u(x,0) = \phi(x), \ x \in \Omega, \\ u_t(x,0) = \psi(x), \ x \in \Omega, \end{cases} \qquad ((1))$$

where $\Omega = (0,1)$ and $Q_T = \Omega \times (0,T), T > 0$.

The motivation is from the study of the space dependent Rayleigh type equation for samll positive $\alpha = \beta = \varepsilon$ in [1]. It models the approximation of the growth of wind-induced (galloping) oscillations in overhead transmission line. This problem has been analyzed by using the perturbation method in Krylov-Bogoliubov [2] and it is shown that a stable limit cycle exists for some initial conditions. Keller and Kogelman [3] used two-time method to determine the asymptotic behavior of the solution when ε is small. The solution will approach one of these periodic solutions as $t \to \infty$. It is also found that for some special initial conditions, the solution tends to a combination of a finite number of periodic solutions and theses solutions are unstable, and only the individual periodic solution is stable. However for ε large , it remains open. For a certain class of nonlinear dampong terms, Haraux [4] gave a more complete study in this respect, and obtained stability and oscillation results. Carpio[5] also proved a global existence to some dissipative wave equations. However, their results are not applicable to our problem. In this paper we shall give the analytical proof of the existence of the solution, and some delicate estimates for the solutions are obtained. Banach fixed point theorem, Sobolev -Poincar?inequality, Gagliardo-Nirenberg inequality are used to prove the existence..

Hereafter we shall assume that

$$\begin{aligned} \phi &\in H_0^1(\Omega) \cap H^2(\Omega), \\ \psi &\in H_0^1(\Omega). \end{aligned}$$

Note that problem (*) can be rewritten in the following form

$$\begin{cases} u'' + Au = \alpha u' - \beta (u')^3 \\ u(0) = \phi \\ u'(0) = \psi \\ u(x,t)|_{\partial\Omega} = 0 \end{cases}, \qquad ((2))$$

here we denote

$$' = \frac{d}{dt}, \ A = -\frac{\partial^2}{\partial x^2}, \ A^{\frac{1}{2}} = -\nabla, \ \|\cdot\| = \|\cdot\|_{L^2(\Omega)}$$

2. Preliminaries

Before proving the existence, we need the following lemmas.

Lemma 1 *(Banach Fixed Point Theorem) If mappiing $S : X \to X$ is a contraction in a complete metric space X. Then there exists a unique fixed point $u \in X$ such that $Su = u \in X$*

Lemma 2 *(Sobolev-Poincare Inequality)*
Let $v \in D\left(A^{\frac{m}{2}}\right)$ and $1 \le p \le \frac{2N}{[N-2m]^+}$ $(1 \le p < \infty$ if $N \le 2m)$
Then the inequality $\|v\|_p \le c_* \left\|A^{\frac{m}{2}} v\right\|$
holds with some constant c_*

Lemma 3 *(Gagliardo-Nirenberg Inequality) Let $1 \le r < p \le \infty$ and $p \ge 2$*
Then, for $v \in D\left(A^{\frac{m}{2}}\right) \cap L^r(\Omega)$, the inequality $\|v\|_p \le c_* \left\|A^{\frac{m}{2}} v\right\|^\theta \|v\|_r^{1-\theta}$
holds with some constant c_* and $\theta = \left(\frac{1}{r} - \frac{1}{p}\right)\left(\frac{1}{r} + \frac{m}{N} - \frac{1}{2}\right)^{-1}$
provided that $0 < \theta \le 1$ if $m - \frac{N}{2}$ is a nonnegative integer.

Theorem 4 *(Existence for linear theory) If A and B have matrix elements in $C([0,T], C_B^k(R^n))$, $k \ge 0$ and f has vector components in $C([0,T], H^k(R^n))$ and $g \in H^k(R^n, R^N)$, $N \ge 1$. then*

$$\begin{aligned} u_t + A(x,t)u_x + B(x,t)u &= f(x,t) \quad in \quad R^n \times (0,T) \\ u(0) &= g \quad in \quad R^n \end{aligned}$$

admit a unique weak solution

$$u \in L^\infty\left((0,T), H^k(R^n, R^N)\right)$$

with

$$u_t \in L^\infty\left((0,T), H^{k-1}(R^n, R^N)\right)$$

Suppose, in addtion that,

$$g \in H^k(R^n, R^N) \quad for \quad k \ge 2 + \frac{n}{2}$$

Then $u \in C^1\left((0,T) \times R^n, R^N\right)$.

3. Existence

We shall prove the local existence and uniqueness of the solution of (1) by using Banach fixed point theorem.

Let

$$D(A) = H^2(\Omega) \cap H_0^1(\Omega),$$
$$D\left(A^{\frac{1}{2}}\right) = H_0^1(\Omega),$$

and

$$X = C^0(0,T; D(A)) \cap C^1\left(0,T; D\left(A^{\frac{1}{2}}\right)\right).$$

Theorem 5 *(local existence and uniqueness). Assume that*

$$e_2(u(0)) \equiv \left\|A^{\frac{1}{2}}\psi\right\|^2 + \|A\phi\|^2 < R^2,$$

then there exists a unique local solution

$$u \in C^0(0,T;D(A)) \cap C^1\left(0,T;D\left(A^{\frac{1}{2}}\right)\right)$$

on $[0,T]$ *of problem (1) for* T *depending on* R *and* $e_2(u(0))$.

Proof : Let

$$e_1(u(t)) = \|u'(t)\|^2 + \left\|A^{\frac{1}{2}}u(t)\right\|^2,$$

and

$$e_2(u(t)) = \left\|A^{\frac{1}{2}}u'(t)\right\|^2 + \|Au(t)\|^2.$$

For $T > 0$ and $R > 0$, we define the two-parameter space $X_{T,R}$ by

$$X_{T,R} = \left\{v \in X : e_2(v(t)) \le R^2 \text{ on } [0,T], v(0) = \phi, v'(0) = \psi, v(t)\mid_{\partial\Omega} = 0 \;\right\}$$

and let the distance function d on $X \times X$ be defined by

$$d(u,v) = \sup_{0 \le t \le T} \left[e_1(u-v)\right]^{\frac{1}{2}}.$$

Then we see that $(X_{T,R}, d)$ is a complete metric space.

For $v \in X_{T,R}$, define

$$Sv = u$$

where u is a unique solution of the linear problem :

$$\begin{cases} u'' + Au - \alpha u' = -\beta (v')^3 \\ u(0) = \phi, u'(0) = \psi, u(x,t)\mid_{\partial\Omega} = 0 \end{cases} \qquad ((3))$$

We also show that S is a contraction mapping from $X_{T,R}$ into itself provided that $T < T^*$ for $R > [e_2(u(0))]^{\frac{1}{2}}$ .By Banach fixed Point Theorem, there exists a unique fixed point u of S in $X_{T,R}$ which is the solution of the problem (1) on $[0,T]$ for arbitrary constant α and β.

Note that for any solution u of (1) defined on $[0,T]$, $T > 0$, after multiplying (1) by $2Au'$ and then integrating over Ω, we get

$$\frac{d}{dt}e_2(u) = 2\alpha\left\|A^{\frac{1}{2}}u'\right\|^2 - 6\beta\left\langle\left(A^{\frac{1}{2}}u'\right)^2, (u')^2\right\rangle$$

If $\frac{d}{dt}e_2(u(t)) \le 0$ on $[0,T]$, then $e_2(u(t))$ is nonincreasing in t. Hence we have

$$e_2(u(t)) \le e_2(u(0)) < R$$

on $[0,T]$. Furthermore, by Poincaré inequality, we have

$$e_1(u(t)) \leq c^2 e_2(u(t)) \leq c^2 R^2$$

whenever the solution u of (1) exists on $[0,T]$.

Theorem 6 *(Global existence) : If α and β are in one of the following cases,*

$$\begin{array}{l} (i)\ \alpha \leq 0 \quad and\ \beta > 0, \\ (ii)\ \alpha < 0 \quad and\ \beta < 0, \quad \alpha \leq 3\beta C R^2, \\ (iii)\ \alpha > 0 \quad and\ \beta > 0, \\ (iv)\ \alpha \geq 0 \quad and\ \beta < 0, \end{array}$$

then we have the global existence of solution for (1).

Next we prove the global uniqueness of the solution of (1).

Theorem 7 *(Global Uniqueness) : Let u and v be two solutions of (1) on $[0,T]$, then $u = v$ on $[0,T]$.*

4. References.

[1] Myerscough,C.J., A simple model of the growth of wind induced oscillations in overhead lines, Journal of Sound and Vibration,28(1973),699-713.

[2] Krylov,N.M. and Bogoliubov N. N., Introduction to nonlinear mechanics, Princeton Univ. Press, (1937).

[3] Keller J.B. and Kogelman S., Asymptotic solutions of initial value problems for nonlinear partial differential equations, SIAM J. Appl. Math.,18 (1970),748-758.

[4] Haraux A., Nonlinear evolution equations : global behavior of solutions, Springer-Verlag, 1981.

[5] Carpio A.., Existence of global solutions to some nonlinear dissipative wave equations, J. Math. Pures et Appl. 73(1994), 471-488.

計畫成果自評：本計畫的收穫是證明了大域解的存在性，結果尚滿意，惟尚有一些未解決的問題，諸如爆炸解，週期解的存在性，週期解的擾亂表現式，有待日後解決。