行政院國家科學委員會專題研究計畫 成果報告

動態財務分析與模擬最佳化在產險公司的應用(2/2)
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行政院國家科學委員會補助專題研究計畫 成果報告

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中文摘要

資產配置對產險公司的營運很重要。資產配置會影響產險公司的收益、風險、財務健全度、甚至保費的釐定。因此不論是產險公司的經理人、股東、監理機關、以及評等公司都很注重產險公司的資產配置。

學術界提供了兩大類的資產配置方式：效率前緣分析以及動態控制法。前者是在單期的架構下分析投資組合的期望報酬率與風險，忽略了其他的統計量，多期來看也通常得不到最佳的結果。動態控制法在理論上無懈可擊，可是只能在極特殊的假設下才能求出封閉解，數量方法也只能處理少數幾個狀態變數，因此在實務上幾乎沒有實用價值。想跳脫效率前緣單期的架構，又不想落入動態控制執行困難窘境的業者，大多先以效率前緣的方法先求出第一期的資產配置，然後隨著市場的變化，再定時或不定時地回歸第一期的配置。此外，財務的文獻在討論資產配置時都沒有考慮到產險的業務，而保險的文獻也沒有人討論到產險公司的最適資產配置。

本計畫利用近年來在作業研究領域有長足發展的模擬最佳化，求解產險公司可能的最適資產配置。我們先開發產險公司的營運模擬程式。這類的程式在歐洲被稱為內部模型，在美國則是用於動態財務分析。程式中產險公司可以投資於現金、各種到期時間的公債、股票、以及不動產，其業務則有長尾與短尾兩大類，其損失發展期間各為十期與三期。模擬的期間是 26 期。寫好產險公司的模擬程式後，我們再以基因演算法以及演化策略法，在國家高速電腦中心的電腦上，以平行處理的方式，藉著多次的模擬，找出可能的最適資產配置。經過仔細的比較後，我們確定利用模擬最佳化所找出的解，比運用效率前緣或回歸法的結果還好。因此，這個計畫的結果除了對保險文現有貢獻之外，還有潛在的實用價值。

本計畫的結果至少可以產生三篇學術論文。第一篇已經投稿出去，第二篇即將完成，第三篇也已經有完整的結果了。

關鍵詞：產險公司、資產配置、模擬最佳化

Abstract

Asset allocation is imperative to property-casualty (P/C) insurance companies. It affects an insurer's return, risk, solvency, and even premium setting. Various stakeholders including managers, shareholders, regulators, and rating agencies pay close attention to the asset allocation of the P/C insurer.

The literature provides two ways for asset allocation: efficient frontier analysis and dynamic control method. The former analyzes a portfolio's return and standard deviation under a single-period framework. This method ignores other statistics of the outcomes and generates sub-optimal results in multiple periods. Dynamic control method is theoretically sound. However, closed-form solutions can be obtained under rare circumstances and numerous methods can handle only few state variables. Investors who want to execute multi-period asset allocation rely on the so-called re-balancing methods. This method has no theoretical justifications though. Furthermore, the finance literature does not take the underwritten businesses into account. The insurance literature, to our knowledge, provides no reference for the optimal asset allocation of P/C insurers.

This project utilizes one of the recent advances in operational research, simulation optimization, to search the optimal asset allocation of a P/C insurer. We first develop a program to simulate the operations of the insurer. The simulated insurer can invest in cash, bonds with different maturities, stock, and real estate. The insurer underwrites both short- and long-tail businesses. The simulation goes on for 26 periods. Then we apply the genetic algorithms (GA) and evolution strategies (ES) upon the developed program to solve the optimization problems. We employ parallel computation to simulate sufficiently large number of simulations to secure the robustness of our optimization search. The resulted

allocations do perform better that the ones obtained using the efficient frontier and re-balancing methods. Our results thus have potential practical value in addition to the contribution to the insurance literature.

This project will results in at least three journal articles. We have submitted one paper to an international journal and are wrapping up the second one. The results for the third paper are in hands already.

Keywords: property-casualty insurance companies; asset allocation; simulation optimization

報告內容

以下的內容分為三節。第一節是一篇由本計畫產生的審查中論文[1]。第二節則是由本計畫產生的第二篇論文，剛完成初稿。第三節是打算用來寫第三篇論文的結果。

[1] 這一篇文章也是本計畫的期中報告內容。

第一節

Combining Dynamic Financial Analysis with Simulation Optimization to Solve the Asset Allocation Problem of the Property-Casualty Insurer [a]

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ABSTRACT

Dynamic financial analysis (DFA) is a useful decision-support system for the insurer, but it lacks optimization capability. The contribution of this paper is that it incorporates a simulation optimization technique into a DFA system. With the ability to optimize, the DFA system proposed in this paper was able to solve the asset allocation problem of a property-casualty insurance company. The simulation optimization technique used herein is a generic algorithm, and the optimization problem is a constrained multi-period asset allocation problem. We find that coupling DFA with simulation optimization resulted in significant improvements over a basic search method. The result was robust across random number sets. Furthermore, the resulting asset allocation changes with the parameters of the risk models as well as the insurer's specifications in a way that is consistent with the differences in the parameters. Incorporating optimization features in DFA is therefore feasible, useful, and robust and should create considerable interest in the insurance industry.

Keywords: dynamic financial analysis; simulation optimization; asset allocation

JEL Classification: G22, C61

I. INTRODUCTION

How to manage a property-casualty (P&C) insurance company is a major issue for various stakeholders including managers, shareholders, policyholders, and regulators. More specifically, the concerns are as to how a number of factors such as asset allocation, trading styles, capital structure, business growth, business allocation, reinsurance arrangement, and other decisions affect the value and the solvency of a P&C company. Assessing these various decisions is difficult, however, because a P&C insurance company is subject to double-sided uncertainties: the liability risk as well as the asset risk. In addition to the uncertainty regarding asset values, P&C insurance companies do not know how much they will have to pay for the products they sold. A comprehensive tool that simultaneously assesses both the liability and the asset risks of a P&C insurer is essential to competent and sound management.

The dynamic financial analysis (DFA) system is promising. A company-wide DFA system can simulate the distribution of an insurer's surplus/equity at some point of time in the future under various assumptions about the insurer's underwriting and investment strategies, the underwriting outcome, and the evolution of the financial markets. An insurer's value and risk/solvency can then be defined upon the simulated surplus distribution, and people can use the simulated surplus distribution to make choices among alternative strategies. More specifically, a DFA system is capable of incorporating an insurer's new businesses, the uncertain payments for the insurance products sold, the insurer's asset allocation/disposition decisions, and the stochastic returns of financial assets to dynamically simulate the evolution of the insurer's financial conditions. Alternative underwriting and/or investment strategies can then be compared based on how they affect the development of the insurer's financial conditions. For instance, an insurer can use a DFA system to assess asset allocation strategies by examining the impacts of alternative strategies on the surplus distribution over a target time horizon. The DFA system therefore can help managers make investment and business decisions in a comprehensive and robust way.

The DFA system has two major advantages over the commonly used financial ratio analysis and other static analyses. First, a DFA system can incorporate future external changes and internal decisions in addition to the information embedded in financial ratios.[2] It is therefore superior to static analyses for profiling an insurer's financial strength. Second, a DFA system explicitly considers the relations among risk factors and financial variables. Risk factors such as interest rates, equity asset prices, and real estate prices are correlated. The values of an insurer's various types of assets are thus correlated with each other as well. Financial variables are further bound by two simple equations:

$$\sum_i Asset_{i,t} - \sum_j Liability_{j,t} = Surplus_t \text{, and} \tag{1}$$

$$\sum_i \Delta Asset_{i,t} - \sum_j \Delta Liability_{j,t} = \Delta Surplus_t \tag{2}$$

, where $Asset_{i,t}$ and $Liability_{j,t}$ represent the values of individual asset and liability items at time t respectively, and $\Delta(.)$ denotes the change of the variable. The first equation depicts the fundamental relations among financial variables at any point in time; the second equation captures the dynamic relations among the variables across time. The financial ratio analysis and other static analyses have difficulties in taking full account of the correlations, fundamental relations, and dynamic relations among the variables.

The construction of a DFA system for the P&C insurance company dates back to almost two decades ago. Insurance and actuarial scholars started conceptual discussions in the late 80s (Pentikainen, 1988; Taylor and Buchanan, 1988; Coutts and Devitt, 1989; Paulson and Dixit, 1989; Taylor, 1991). The British Institute of Actuaries Working Party on Insurance Solvency and actuaries soon developed P&C insurance company simulation models that could be used to evaluate the solvency of a company (Daykin et al., 1989; Daykin and Hey, 1991; Daykin, Pentikainen, and Pesonen, 1994). The Casualty Actuarial Society of the United

[2] The information embedded in financial ratios is taken into account in the DFA system in the form of the initial position inputs.

States embarked upon a long-term, multi-stage project entitled "Dynamic Financial Analysis" in the mid 90s. Starting from identifying risk factors and variables, this project not only developed general specifications for insurance company financial models but also studied refined issues such as model parameterization, result interpretation, and management/strategic usage. The potential of a DFA model was demonstrated by Cummins, Grace, and Phillips (1999) in which the scenario analysis conducted using a simple cash flow model outperformed the early warning and capital requirement systems employed in the United States for the P&C insurance company.

The DFA system, albeit powerful, tells us only which proposed strategy is better. It cannot tell us what the optimal strategy is. The DFA system generates surplus distributions, given users' input about initial positions and strategies. It does not have the mechanism/algorithm to search for the optimum. Managers therefore have to make educated guesses on what the optimal strategy looks like and employ the trial-and-error method to shoot for a good strategy. Trying all possible strategies to seek for the optimum is infeasible due to the large number of decision variables. A DFA system without an optimization mechanism is therefore incapable of helping managers maximize the shareholders' value. The goal of this paper is to illustrate how to couple the technique of simulation optimization with a DFA system so that an insurer can use the improved DFA system for making optimal decisions.

Simulation optimization is the process of determining the values of the controllable input variables that optimize the values of the stochastic output variables generated by a simulation model. The controllable input variables, also called decision variables, in the case of a DFA may include asset allocation, trading frequency, rebalancing interval, capital structure, business growth, business allocation, and reinsurance arrangement.[3] The output variables, also called the response variables, are usually a function of the expected value of simulated

[3] A simulation model might also have parameters that are not controllable. For instance, the financial market and the underwriting market parameters are uncontrollable inputs in the DFA system.

surplus, insolvency probability, and other concerns of the board (e.g., meeting the capital requirement). The simulation model itself (a DFA system in this paper) can be thought of as a complex function mapping controllable input values to response values.[4] In short, the simulation optimization problem can be characterized as a stochastic search over a feasible exploration region (Keys and Rees, 2004).[5]

Tekin and Sabuncuoglu (2004) classified the techniques for simulation optimization into two main headings: local optimization and global optimization. Local optimization techniques assume that response values have a uni-modal surface. Some of them are iterative while some require gradient information. Therefore, when the response surface is high-dimensional, discontinuous, and/or non-differentiable, local optimization techniques are often trapped into a local optimum and fail to find the optimal solution. On the other hand, global optimization techniques such as evolutionary algorithms, simulated annealing, and tabu search can be applied to these types of problems, and they are designed for problems with multi-modal response surfaces.

The contribution of this paper is applying one of these global optimization techniques to a DFA system to solve the asset allocation problem of the property-casualty insurance company. Our DFA system contains four asset classes (cash, bonds, stocks, and real estate) and two types of insurance businesses (long- and short-tail businesses) to capture the essence of the insurance company's operations. Although the DFA system is simple when compared with commercial packages, it is complex enough to preclude one from finding optimal decision

[4] Due to its stochastic nature, repeated runs of the model lead to different outputs even when using the same values of controllable inputs. The average value of the output is often calculated, and the deviations from the average are subsequently analyzed.

[5] The feasible region is defined by the practical limits on the ranges of the controllable inputs. Examples of practical limits include short-sale constraints and upper bounds on portfolio weights faced by most financial institutions. The optimization problem is difficult to solve for several reasons. First, the function represented by the simulation model is almost always unknown. Second, the deviations from the average are usually significant, heterogeneous over the feasible region, and not normally distributed. Third, the feasible region is usually large because of the large number of controllable variables.

variables analytically.[6] We therefore resort to some of the recent developments in simulation optimization. We choose one of the most popular evolutionary algorithms (EAs), the genetic algorithms (GAs), to optimize our DFA system. EAs work on a population of solutions in such a way that poor solutions become extinct while good solutions evolve to reach for the optimum. The most popular EAs are GAs, evolutionary programming (EP), and evolution strategies (ES). EP and ES have not yet been widely used in simulation optimization, but GAs have been successfully applied to the optimization problems arising in complex manufacturing systems (Tekin and Sabuncuoglu, 2004). In our simulation optimization problem, the objective function incorporates the expected discounted surplus as well as the insolvency probability. This objective function will result in an asset allocation that balances return with risk. The optimization problem is formulated as a multi-period one with short-sale constraints.[7]

Our results show that the application of simulation optimization to DFA is feasible, useful, and robust. Our GA introduces a significant improvement over a basic search method. The resulting "optimal" asset allocations look reasonable without extreme positions.[8] Furthermore, our method is robust across different random numbers. The value of the objective function is insensitive to the generated random numbers. Different parameter sets result in different optimal asset allocations, as expected, and the changes in the optimal solutions are comprehensible with respect to the differences in the parameters. Therefore it can be concluded that the application of simulation optimization in DFA is successful.

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[6] The impossibility is due to three reasons. First, the system contains several types of stochastic processes. The function represented by the simulation model is thus unknown. Second, the variations of the outcomes generated by the system are significant, heterogeneous over the feasible region, and not normally distributed. Third, the system has 12 controllable variables over real intervals. The feasible region is therefore large.

[7] Such a problem is difficult to solve. It can be attacked by the methods of dynamic programming, and the solutions are characterized by the Hamilton-Jacobi-Bellman (HJB) partial differential equations. However, the HJB equation has only been solved in few specific cases. Even if the solution can be obtained, the required long-winded technicalities are awkward for practical uses. The short-sale constraints make the problem even more difficult.

[8] Readers should be aware that simulation optimization is a heuristic search method. The existence of the optimal solution is not proven, and there is no verification theorem to show that the resulted solution from simulation optimization is at least as good as all other solutions. The word "optimal" is used loosely in this paper to mean "the best known solution."

The rest of this paper is organized as follows. Section 2 describes our DFA system, including the setting of the financial markets and insurance markets, the dynamics of the representative insurer's financial positions, and the optimization problem of the insurer. In Section 3 our genetic algorithm is described in detail. It starts with an introduction to simulation optimization and is followed by a brief review on genetic algorithms and a detailed description of our proposed algorithm. The application results are discussed in Section 4. We first provide the simulated interest rates, equity index, and real estate index to display some outputs of our DFA system. Then we demonstrate a basic searching method for the single-period asset allocation problem and end section 4 with analyzing the results using our GA. Finally, in Section 5 we make our summaries and draw our conclusions.

II. THE DYNAMIC FINANCIAL ANALYSIS SYSTEM

A. The financial markets and the insurance markets

In this section, we set up five types of markets and specify their stochastic processes. We assume that the risk-neutral process for the one-year spot rate at time *t*, *r(t)*, is:

$$dr(t) = q(m - r(t))dt + \vec{\sigma}_r \sqrt{r(t)} d\vec{W} \tag{3}$$

, where *t* is zero or a positive integer, m stands for the long-term average of spot rates, q reflects the speed of mean reverting ($0 < q < 1$), $\vec{\sigma}_r = [v \quad 0 \quad 0 \quad 0 \quad 0]$, and $d\vec{W} = [dW_r \quad dW_S \quad dW_{RE} \quad dW_{LR(L)} \quad dW_{LR(S)}]'$. $d\vec{W}$ represent the differentials of five-dimension Wiener processes including the processes of the one-year spot rate (*r*), the equity index (*S*), the real estate index (*RE*), the loss ratio of the long-tail insurance liabilities (*LR(L)*), and the loss ratio of the short-tail lines (*LR(S)*). It has a correlation matrix $\Re$ specifying the correlations among the Wiener processes. The mapping from short rates to Treasury bond prices has been derived in Cox, Ingersoll, and Ross (1985): the price at time *t* of a default-free zero-coupon bond that pays \$1 at time *T* equals

$$P_T(t,r) = A_0(T-t)e^{-B(T-t)r} \tag{4}$$

, where T is a positive integer, $T \geq t$, $B(x) = \dfrac{2(e^{rx}-1)}{(r+q)(e^{rx}-1)+2\gamma}$,

$$A_0(x) = \left[\frac{2\gamma e^{\frac{x}{2}(q+\gamma)}}{(q+\gamma)(e^{rx}-1)+2\gamma}\right]^{\frac{2qm}{\sigma^2}}$$

, and $\gamma = \sqrt{q^2 + 2v^2}$.

The equity index is assumed to evolve according to the following interest-rate-adjusted geometric Brownian motion process:

$$\frac{dS(t)}{S(t)} = (r(t) + \pi_S)dt + \vec{\sigma}_S d\vec{W} \quad (5)$$

, where the constant parameter π_S denotes the risk premium on the stock index investment, and $\vec{\sigma}_S = [0 \quad \sigma_S \quad 0 \quad 0 \quad 0]$. We assume that the real estate index follows a geometric Brownian motion:

$$\frac{dRE(t)}{RE(t)} = \mu dt + \vec{\sigma}_{RE} d\vec{W} \quad (6)$$

, where the constant parameter μ denotes the expected return of the real estate investment per period with continuous compounding, and $\vec{\sigma}_{RE} = [0 \quad 0 \quad \sigma_{RE} \quad 0 \quad 0]$.

In the insurance markets, insurers underwrite both long-tail and short-tail businesses. We assume that the loss ratio of the long-tail businesses follows:

$$dLR(L)(t) = \vec{\sigma}_{LR(L)} \times d\vec{W} \quad (7)$$

, where $\vec{\sigma}_{LR(L)} = [0 \quad 0 \quad 0 \quad \sigma_{LR(L)} \quad 0]$. The loss ratio of the short-tail lines has a similar process to equation (7) with a different volatility $\vec{\sigma}_{LR(S)} = [0 \quad 0 \quad 0 \quad 0 \quad \sigma_{LR(S)}]$.

The parameters for the above five models are specified as follows.

Model Parameters			
Short Rate	$m = 6\%$	$q = 0.3$	$v = 2\%$

Equity Index	$\pi_S = 6\%$	$\sigma_S = 20\%$
Real Estate Index	$\mu = 15\%$	$\sigma_{RE} = 35\%$
Loss Ratio (Long)	mean = 75%	$\sigma_{LR(L)} = 30\%$
Loss Ratio (Short)	mean = 80%	$\sigma_{LR(L)} = 25\%$

The starting value of the short-term interest rate is 6%. Furthermore, the correlation matrix $\Re$ is specified as follows.[9]

	dW_S	dW_r	$dW_{LR(L)}$	dW_{RE}
dW_S	1	-0.31	-0.19	0.36
dW_r	-0.31	1	-0.004	-0.03
$dW_{LR(L)}$	-0.19	-0.004	1	-0.47
dW_{RE}	0.36	-0.03	-0.47	1

B. The dynamics of the insurer's financial status

Suppose that a newly established property-casualty insurer starts to underwrite insurance businesses with a surplus of *IS(0)* million dollars. It receives premiums of *IP(0)* million dollars in cash at the beginning of year 1 with *B(0)* ($0 \le B(0) \le 1$) being the proportion of the businesses in the long-tail lines. To underwrite these businesses, the insurer incurs and pays underwriting expenses in cash and upfront. The underwriting expense ratios of the long- and short-tail businesses are assumed to be *Exp(L)* and *Exp(S)* respectively, where both ratios are positive but smaller than one.

The remaining cash and the initial surplus are then invested in cash, Treasury bonds, equity index, and real estate index with the proportion vector $\vec{\theta}(0)$, where

[9] The correlation coefficients are estimated using the historical data on S&P 500 index, Indexes of All Publicly Traded REITs, Treasury Bill Rates, and the loss ratios published in *Best's Aggregates and Averages*. The sampling period is from 1972 to 1999. $dW_{LR(S)}$ is not in the matrix because we assume that the loss ratio of short-tail businesses is independent of other processes.

$\vec{\theta}(0) = [\theta_1(0) \quad \theta_2(0) \quad \theta_3(0) \quad \theta_4(0)]'$, $\theta_1(0)$, $\theta_2(0)$, $\theta_3(0)$, and $\theta_4(0)$ is the proportion of the wealth invested in cash, equity index, Treasury bonds, and real estate index respectively. Obviously $\sum_{i=1}^{4}\theta_i(t) = 1$. We further assume that $\theta_i(t) \geq 0$ since insurers are almost always subject to short-sale constraints from regulation. The maturity of invested bonds ranges from one year to fifteen years, and the invested proportions are assumed to be even across the maturities for the sake of simplicity. Assuming that the fair value of the reserves equal to the premiums written net of expenses, we get the following balance sheet of the insurer at the beginning of year 1:

Assets		Liabilities and Surplus	
Cash	\$ ($\theta_1(0)$ *Total Assets)	Liabilities of Long-Tail Businesses	\$ *B(0)* * *IP(0)* * (1-*Exp(L)*)
Stocks	\$ ($\theta_2(0)$ *Total Assets)	Liabilities of Short-Tail Businesses	\$ (1- *B(0)*) * *IP(0)* * (1-*Exp(S)*)
Treasury Bonds	\$ ($\theta_3(0)$ *Total Assets)		
Real Estate	\$ ($\theta_4(0)$ *Total Assets)	Surplus	\$ *IS(0)*
Total Assets	\$ (*IS(0)* + *B(0)***IP(0)**(1- *Exp(L)*) + (1-*B(0)*)**IP(0)**(1- *Exp(S)*))	Total Liabilities and Surplus	\$ (*IS(0)*+ *B(0)* * *IP(0)* * (1- *Exp(L)*) + (1-*B(0)*) * *IP(0)* * (1- *Exp(S)*))

At the end of the year, investment returns and loss ratios are realized according to the stochastic models in section A.[10] To account for loss development and business growth we assume that the insurer's long-tail businesses grow *G(L)* annually and have a ten-year development period with a loss development function $D_L(dy)$, where dy = 1, 2, 3, …, or 10, $0 \le D_L(dy) \le 1$, and $\sum_{dy=1}^{10} D_L(dy) = 1$. The short-tail businesses have an annual growth rate of *G(S)* and a three-year development period with a loss development function $D_S(dy)$, where dy = 1, 2, or 3, $0 \le D_S(dy) \le 1$, and $\sum_{dy=1}^{3} D_S(dy) = 1$. Simulated loss ratios represent a multiple of the ultimate loss divided by the premiums written, where the ultimate losses for the businesses written in any given year are defined as the total payments across all development years paid for the written businesses[11]. More specifically, the ultimate losses for the businesses written in year *t* equal $\frac{\text{Simulated Loss Ratio} \times \text{Premiums Written in Year t}}{\text{An Adjustment Factor}}$.

The loss payment in development year *dy* for the businesses written in year *t* is then equal to the *t*-th year's ultimate loss times $D_L(dy)$ or $D_S(dy)$.

To pay losses, the insurer sells assets proportionally. Specifically, we assume that the insurer sells each type of invested assets, including cash, Treasury bonds, stocks, and real estate by the proportion of the asset's market value to the total asset's value. The asset allocation of the insurer will thus be unaffected by the sale of assets. We then deduct the amount of losses paid from reserves and attain the year-end balance sheet for year 1.[12]

At the beginning of year 2, the insurer underwrites *IP(1)* million dollars of businesses, pays underwriting expenses for long- and short-tail businesses, and invests the net amount in

[10] We assume that the return on cash is *r(t)*.

[11] The multiple, also called the adjustment factor in the paper, is to account for the effect of growth and time value of money. For an insurer that does not have growth in premiums written, calendar-year loss ratios are equal to the ratio of the ultimate losses to the premiums written. For a growing insurer, however, calendar-year loss ratios will be less than the ratio of the ultimate losses to the premiums written because the denominators of loss ratios grow with time. Furthermore, time value of money should be considered.

[12] Reserves might be smaller than loss payments in extreme cases. We set reserves as zero in these cases and deduct the deficit from surplus.

cash, Treasury bonds, stocks, and real estate with the proportion vector $\vec{\theta}(1)$.[13] Our simulation model then generates investment returns and loss ratios for year 2. The insurer pays losses at the end of year 2 by selling all types of assets proportionally. Reserves are reduced by loss payments, and we obtain the year-end balance sheet for year 2 thereafter. Similar procedures are repeated for twenty-five years, or, repeated until the insurer becomes insolvent. The insurer is deemed insolvent whenever its surplus (*IS(t)*), the difference between the market value of assets and the fair value of reserves, is smaller than zero.

The parameters of the representative insurer are set as follows: *IS(0)* = 120, *IP(0)* = 200, *B(0)* = 50%, *Exp(L)* = 25%, *Exp(S)* = 20%, *G(L)* = 5%, *G(S)* = 4%, and

dy	1	2	3	4	5	6	7	8	9	10
$D_L(dy)$ (%)	50	30	10	5	3	1	0.5	0.3	0.1	0.1
$D_S(dy)$ (%)	80	15	5							

.[14] The above parameters and the parameters of the underlying risk models are chosen so that the insurer has an "adequate" insolvency probability to facilitate subsequent analyses. We tried various sets of parameters and learned that the key variables to the insolvency probability are the initial premium-to-surplus ratio, sum of the expense ratio and expected loss ratio, growth rate, returns of investments, and volatilities of risks. As expected, higher leverage ratios, combined ratios, and/or volatility of risks result in higher insolvency probabilities while higher expected investment returns lead to fewer bankruptcies.

C. The optimization problem

We assume that the insurer's objective is to maximize a utility function over the time horizon [*0, H*]. The utility function consists of two components: expected discounted surplus

[13] Notice that *IP(t+1) = IP(t)*B(t)**(1+*G(L)*) + *IP(t)**(1-*B(t)*)*(1+*G(S)*).

[14] After a simple spreadsheet work, we obtain an adjustment factor of 0.9449 for the long-tail businesses given a growth rate of 5%, a discount rate of 7%, and the specified $D_L(dy)$. The adjustment factor for the short-tail businesses is 0.9905 given a growth rate of 4%, a discount rate of 7%, and the specified $D_S(dy)$.

and ruin probability. The insurer prefers high expected discounted surplus but low ruin probability. More specifically, the optimization problem of the insurer is:

$$\max_{\bar{\theta}(t)} \frac{\sum_{i=1}^{I}[\frac{1}{H}(\sum_{t=1}^{H}\frac{IS(t)}{(1+s)^{t}})]}{I} - k(ruin\_probability - x) \tag{8}$$

, where I is the number of the simulated paths in which no insolvency occurs, s is a constant chosen subjectively by the insurer to discount future surplus $IS(t)$, k is also a constant chosen by the insurer to reflect the relative importance of excessive insolvency probability to expected discounted surplus, and x is the tolerable insolvency probability of the insurer.

The decision variable used to maximize the objective function is the asset allocation $\bar{\theta}(t)$. Allocating more funds to high-risk types of assets may result in higher expected surplus. It will however also result in higher ruin probability at the same time, which may not be optimal. A low ruin probability can be achieved by allocating more funds to low-risk assets. Such a strategy may not generate adequate returns for shareholders on the other hand. Therefore, the optimization problem can be deemed as a search for the optimal balance between risk and return through asset allocation.

In the following simulation, we set H at 25 years and the number of simulated paths at 5,000. The discount rate for future surplus per period s is assumed to be 3%, k is chosen to be 4×10^{10}, and x = 2%. Without loss of generality of the multi-period asset allocation problem, we reduce the optimization problem from 25 years to 4 periods for the sake of computation time.[15] More specifically, the insurer makes asset allocation decisions at t = 0, 6, 12, and 18, and keeps the allocation the same as the previous year's at all other times. The number of controllable variables is thus reduced to 12. A single-period asset allocation problem will have only three controllable variables, which may be solved using other simpler techniques.

III. THE SIMULATION OPTIMIZATION TECHNIQUES

[15] The last period starting from the fourth asset allocation is therefore 7 years.

A. An introduction to optimization via simulation

To solve the optimization problem set up in section II, we make use of the simulation optimization techniques. Optimization in the field of operations research has long been synonymous with mathematical programming. Due to the rapid advances in computational efficiency, there are now many techniques to optimize stochastic systems via simulation. Generally the problem setting is the following parametric optimization problem:

$$\max_{\theta \in \Theta} J(\theta) \tag{9}$$

, where $J(\theta) = E[L(\theta, \omega)]$ is the performance measure of the problem, $L(\theta, \omega)$ is called the sample performance, ω represents the stochastic effects of the system, θ is a *p*-vector of controllable variables, and Θ is the constraint set on θ. Let us also define the optimum as $\theta^* = \arg\max_{\theta \in \Theta} J(\theta)$.

Various simulation optimization techniques have been proposed to solve the above optimization problem. Several survey papers such as Fu (1994), Andradottir (1998), and Tekin and Sabuncuoglu (2004) have provided comprehensive coverage on the foundations, theoretical developments, and applications of these techniques. Existing techniques can be classified into two types: local optimization and global optimization. Local optimization techniques are further classified in terms of discrete and continuous decision spaces.[16] Figure 1, copied from Tekin and Sabuncuoglu (2004), demonstrates the aforementioned classification scheme.

The major difference between local and global optimization techniques lies in the assumption about the shape of the response value surface, i.e., uni-modal or multi-modal. Local optimization techniques are therefore not suitable for those cases in which the function represented by the simulation model is complex and multimodal. These algorithms are

[16] In a discrete space, decision variables take a discrete set of values such as the number of machines in a system. The feasible region in a continuous space, on the other hand, consists of real-valued decision variables such as the release time of factory orders.

usually trapped in a local optimum and generate poor solutions, without an effective method to find good initial solutions. Global optimization techniques are developed to help a search escape from the local optimum. In the SCI and SSCI database we found over four thousand technical papers demonstrating that global optimization techniques such as tabu search, simulated annealing, and evolutionary algorithms can help the search escape from local optimum and produce better solutions.

Among the global optimization techniques, we chose evolutionary algorithms (EAs) for the DFA system. Using EAs in simulation optimization is on the increase lately because they require no restrictive assumptions or prior knowledge about the shape of the response surface (Back and Schwefel, 1993). In general, an EA has the following procedures: generate a population of solutions, evaluate these solutions through a simulation model, perform the selection, apply genetic operators to produce new offspring, and insert the new offspring into the population. These steps are repeated until some stopping criterion is reached.

The most popular EAs are genetic algorithms (GAs), evolution programming (EP), and evolution strategies (ES). These algorithms differ from each other in the representation of individuals, the design of variation operators, and the selection of their reproduction mechanisms. In general, each point in the solution space is represented by a string of values for the decision variables. The crossover operator breaks the strings representing two members of the population and exchanges certain portions of the strings to create two new strings. The mutation operator selects a random position in a string and changes the value of that variable with a pre-specified probability. Appropriate crossover and mutation operators can reduce the probability of being trapped in a local optimum.

Among the GAs, EP, and ES, we chose to employ a GA to optimize the DFA system. The main reason for the choice was that GAs have found more applications for the optimizing problems in complex systems than either the EP or the ES. Also, we have applied GAs to several discrete optimization problems before (Chen et al. 1995, Chen et al. 1996, Chen et al.

2003). The following section gives a brief introduction on GAs and describes our GA in detail.

B. Genetic algorithms

The search procedure of GAs combines reproduction and recombination to mimic the process of natural evolution. An optimization problem solved by GAs can be explained as follows. The solution space of the problem is viewed as the environment of evolution. A solution of the problem is a member of a species in the environment. A generation of the species is presented as a population of solutions. Darwin's concept of survival of the fittest is then applied to the solutions in the population. The objective value of a solution is a measure of its fitness. The better the fitness of the solution, the higher the probability that the solution can be chosen as a parent to produce new solutions (offspring) for the next population (generation). Genetic operators (usually crossover and mutation) have to be applied to the chosen parents to produce offspring. As this process continues for generations, the fitness of the members (objective values of solutions) improves.

Based on this explanation, the procedure of a basic genetic algorithm can be described as follows.[17] Let $S(t)$ denote the population in the t-th generation, $s_i(t)$ the *i-th* member in $S(t)$, $f(s_i(t))$ the fitness of $s_i(t)$, *Totfit* the sum of $f(s_i(t))$ in $S(t)$, *popsize* the population size, and *maxgem* the maximum number of generations for convergence. Then a GA usually has the following steps.

Step 1: Generate an initial population, $S(t)$, where $t = 0$.

Step 2: Calculate the fitness value for each member, $f(s_i(t))$, in population $S(t)$.

Step 3: Calculate the selection probability for each member, which is defined as $f(s_i(t))/Totfit$.

Step 4: Select a pair of members (parents) randomly according to the selection probability.

Step 5: Apply genetic operators to the parents to produce the offspring for the next population, $S(t+1)$. If the size of the new population is equal to *popsize,* then go to Step 6;

[17] The following descriptions are mainly drawn from Chen et al. 1995 and Chen et al. 1996.

otherwise, go to Step 4.

Step 6: If the current generation, *t+1,* is equal to *maxgen,* then stop; else go to Step 2.

According to the above basic procedure, the application of a GA must consider the following factors: (1) representation of a solution, (2) initial population, (3) selection probability, (4) genetic operators, (5) termination criterion, and (6) three parameters: population size, crossover rate, and mutation rate. In the following, each of these factors and parameters will be discussed in detail for a GA to generate the optimal asset allocations through our DFA system.

1. Representation of a solution

A solution for GA application is usually represented by a row vector. The value of an element in the vector refers to the allocation to an asset in a period. The number of elements in the vector depends upon the number of investable assets considered in a period and the number of periods considered in a problem. Since four assets are considered in four periods with the constraint that the sum of the allocations is equal to one, twelve elements are included in the vector in the current application.

2. Initial population

The initial population of our GA is randomly generated as most applications of GAs are. Since the value of an element in the vector is in the range of [0, 1], we first generate three random numbers from the uniform [0, 1] distribution. If the sum of the three generated elements is greater than one, then the elements will be multiplied by 0.9 consecutively until their sum is less than or equal to one. We call this procedure a feasibility-keeping procedure. The purpose of the procedure is to ensure that the fourth element will not be negative and violate the short-sale constraint. Note that the feasibility-keeping procedure has to be implemented to all four periods in each vector.

3. Selection probability

The selection probability of a member in a population should generally project the performance measure of the member. Usually, a member with a better fitness in a population would have a higher probability of being selected. Since the candidate problem is to find an asset allocation that maximizes a specified utility function, the value of the utility function is used as the fitness value of the corresponding asset allocation. The larger the value of the utility function, the higher the probability the corresponding set of allocations will be selected. The following procedure is an application of the well-known roulette-wheel selection scheme for calculating the selection probability of a member in a population.

Step 1: Calculate the fitness *f(i)* for each member in the population.

Step 2: Calculate the total fitness, *Totfit,* of all the members in the population.

Step 3: Calculate the selection probability for each member that is equal to *f(i)/Totfit.*

A complementary selection strategy (elitist strategy) is also considered in the current application. More specifically, the member with the best fitness value in each population will always survive and automatically become a member in the next generation. The purpose is to preserve the best solution so that the search always covers certain good solution regions.

4. Genetic operators

Genetic operators are performed on the parents to generate offspring. Crossover and mutation are two common genetic operators of GAs.

4.1 Crossover

An effective crossover operator, BLX-0.5 (Eshelman & Schaffer, 1993), is used in this research. Two selected parents, vectors A and B, are given and denote the values of an element in A and B as x and y respectively. The BLX-0.5 is implemented to x and y in the following procedure to produce a value z for the element in the offspring generated by A and B.

Step 1: Let $\Delta = 0.5 * |y-x|$.

Step 2: Randomly generate z from the range of $(x-\Delta, y+\Delta)$ if $x \le y$; let $x-\Delta = 0$ if $x-\Delta < 0$, and let $y+\Delta = 1$, if $y+\Delta > 1$. Otherwise, randomly generate z from the range of $(y-\Delta, x+\Delta)$; let

y-Δ = 0 if y-Δ < 0, and let x+Δ = 1 if x+Δ > 1.

For example, let $(x_1, x_2) = (0.5, 0.2)$ be the values of the first two elements in A and let $(y_1, y_2) = (0.1, 0.8)$ be the values of the first two elements in B. The values of the first two elements, (z_1, z_2) of the offspring of A and B can then be generated as follows. Let $\Delta_1 = 0.5 * |y_1 - x_1| = 0.5 * |0.1 - 0.5| = 0.2$ and $\Delta_2 = 0.5 * |y_2 - x_2| = 0.5 * |0.8 - 0.2| = 0.3$. Then randomly generate z_1 from the range of $(y_1\text{-}\Delta_1, x_1+\Delta_1) = (0.1\text{-}0.2, 0.5+0.2) = (0, 0.7)$ and randomly generate z_2 from the range of $(x_2\text{-}\Delta_2, y_2+\Delta_2) = (0.2\text{-}0.3, 0.8+0.3) = (0, 1.0)$.

4.2 Mutation

When a solution is produced by crossover, a mutation operator is applied to the solution. Michaleicz (1996) developed a non-uniform mutation operator and showed that the operator outperformed other mutation operators after performing thorough experiments. A non-uniform mutation operator is applied in this GA application by following the procedure.

Step 1: Randomly select k elements out of the 12 elements in the solution, where $k = 1 + Int[rnd*12]$ and Int is an integer function.

Step 2: Apply the non-uniform mutation operator to each of the k elements. It is given that element i is one of the k elements and z_i^t is the value of element i in the current generation t. The value of element i in generation t+1, z_i^{t+1}, is generated as follows: $z_i^{t+1} = z_i^t + \Delta(t, 1.0 - z_i^t)$ if $rnd < 0.5$; otherwise $z_i^{t+1} = z_i^t - \Delta(t, z_i^t)$, where $\Delta(t, v) = v * (1.0 - rnd^b)$, $b = (1.0 - t/T)^5$, and T is equal to $maxgen$ (the maximum number of generations for convergence).

Note that $b = (1.0 - t/T)^5$ is approaching 0 when t is close to T; when b is approaching 0, (t, v) also approaches 0. Michaleicz (1996) pointed out that this property causes the non-uniform mutation operator to search uniformly the solution space initially (when t is small) and locally at later stages. Note also that after applying the mutation operator to the solution, the feasibility-keeping procedure has to be implemented in the solution to maintain its feasibility.

4.3 Population size, crossover rate, and mutation rate

Population size (*popsize*) is the number of members generated in each generation. Crossover rate is the probability that a crossover operator applies to the chosen parents, and the mutation rate is the probability that a mutation operator applies to the offspring. The population size of 60, as used in all the examples in Michaleicz (1996), is also used in the current application. Both crossover rate and mutation rate are set to be one after several trial runs for the candidate problem.

5. Termination Criteria

The maximum number of generations (*maxgen*) is the most widely used termination criterion for GAs. It is usually determined by trial-and-error. We found that our GA converged within 2000 iterations in all the trial runs in the current application. Therefore, *maxgen* is set to be 2000.

We are now in a position to present the procedure of applying our GA to the DFA.

Step 0: Let *popsize* = 60, *maxgen* = 2000, and $t = 0$.

Step 1: Generate initial population with twelve-element solutions (vectors) $\boldsymbol{V_i}$ ($i = 1, 2, \ldots,$ *popsize*). For each solution, call *rnd* to generate an asset allocation to each of its elements and apply the feasibility-keeping procedure to satisfy the constraint.

Step 2: Calculate the fitness, $f(\boldsymbol{V_i})$, for each solution by conducting the DFA simulation with the asset allocations given in $\boldsymbol{V_i}$.

Step 3: Select two parent solutions based on their fitness. The selection probability of a parent solution $\boldsymbol{V_i}$ is $p(i) = f(\boldsymbol{V_i})/Totfit$.

Step 4: Apply crossover operator, BLX-0.5, to the selected parent solutions.

Step 5: Apply the non-uniform mutation operator to the solution generated in Step 4, and apply the feasibility-keeping procedure to maintain the feasibility of the solution.

Step 6: If the total number of offspring solutions generated is equal to *popsize*, go to Step 7; otherwise, go to Step 3.

Step 7: Let $t = t + 1$. If t is equal to *maxgen*, stop; otherwise go back to Step 2.

The optimization program is coded in C language and is executed using a Linux platform (the OS is the RedHat AS 3.0). The CPU is Intel Itanium 21.5GHz. One of the features of this optimization program is that the dynamics memory allocation is used for all the necessary variables. Therefore, the required memory used is around 70MB when running the program. The program is compiled using the Intel C/C++ compiler in the Linux System, yet it can also be compiled and executed using the Microsoft Window XP platform with Visual Studio C/C++ compiler. For 2000 iterations, this program took around 18,200 seconds to finish on average.

IV. RESULTS

A. General description of the DFA's simulation results

The simulated results of the financial markets are shown in Figures 2 to 4.[18] We plot the simulated 1st percentile, 25th percentile, mean, 75th percentile, and 99th percentile values of each year in these figures. The mean of the simulated short rates remains at the long-term average value of 6%, with fifty percent of the simulated short rates falling within the range of 5.5% to 6.5%. The 1st percentile simulated short rates are about 4.5% while the 99th percentile values are around 7.7%. The means of the equity index and real estate index display upward trends and the ranges of the simulated values widen as the simulation goes on, which is consistent with the specified stochastic processes.[19]

[Insert Figures 2 – 4 Here]

B. Results of a basic search method

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[18] The simulation results of the insurance markets are not reported because the simulation is straightforward. The loss ratio of each year is drawn from a normal distribution.

[19] We have also inspected the figures of the equity returns and real estate returns (not shown in the paper). The 25th percentile, mean, 75th percentile values of equity returns remain at -0.1%, 12%, 24% respectively across time. The minima and maxima of equity returns exhibit some bumps around -60% and 80%, respectively. Real estate returns display similar features with higher means and wider variation ranges.

Before using the simulation optimization technique, we tried out a basic search method in a single-period framework as follows. We list all possible asset allocations using the grid size of 20%. Inserting these allocations into the DFA system and assuming that these allocations are kept to the end of the simulation, we obtain the values of the objective function. These values along with their associated average discounted surpluses as well as ruin probabilities are shown in Table 1.

[Insert Table 1 Here]

Table 1 shows that the best asset allocation is $\vec{\theta}^*(t) = [0 \quad 0.4 \quad 0.4 \quad 0.2]'$. It results in an objective function value of 1,137,275,044 with an average discounted surplus of $881,275,044 and a ruin probability of 1.36%. Although the zero-cash allocation looks odd, it is reasonable because we did not consider liquidity in the model setting. Furthermore, new premiums can cover the loss payments in most cases. The runner-up is $\vec{\theta}(t) = [0.2 \quad 0.4 \quad 0.2 \quad 0.2]'$ with an objective function value of 1,132,889,250, an average discounted surplus of $876,889,250, and a ruin probability of 1.36%. The runner-up allocation generates a little bit less expected surplus, which is reasonable because the return on cash on average is smaller.[20] Number three is $\vec{\theta}(t) = [0.4 \quad 0.4 \quad 0 \quad 0.2]'$ which results in an objective function value of 1,097,049,222, an average discounted surplus of $873,049,222, and a ruin probability of 1.44%. The higher insolvency probability is probably because cash does not generate adequate returns. Number four is $\vec{\theta}(t) = [0 \quad 0.2 \quad 0.6 \quad 0.2]'$. This asset allocation produces an objective function value of 1,022,434,859, an average discounted surplus of $710,434,859, and a ruin probability of 1.22%. Although it generates the smallest ruin probability among all the asset allocations, it produces an inferior average discounted

[20] Remember that the return from cash is the one-year short rate. Since the simulated yield curve is usually upward-sloping, the one-year bond is smaller than longer-maturity bonds. The insolvency probability of the runner-up is the same as the number-one choice implies that the risk of the bond portfolio is small.

surplus compared to the top three allocations. The fifth winner is $\bar{\theta}(t) = \begin{bmatrix} 0 & 0.4 & 0.2 & 0.4 \end{bmatrix}'$ which results in an objective function value of 1,017,769,551, an average discounted surplus of $1,225,769,551, and a ruin probability of 2.52%. Allocating more assets to higher-risk equities produces a significantly higher average surplus, but results in a higher ruin probability.

The ranking of the top five asset allocation looks reasonable. However, we do not spot any pattern revealing between which two asset allocations the optimal allocation might be. Weighting three variables to balance a higher average surplus with lower ruin probability is rather difficult. Reducing the grid size of 20% is therefore the way to go if we do not have any algorithm to search for the optimum. However, this reduction will increase the possible asset allocation dramatically. When the grid size is 20%, the total number of asset allocations is 56. When the grid size is 10%, the number increases to 286. Five percent of grid size will produce 1,771 combinations while one percent will result in 176,851 allocations. Therefore, the basic search method is not feasible in finding the optimal asset allocation even though it is intuitive and instructive.

C. Results of the genetic algorithm

Our GA produces a significantly better result than the basic search method. The value of the objective function is 1,343,396,299 implying an 18% improvement over the above basic search method. The GA results in a significantly higher average discounted surplus ($1,055,396,299 that is 20% higher than that of the winner in section B) and a lower ruin probability (1.28%). The optimal asset allocation is as follows:

t	0	6	12	18
Cash	16.1906%	2.7821%	14.4054%	3.3340%
Stock	32.4451%	45.2235%	47.1426%	40.3112%
Bond	29.6427%	12.7891%	5.7029%	7.0734%

Real Estate	21.7216%	39.2053%	32.7491%	49.2814%

. We plotted the resulting 1st percentile, 25th percentile, mean, 75th percentile, and 99th percentile surplus over time from the optimal asset allocation in Figure 5.

[Insert Figure 5 Here]

To secure the robustness of our application of GA to DFA, we tried two other sets of random numbers and two other sets of parameters of the underlying risk models.[21] Using different sets of random number results in small changes in the objective function value.[22] The resulting values are 1,338,413,716 and 1,358,066,783. Since these values represent a -0.37% and 1.09% difference respectively from the benchmark case described in the previous one paragraph, our application is robust across random numbers.

The other two sets of parameters generate significantly different results. Alternative parameter set 1 resulted in an objective function value of 1,445,571,649, an average discounted surplus of $829,571,650, and a ruin probability of 0.46%. The optimal asset allocation is as follows:

t	0	6	12	18
Cash	34.4349%	18.2585%	0.1492%	2.5802%
Stock	19.6516%	15.9641%	36.8198%	49.8485%
Bond	17.8915%	30.4849%	2.1546%	10.3742%
Real Estate	28.0221%	35.2925%	60.8764%	37.1971%

. The features of this parameter set include fairly profitable but highly volatile financial as well as insurance markets with a moderately positive correlation between the long-tail insurance and financial markets. The position correlation possibly contributes to the lower

[21] The alternative parameter sets are described in the appendix.
[22] The two alternative parameter sets are specified rather arbitrarily. We intentionally make them "unreasonable" to see whether our GA can still find solutions under odd settings. The random number set used for these two alternative parameter sets is the same as the one used in the benchmark case.

ruin probability compared to that in the benchmark case and thus generates a higher objective function value. The lower average discounted surplus might be as a result from the relatively conservative investments during the first twelve years.

Alternative parameter set 2 results in an objective function value of 998,583,316, an average discounted surplus of $198,583,316, and zero ruin probability. The optimal asset allocation is as follows:

t	0	6	12	18
Cash	0.0010%	0.0010%	0.0010%	0.0183%
Stock	14.3156%	17.6433%	25.7770%	21.0308%
Bond	0.0010%	0.0013%	0.0053%	0.1233%
Real Estate	85.6824%	82.3544%	74.2167%	78.8276%

. The features of alternative parameter set 2 include highly positive correlations between the long-tail insurance and the financial markets, relatively safe insurance markets, a fairly profitable but highly volatile stock market, and a low-return but high-risk bond market. The allocations to bonds and cash are thus minimal. Most of the funds are allocated to real estate with some funds to stocks for higher returns. The average discounted surplus under parameter set 2 is the smallest among the three parameter sets because the discount rate for surplus is the highest (45% vs. 3% and 15%). The lower returns in the financial markets might also contribute to the smallest average discounted surplus. The zero insolvency probability is probably due to the low risk in the insurance markets and the high correlation between the financial and long-tail insurance markets. Finally, the significantly lower average discounted surplus under parameter set 2 compared to the other two parameter sets results in the smallest objective among the three parameter sets.

V. SUMMARIES AND CONCLUSIONS

Managing an insurance company is more difficult than managing other types of companies because an insurer faces not only asset risks but also liability risks. The DFA system is a promising tool for the insurer. It takes full account of the static and dynamic relations among asset variables and liability variables. The major output of a DFA system is the distribution of an insurer's future surplus that can be further used to compare alternative asset allocations, business strategies, and reinsurance arrangements, among others. Insurance regulators can use a DFA system to perform an early warning analysis as well as set up minimal capital requirements.

The main drawback of the DFA system is the lack of an optimization mechanism. Users can perform only comparative analysis with no way of knowing what the optimal strategy is. Simulation optimization is receiving considerable interest in the field of operations research and may be a nice complement to the DFA system. By incorporating optimization features in a DFA system, the DFA system turns from a descriptive model into an operational tool to solve various decision-making problems. The contribution of this paper is coupling a DFA system with a simulation optimization technique and applying the combination to the asset allocation problem of a property-casualty insurance company.

We first built up a simply DFA system in which an insurer underwrites both short- and long-tail businesses and invest in four types of assets. Then we formulated the asset allocation problem as a multi-period one instead of a single-period one. A multi-period asset allocation is superior because the accumulation of a sequence of single-period optimal decisions across periods may not be optimal for these periods taken as a whole. We also considered the short-sale constraints faced by insurers when making investments. The capability of solving a constrained multi-period problem illustrates the advantage of simulation optimization, although we must keep in mind that the found solution as a result of simulation optimization cannot be proved to be the optimum. The simulation optimization technique used in this paper is a generic algorithm.

We successfully incorporated a generic algorithm into a DFA system and performed a search for the optimal asset allocation of a property-casualty insurer in this paper. The resulting asset allocation was a significantly higher value of the objective function compared to the allocation found from a basic search method. The optimal allocation produced a higher average discounted surplus and a lower ruin probability. Using different sets of random number generated similar values of objective function and demonstrated the robustness of our coupling across random numbers. The optimal asset allocation is sensitive to the parameters of financial and insurance market models, with the changes being consistent with the differences in the parameters. Therefore, insurance companies that are using or are interested in DFA should learn one of the simulation optimization techniques to equip their DFA systems with optimization features.

References

Andradottir, S., 1998. Simulation Optimization. In: Handbook of Simulation: Principles, Methodology, Advances, Applications, and Practice. Wiley, New York, pp. 307-333.

Back, T., Schwefel, H. P., 1993. An overview of evolutionary algorithms for parameter optimization. Evolutionary Computation 1, 1-23.

Chen, C. L., Vempati, V. S., Aljaber, N., 1995. An application of genetic algorithms for flow shop problems. European Journal of Operational Research 80, 389-396.

Chen, C. L., Neppalli, V. R., Aljaber, N., 1996. Genetic algorithms applied to the continuous flow shop problem. Computers and Industrial Engineering 30, 919-929.

Chen, C. L., Lin, R. H., Zhang, J., 2003. Genetic algorithms for MD-optimal follow-up designs. Computers & Operations Research 30, 233-252.

Coutts, S. M., Devitt, R., 1989. The Assessment of the Financial Strength of Insurance Companies by a Generalized Cash Flow Model. In: Financial Models of Insurance Solvency. Kluwer Academic Publishers, Boston, pp. 1-36.

Cox, J. C., Ingersoll, J. E., Ross, S. A., 1985. A theory of the term structure of interest rates.

Econometrica 53, 385-407.

Cummins, J. D., Grace, M. F., Phillips, R. D., 1999. Regulatory solvency prediction in property-liability insurance: risk-based capital, audit ratios, and cash-flow simulation. Journal of Risk and Insurance 66, 417-458.

Daykin, C. D., Bernstein, G. D., Coutts, S. M., Devitt, E. R. F., Hey, G. B., Reynolds, D. I. W., Smith, P. D., 1989. The Solvency of a General Insurance Company in Terms of Emerging Costs. In: Financial Models of Insurance Solvency. Kluwer Academic Publishers, Boston, pp. 87-149.

Daykin, C. D., Hey, G. B., 1991. A Management Model of a General Insurance Company Using Simulation Techniques. In: Managing the Insolvency Risk of Insurance Companies. Kluwer Academic Publishers, Boston, pp. 77-108.

Daykin, C. D., Pentikainen, T., Pesonen, M., 1994. Practical Risk Theory for Actuaries. Chapman & Hall, London.

Eshelman, L. J., Schaffer, J. D., 1993. Real-Coded Genetic Algorithms and Interval Schemata. In: Foundations of Genetic Algorithms. Morgan Kaufmann Publishers, San Mateo, pp. 187-202.

Fu, M. C., 1994. Optimization via simulation: a review. Annals of Operations Research 53, 199-247.

Keys, A. C., Rees, L. P., 2004. A sequential-design metamodeling strategy for simulation optimization. Computers and Operations Research 31, 1911-1932.

Michalewicz, Z., 1996. Genetic Algorithms + Data Structures = Evolution Programs, 3rd ed. Springer-Verlag, Berlin.

Paulson, A. S., Dixit, R., 1989. Cash Flow Simulation Models for Premium and Surplus Analysis. In: Financial Models of Insurance Solvency. Kluwer Academic Publishers, Boston, pp. 37-56.

Pentikainen, T., 1988. On the Solvency of Insurers. In: Classical Insurance Solvency

Theory. Kluwer Academic Publishers, Boston, pp. 1-48.

Taylor, G., 1991. An Analysis of Underwriting Cycles and their Effects on Insurance Solvency. In: Managing the Insolvency Risk of Insurance Companies. Kluwer Academic Publishers, Boston, pp. 3-76.

Taylor, G. C., Buchanan, R., 1988. The Management of Solvency. In: Classical Insurance Solvency Theory. Kluwer Academic Publishers, Boston, pp. 49-155.

Tekin, E., Sabuncuoglu, I., 2004. Simulation optimization: a comprehensive review on theory and applications. IIE Transactions 36, 1067-1081.

Table 1: The results of the basic research method

Cash	Stock	Bond	Real Estate	Value of the Objective Function	Average Discounted Surplus	Ruin Probability
1	0	0	0	-72,581,897	351,418,103	0.0306
0.8	0.2	0	0	551,978,871	463,978,871	0.0178
0.8	0	0.2	0	-46,112,487	353,887,513	0.0300
0.8	0	0	0.2	552,303,659	552,303,659	0.0200
0.6	0.4	0	0	637,421,327	605,421,327	0.0192
0.6	0.2	0.2	0	570,938,385	466,938,385	0.0174
0.6	0.2	0	0.2	971,157,610	699,157,610	0.0132
0.6	0	0.4	0	12,156,152	356,156,152	0.0286
0.6	0	0.2	0.2	587,409,029	555,409,029	0.0192
0.6	0	0	0.4	336,229,618	840,229,618	0.0326
0.4	0.6	0	0	280,468,735	776,468,735	0.0324
0.4	0.4	0.2	0	664,738,139	608,738,139	0.0186
0.4	0.4	0	0.2	1,097,049,222	873,049,222	0.0144
0.4	0.2	0.4	0	589,926,635	469,926,635	0.0170
0.4	0.2	0.2	0.2	975,076,921	703,076,921	0.0132
0.4	0.2	0	0.4	865,202,808	1,017,202,808	0.0238
0.4	0	0.6	0	70,408,056	358,408,056	0.0272
0.4	0	0.4	0.2	614,605,036	558,605,036	0.0186
0.4	0	0.2	0.4	379,816,127	843,816,127	0.0316
0.4	0	0	0.6	-1,437,343,176	1,234,656,824	0.0868
0.2	0.8	0	0	-979,818,603	988,181,397	0.0692

0.2	0.6	0.2	0	331,768,042	779,768,042	0.0312
0.2	0.6	0	0.2	920,627,902	1,072,627,902	0.0238
0.2	0.4	0.4	0	707,867,321	611,867,321	0.0176
0.2	0.4	0.2	0.2	1,132,889,250	876,889,250	0.0136
0.2	0.4	0	0.4	996,985,281	1,220,985,281	0.0256
0.2	0.2	0.6	0	585,128,292	473,128,292	0.0172
0.2	0.2	0.4	0.2	1,002,693,284	706,693,284	0.0126
0.2	0.2	0.2	0.4	916,984,155	1,020,984,155	0.0226
0.2	0.2	0	0.6	-262,968,275	1,425,031,725	0.0622
0.2	0	0.8	0	120,775,989	360,775,989	0.0260
0.2	0	0.6	0.2	641,808,556	561,808,556	0.0180
0.2	0	0.4	0.4	423,394,768	847,394,768	0.0306
0.2	0	0.2	0.6	-1,377,259,696	1,238,740,304	0.0854
0.2	0	0	0.8	-4,699,438,942	1,764,561,058	0.1816
0	1	0	0	-3,230,097,737	1,249,902,263	0.1320
0	0.8	0.2	0	-881,219,802	990,780,198	0.0668
0	0.8	0	0.2	192,104,867	1,304,104,867	0.0478
0	0.6	0.4	0	351,750,776	783,750,776	0.0308
0	0.6	0.2	0.2	948,987,876	1,076,987,876	0.0232
0	0.6	0	0.4	608,774,850	1,456,774,850	0.0412
0	0.4	0.6	0	727,242,728	615,242,728	0.0172
0	**0.4**	**0.4**	**0.2**	**1,137,275,044**	**881,275,044**	**0.0136**
0	0.4	0.2	0.4	1,017,769,551	1,225,769,551	0.0252
0	0.4	0	0.6	-153,875,069	1,662,124,931	0.0654
0	0.2	0.8	0	604,121,085	476,121,085	0.0168

0	0.2	0.6	0.2	1,022,434,859	710,434,859	0.0122
0	0.2	0.4	0.4	953,078,642	1,025,078,642	0.0218
0	0.2	0.2	0.6	-187,400,811	1,428,599,189	0.0604
0	0.2	0	0.8	-3,183,750,687	1,968,249,313	0.1488
0	0	1	0	131,373,027	363,373,027	0.0258
0	0	0.8	0.2	661,117,051	565,117,051	0.0176
0	0	0.6	0.4	451,314,002	851,314,002	0.0300
0	0	0.4	0.6	-1,309,777,975	1,242,222,025	0.0838
0	0	0.2	0.8	-4,639,208,243	1,768,791,757	0.1802
0	0	0	1	-8,421,281,758	2,482,718,242	0.2926

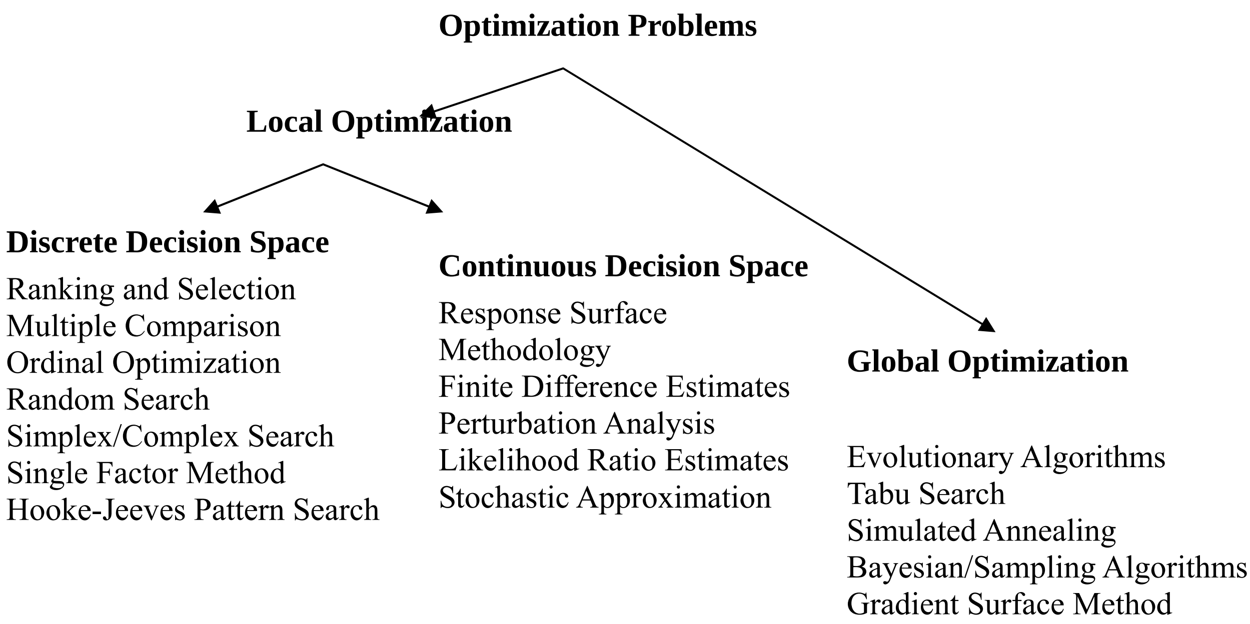

Figure 1: Classification of optimization methodologies

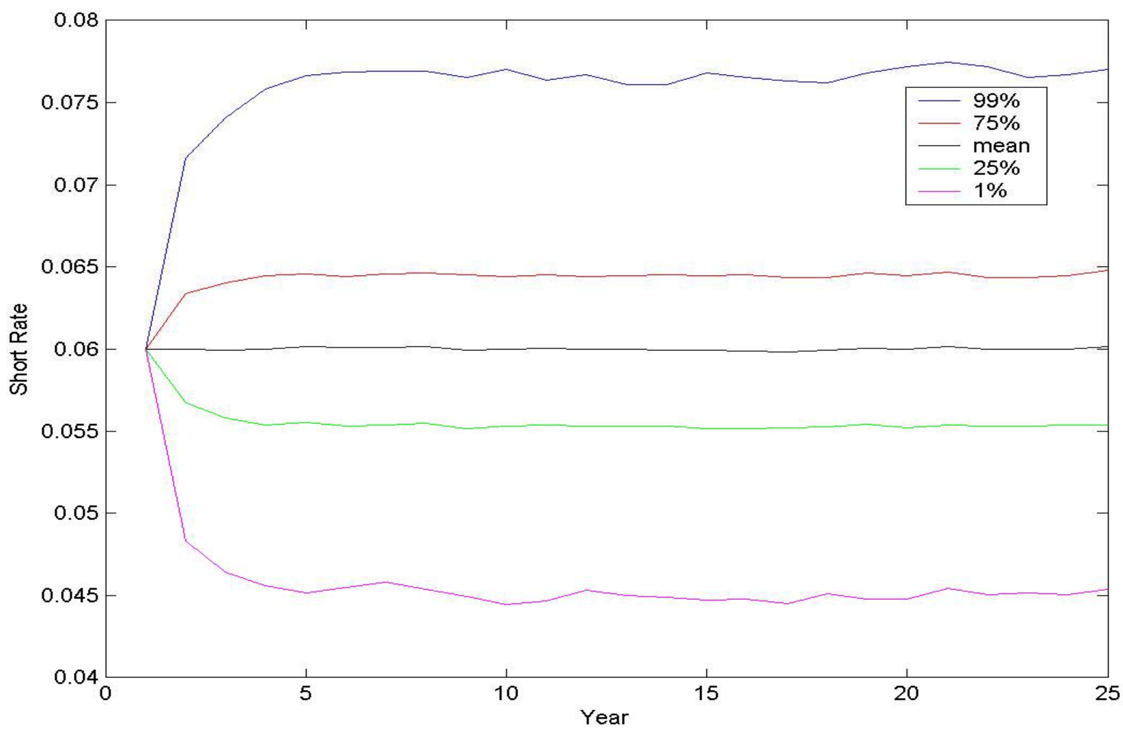

Figure 2: The simulated short rate statistics along with time

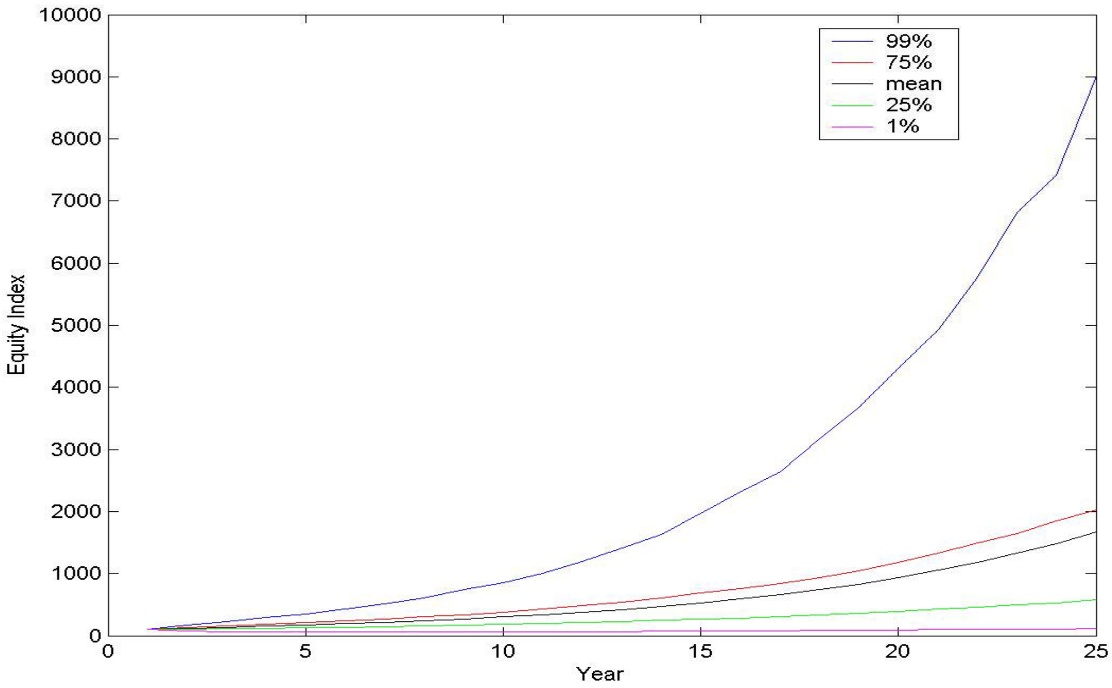

Figure 3: The simulated equity index statistics along with time

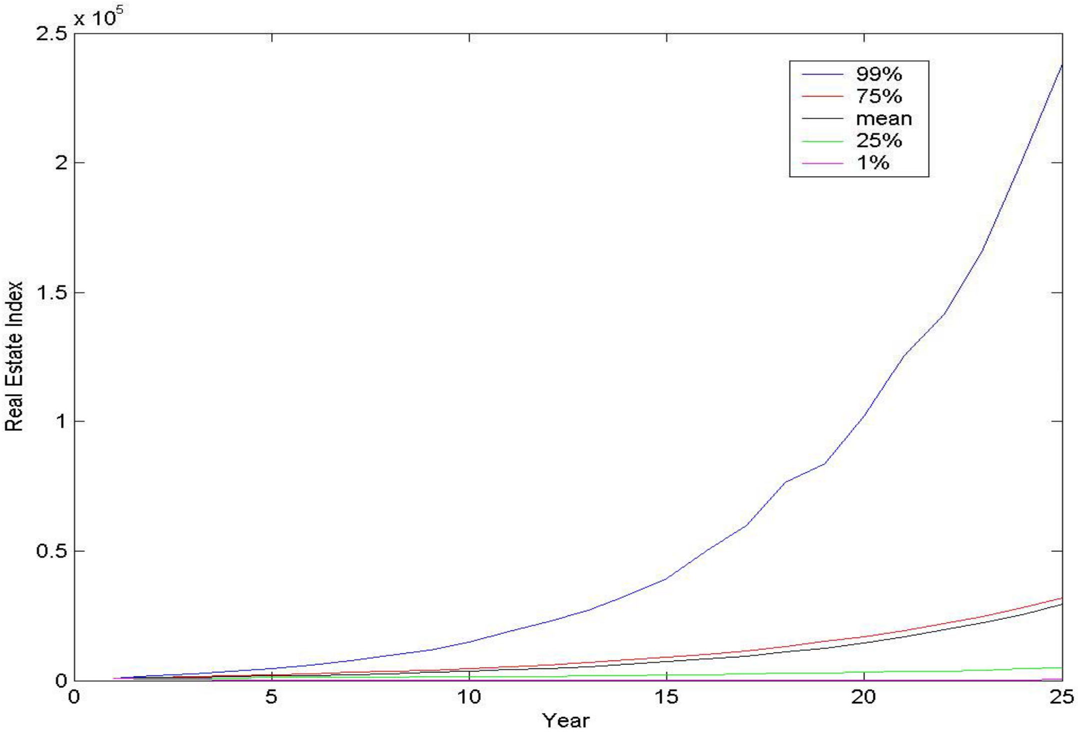

Figure 4: The simulated real estate index statistics along with time

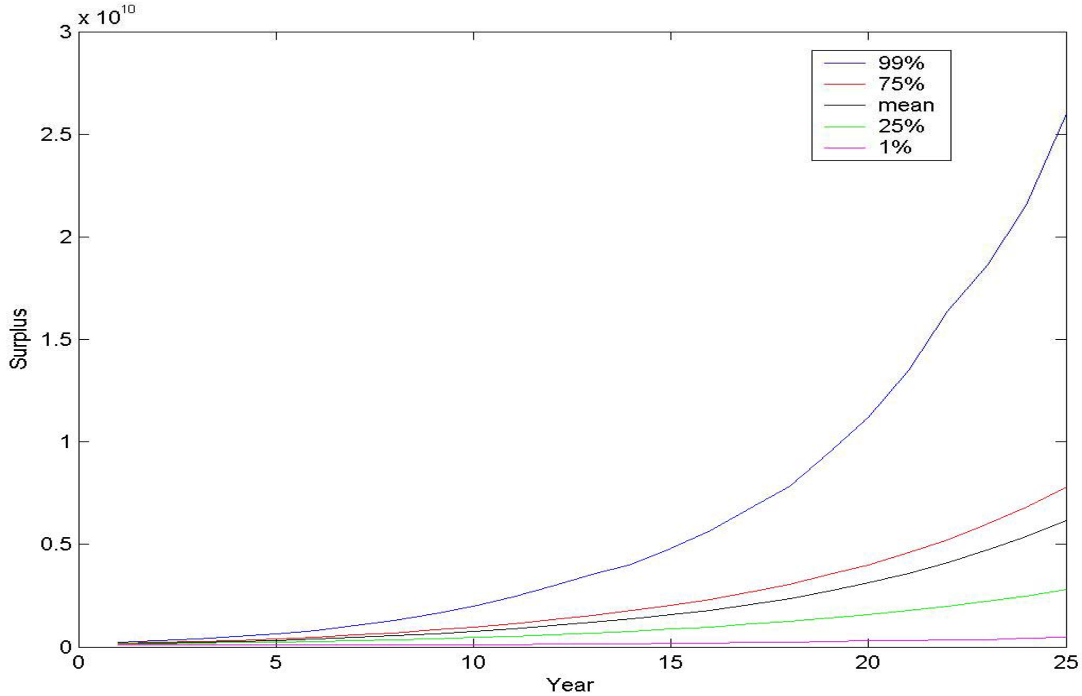

Figure 5: The simulated surplus statistics under the optimal asset allocation

Appendix: Alternative Parameters

A. Alternative Parameter Set 1

Model Parameters			
Short Rate	m = 20%	q = 0.1	v = 10%
Equity Index	π_S = 10%	σ_S = 35%	
Real Estate Index	μ = 30%	σ_{RE} = 60%	
Loss Ratio (Long)	mean = 30%	$\sigma_{LR(L)}$ = 100%	
Loss Ratio (Short)	mean = 20%	$\sigma_{LR(L)}$ = 80%	

The starting value of the short-term interest rate is 1%. The correlation matrix $\Re$ is:

	dW_S	dW_r	$dW_{LR(L)}$	dW_{RE}
dW_S	1	0.60	0.60	0.60
dW_r	0.60	1	0.60	0.60
$dW_{LR(L)}$	0.60	0.60	1	0.59
dW_{RE}	0.60	0.60	0.59	1

. The parameters of the representative insurer are set as follows: $IS(0)$ = 100, $IP(0)$ = 110, $B(0) = \frac{10}{11}$, $Exp(L)$ = 50%, $Exp(S)$ = 50%, $G(L)$ = 2%, $G(S)$ = 1%, and

dy	1	2	3	4	5	6	7	8	9	10
$D_L(dy)$ (%)	10	10	10	10	10	10	10	10	10	10
$D_S(dy)$ (%)	30	40	30							

. The discount rate for future surplus is assumed to be 15% while the discount rate for the reserves is set at 1%. The new investments to bonds are allocated one-third to one-year bonds, one-third to seven-year bonds, and one-third to fifteen-year bonds.

B. Alternative Parameter Set 2

Model Parameters			
Short Rate	m = 2%	q = 0.7	v = 6%
Equity Index	π_S = 20%	σ_S = 50%	
Real Estate Index	μ = 10%	σ_{RE} = 13%	
Loss Ratio (Long)	mean = 90%	$\sigma_{LR(L)}$ = 20%	
Loss Ratio (Short)	mean = 95%	$\sigma_{LR(L)}$ = 10%	

The starting value of the short-term interest rate is 12%. The correlation matrix $\Re$ is:

	dW_S	dW_r	$dW_{LR(L)}$	dW_{RE}
dW_S	1	0.99	0.99	0.99
dW_r	0.99	1	0.99	0.99
$dW_{LR(L)}$	0.99	0.99	1	0.99
dW_{RE}	0.99	0.99	0.99	1

. The parameters of the representative insurer are set as follows: *IS(0)* = 250, *IP(0)* = 1,100, *B(0)* = $\frac{1}{11}$, *Exp(L)* = 10%, *Exp(S)* = 5%, *G(L)* = 12%, *G(S)* = 20%, and

dy	1	2	3	4	5	6	7	8	9	10
$D_L(dy)$ (%)	0.1	0.1	0.3	0.5	1	3	5	10	30	50
$D_S(dy)$ (%)	5	15	80							

. The discount rate for future surplus is assumed to be 45% while the discount rate for the reserves is set at 10%. The new investments to bonds are all allocated seven-year bonds.

第二節

Applying Simulation Optimization to the Asset Allocation of a Property-Casualty Insurer [a]

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I. INTRODUCTION

Asset allocation is essential to the solvency of a property-casualty insurance company. A proper asset allocation generates adequate returns with non-excessive risk while provides liquidity for claim payments. A poor allocation, on the other hand, may result in inadequate returns, excessive risks, and/or illiquidity. A poor allocation is bad not only to shareholders but also to customers since the inefficiency in investing premiums will ultimately result in an undue insolvency probability of the serving insurer and/or high premium rates if the market is imperfect. Therefore, asset allocation should be regarded as an integrated part of a property-casualty insurer's core businesses.

The literature offers a property-casualty insurer two major ways to tackle the asset allocation problem. The first way follows the renowned mean-variance analysis of Markowitz (1952). In its simplest form that ignores the underwritten businesses, this method comes out of an efficient frontier representing the best portfolios in terms of the return-risk tradeoff. Many software packages, even the Microsoft Excel, can generate the efficient frontier instantly whenever the mean vector and covariance matrix of the asset returns are given. A more comprehensive method is taking the liability side into consideration explicitly in the mean-variance analysis (e.g., Sharpe, 1990; Craft, 2005; Chiu and Li, 2006). This consideration is essential to the asset-liability management of life insurers, property-casualty insurers, and pension funds. The mean-variance analysis, however, is subject to two fundamental flaws: the single-period framework and the inappropriate utility function assumed for the investor (Brennan, Schwartz, and Lagnado, 1997). The solution to a static portfolio choice problem can be very different from the solution to a multi-period problem except under restrictive conditions on utility functions and/or asset returns (Campbell, 2000).

The second way to construct optimal portfolios originated from Merton (1971; 1990). The literature along this line formulates the asset allocation problem as a stochastic optimal control problem and the solutions are characterized by the Hamilton-Jacobi-Bellman (HJB) partial differential equations. Solving the highly non-linear partial differential equations is, however, rather difficult. Algebraic solutions can be obtained in very special cases only. Furthermore, the number of state variables must be small since the size of the stochastic optimal control problem grows exponentially with the number. Cox and Huang (1989) made conceptual progress by showing that one can apply the martingale representation theory to reduce the stochastic dynamic programming problem to a static problem in complete markets. Few closed-form solutions have been available outside the simplest cases, however. Furthermore, complex hedging terms are difficult to evaluate numerically. The empirical applicability of the intertemporal portfolio choice method is therefore severely limited.

We in this paper take advantage of the recent progresses in the techniques of simulation optimization to tackle the asset allocation problem faced by a property-casualty insurer in a heuristic way. We formulate the asset allocation problem as a simulation optimization problem (Tekin and Sabuncuoglu, 2004) in which the insurer's future surplus and risk are generated by a simulation model and a global optimization technique, multi-phase evolutionary strategies (MPES), is used to search the optimal asset allocation. The model simulates the dynamics of a property-casualty insurer that underwrites both short-tail and long-tail line of businesses and invests in five types of risky assets. The simulation generates 10,000 paths of 25-year changes in which the insurer re-allocate its assets every five years with no-short-sale constraints, which makes our problem a multi-period one. The objective function of the insurer involves not only average discounted future surplus but also insolvency probabilities and thus is non-linear.

To examine the potential gain of using simulation optimization, we compare our results with those generated by the commonly used ad hoc multi-period strategies, the so-called re-balancing methods. The re-balancing can be done periodically (called periodical re-balancing in the following) or triggered by the deviations of the asset proportions from the first-period allocations that exceed a pre-specified interval (called interval re-balancing in the following). The first-period asset allocation is usually obtained by trying all possible combinations of allocations in which the allocations are restricted to be multiples of certain proportions like 10% or 20% (we call this method as grid search in the following). We also employ MPES to produce the first-period asset allocation for the re-balancing methods to see how much improvement MPES can be over the grid search method. We set up two comparison criteria: the value of the objective function and the position of the generated "efficient frontier." The comparison based on the objective function's value is intuitive, but any specific objective function represents only one combination of expected return and risk. We thus extend our comparisons to a series of return-risk pairs to see the relative positions of the resulted efficient frontier.

Our results show that the periodical re-allocation using MPES produces significant improvements over other strategies. The MPES re-allocation strategy results in the highest value of the objective function. It outperforms the interval re-balancing using grid search to determine the first-period asset allocation by 16.49%. The improvements of MPES re-allocation over grid-search periodical re-balancing, MPES periodical re-balancing, and MPES interval re-balancing are 16.16%, 10.04%, and 8.97%. With regard to the relative positions of the efficient frontier, the MPES re-allocation strategy remains to be the best, followed by the MPES interval re-balancing, MPES periodical re-balancing, grid-search interval rebalancing, and grid-search periodical rebalancing strategies. In short, the reallocation using MPES is the best and all the strategies using MPES perform better than the strategies using the grid search method.

The rest of this paper is organized as follows. Section 2 describes our simulation model, including the setting of the financial markets and insurance markets. Section 3 depicts the dynamics of the representative insurer's financial positions. The investment strategies used for comparisons are described in Section 4, and the optimization problem of the insurer is formulated in Section 5. In Section 6 we explain the proposed searching technique MPES in detail. Comparisons are displayed in Section 7. Five investment strategies are evaluated by the values of the objective function and the efficient frontiers that they can generate. Section 8 contains summaries and conclusions.

II. STOCHASTIC INVESTMENT AND INSURANCE MARKETS

In this section, we set up the stochastic models of the investment and insurance markets used to simulate asset prices and underwriting losses.

A. Investment Markets

The insurer could invest five different types of assets. These assets include stocks, bonds, three types of alternative investments that are individual with "high-return and high-risk", "low- return and high-risk" and "low-return and low-risk". The bonds could be separated as one-year maturity, two-year maturity… and fifteen-year maturity, with total fifteen kinds of short and long maturity bonds.

The entire simulation horizon is T_n , and all the models would be built as the dynamics form at a certain period t ($t \le T_n$). All the volatilities of the asset models yield Wiener processes, so we use $d\vec{W}$ to present the differential of those processes. The correlations among asset price changes and losses claims of insurance are denoted as R.

$$d\vec{W} = [dW_r \quad dW_{St} \quad dW_{AI^{hh}} \quad dW_{AI^{ll}} \quad dW_{AI^{lh}} \quad dW_{lr^l} \quad dW_{lr^s}]' \text{, where}$$

r : short rate

St: Stock

AI^{hh} : high-return and high-risk alternative investment

AI^{ll} : low-return and low-risk alternative investment

AI^{lh} : low-return and high-risk alternative investment

lr^l : loss ratio of long-tail business

lr^s : loss ratio of short long-tail business

1. Bond Markets

The variation of the short rate usually affects both bond market and stock market. To simulate the variation of the short rate, we set up the CIR (Cox, Ingersoll, and Ross, 1985) model as short rate model. That is $dr_t = b(a - r_t)dt + \vec{\sigma}_r \sqrt{r_t} d\vec{W}$, where a is long-term average of the short rate and $0 < b < 1$ denotes the mean reversion rate. The volatility of the evolving process is $\vec{\sigma}_r = [v \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$.

In the bond market, we also adopted CIR model and the price of the bond is mapping from the short rate. The price of a default-free zero-coupon bond at time *t* for the delivery of \$1 at time *T* as:

$$P_{t,r}(Bd) = A_0(T - t)e^{-B(T-t)r}$$

, where $B(x) = \dfrac{2(e^{\gamma x} - 1)}{(\gamma + b)(e^{\gamma x} - 1) + 2\gamma}$, $A_0(x)\left[\dfrac{2\gamma e^{\frac{x}{2}(b+\gamma)}}{(b+\gamma)(e^{\gamma x} - 1) + 2\gamma}\right]^{\frac{2ba}{\sigma^2}}$, $T \geq t > 0$

and $\gamma = \sqrt{b^2 + 2v^2}$.

2. Stock Markets

We assumed that the dynamics of stock index follows the interest-rate-adjusted geometric Brownian motion as below. The dSt_t denotes the change in the stock market index within a time period *t*. The ℓ_{St} denotes the risk premium and the volatility of the index return $\vec{\sigma}_{St} = [0 \quad \sigma_{St} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$.

$$\frac{dSt_t}{St_t} = (r_t + \ell_{St})dt + \vec{\sigma}_{St} d\vec{W}$$

3. Alternative Investment Markets: High-return and High-risk

The adjustment of AI^{hh} index moves according to geometric Brownian motion. The primary feature of this model is high risk and high return, hence its σ and μ is relative high to other alternative investment models. The index of this asset model is listed as follows:

$$\frac{dAI^{hh}}{AI^{hh}(t)} = \mu_{AI^{hh}} dt + \vec{\sigma}_{AI^{hh}} d\vec{W}$$

, where $\mu_{AI^{hh}}$ is the expected return of AI^{hh} and the volatility is indicated as $\vec{\sigma}_{AI^{hh}} = [0 \quad 0 \quad \sigma_{AI^{hh}} \quad 0 \quad 0 \quad 0 \quad 0]$.

4. Alternative Investment Markets: Low-return and Low-risk

Changes of this asset price also follow geometric Brownian motion. The parameters σ and μ are set purposely to be relatively low to other alternative investment models. The associated process is as follows:

$$\frac{dAI^{ll}(t)}{AI^{ll}(t)} = \mu_{AI^{ll}} dt + \vec{\sigma}_{AI^{ll}} d\vec{W} ,$$

where $\mu_{AI^{ll}}$ is the expected return of high-return and high-risk alternative investment and the volatility is indicated as $\vec{\sigma}_{AI^{ll}} = [0 \quad 0 \quad 0 \quad \sigma_{AI^{ll}} \quad 0 \quad 0 \quad 0]$.

5. Alternative Investment Markets: Low-return and High-risk

The motivation of AI^{lh} index still complies with geometric Brownian motion. This kind of investment usually brings high risk and low return and hence σ is relative high but μ is relative low to the entire alternative investment models. One characteristic of the model is with negative correlation among the assets. It is a hedging asset and could be used to regulate the high-risk assets. The model of this assets index is depicted as:

$$\frac{dAI^{lh}(t)}{AI^{lh}(t)} = \mu_{AI^{lh}} dt + \vec{\sigma}_{AI^{lh}} d\vec{W}$$

, where $\mu_{AI^{lh}}$ is the expected return of AI^{lh} and the volatility is $\vec{\sigma}_{AI^{lh}} = [0 \quad 0 \quad 0 \quad 0 \quad \sigma_{AI^{lh}} \quad 0 \quad 0]$.

B. Insurance Markets

The insurer has a long-tail business and a short-tail business here. We particularly considered two issues about the timing and the amount of claim payments in the long-tail and short-tail businesses.

The timing of claim payments is also called claims tail. In some insurance contracts, a considerable proportion of claim payments are paid slowly because of the negotiations between the insurer and their customers. The lag between the time that coverage is sold and the time that claims are paid is the called claims tail. "Long-tailed" business such as liability and workers' compensation insurance in which most claims are paid in several years after the coverage period. "Short-tailed" business likes property insurance coverage and coverage for employee medical costs under group health insurance contracts in which large proportion of claim are paid during the year of coverage or the year after. (Harrington and Niehaus, 1999). We call the span of claim payments as the loss development periods in this study.

Whether long-tail or short-tail business, total claim amount of every new coming business is different from each other because the uncertainty of loss ratio always associates with the insurance business. We need to construct the loss ratio models for the amount of the payment based on this uncertain feature. The following models are related to the timing and the amount of claim payments for the long-tail and short-tail lines of businesses.

1. Long-Tail Businesses

The loss ratio of long-tail business claim payments at each period is simulated using normal distributions with positive parameters ($\mu_{lr^l}, \sigma_{lr^l}$). The loss development will vary with different proportions d_i^l in different period. We assumed the development periods of the long-tail business as t^l and it has to satisfy $\sum_{i=1}^{t^l} d_i^l = 1$ where $0 \le d_i^l \le 1$.

2. Short-Tail Businesses

The variation of loss development across periods in this market is following normal distributions with parameters ($\mu_{lr^s}, \sigma_{lr^s}$). In addition, the loss development of each period is alterable with different proportions d_i^s. We assumed the development periods of short-tail business is t^s and it has to satisfy $\sum_{i=1}^{t_S} d_i^s = 1$ where $0 \le d_i^s \le 1$.

III. THE DYNAMICS OF THE INSURER'S OPERATIONS

The insurance company has two principle operations about insurance business and assets investment proceeding over the time horizon T_n (from $t = 0$ to $t = T_n - 1$, see the time axis in Figure 1). These operations that affect the positions of the assets usually occur at the beginning and the end of period *t*. At the beginning of the period, three sub operations include receive the premiums, list reserves and increase investable funds and allocate investable funds (the rules of allocation strategies would be discussed in the next section). Once it comes to the end of the period, three sub operations include recalculate the asset values, claims realized and deduct claim payments from assets. Therefore, the positions of the asset would be changed again after paying for the losses. All above activities would be separated and described as insurance business and asset investments. In the figure we use italic text to denote the insurance activities of the insurance businesses and use bold text to denote the investment activities of the assets. We describe these two kinds of activities more detail in the following two sections.

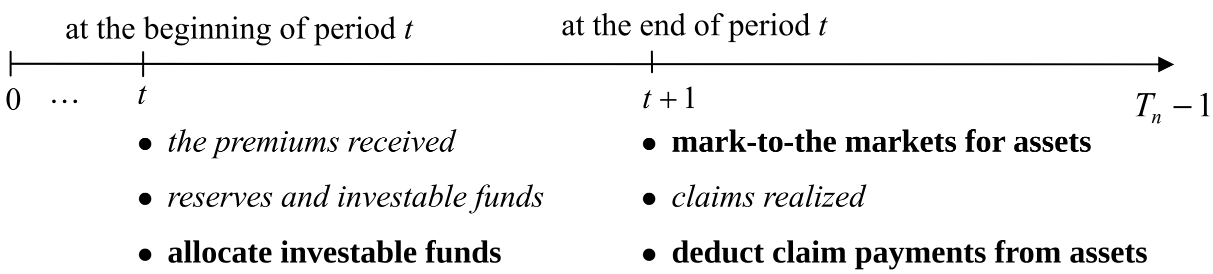


Figure 1: The business and investment activities of company operations in period *t* over the simulated horizon

A. Insurance Activities

The operations of insurance business generally consist (1) receive the premiums (2) calculate reserves and list it on the balance sheet and increase investable funds and (3) claims realized. These operations have some relationships on the procedures. First, the company receives insurance premiums from their customers and then lists the amount of premiums as the reserve in the liability. At the same time, the cash from premium collective could become the investment funds to be put into the financial markets that would be listed as the assets of the company. All claims are usually paid at the end of the year. As the company paid out the claims, the reserve of liability will decrease. We will describe these activities more elaborately in the following subsections.

1. Premiums Received

We assumed that the business revenue of insurer grows with a constant rate. Under the constant growing condition, premium incomes of this period will increase $(1+g^l)$ or $(1+g^s)$ based on the income of last period. Therefore we can express the total insurance premium income of each period as:

$$\Pr m_t = \Pr m_{t-1} * \lambda_{t-1} * (1+g^l) + \Pr m_{t-1} * (1-\lambda_{t-1}) * (1+g^s)\text{, where}$$

$\Pr m_t$: total premium income in period t

λ_t : a proportion of long-tail premium income to total insurance premium income

g^l (g^s): a constant, long-tail (short-tail) business growing rate in each period and $0<g^l<1$ ($0<g^s<1$).

2. Reserves and Investable Funds

During each period, insurer deducts a given expenses from the business revenues and invests all of them into the financial markets afterwards. The net premium income, the reserve or the investment funds, could be depicted as follows.

$$Fund_t = \operatorname{Re} serve_t = \Pr m_t * \lambda_{t-1} * (1-f^l) + \Pr m_t * (1-\lambda_{t-1}) * (1-f^s)\text{, where}$$

f^l (f^s): a constant, a ratio of related expense for the long-tail business (short-tail business)

3. Claims Realized

Both of the insurance businesses receive the premium income at the beginning of each period, and pay the loss for customers at the end of a period. We should consider the old businesses were accepted in the prior period but their losses would be paid afterwards when we calculate the claim payment of this period. This would lead to that the claim the insurer has to pay includes the new coming business and prior business would be paid by the part of this period. Therefore, the total losses paid at the end of period t is

$$Loss_t = \sum_{i=\max(1,t-t^l+1)}^{t} \Pr m_i^l \times lr_i^l \times d_{t-i+1}^l + \sum_{i=\max(1,t-t^s+1)}^{t} \Pr m_i^s \times lr_i^s \times d_{t-i+1}^s$$, where

$lr_i^l\,(lr_i^s)$: loss ratio of long-tail (short-tail) business in period i.

B. Investment Activities

We simulate the changes in the asset positions during each period at three time points, including the beginning of this period, before and after claims realized at the end of this period. At the beginning of period t, the insurer allocates the investable funds. At the end of period t, the insurer mark-to-the markets for the asset before claims realized and it deducts the claim payments from the assets. The asset positions undulate continually at these three time points in all simulation periods. To understand the performance of the simulation periods, we calculate the profit or the loss at the end of every period t of the whole horizon T_n . Whether profit or loss happens, the shareholders absorb them all. Nevertheless, while the company cannot afford the losses, the company would be insolvent and forced to stop simulating the path. In this section, we adopted the balance sheet to express the changes of all asset positions.

Notations

t: a certain period $t = 0, \cdots, T_n - 1$

ζ : a set of periods for making asset allocation strategies, where $0 \le \zeta \le T_n - 1$ and $\zeta \in \mathrm{Z}$

a: the time after claim realized

b: the time before claim realized

x_i : asset class i , where $i = 1,\cdots,5$ corresponding to St, Bd, AI^{hh} , AI^{lh} and AI^{ll}

k : k-th type of bond, the bond is one-year maturity bond when $k = 1$ and so on

k_n : number of the types of bonds

bp_k : allocation of the k-year maturity bond to the all bonds

q_{t,x_i} : quantity of asset class x_i in period t

$q_{t,x_i,k}$: quantity of k-th type of asset x_i in period t

p_{t,x_i} : unit price of asset class x_i in period t

$p_{t,x_i,k}$: unit price of k-th type of asset x_i in period t

v_{t,x_i} : value of the asset class x_i in period t

$Cptl$: capital in the initial period as the company established

θ_{t,x_i} : allocation of asset class x_i in period t

$\theta^{'}_{t,x_i}$: new allocation of asset class x_i in period t

$\mathrm{Re}serve_t$: reserve for the claim in period t

A_t : total asset value in period t

L_t : total liability in period t

E_t : total equity in period t

1. At the Beginning of Period t — Allocate Investable Funds

The company receives the premium income at the beginning of every period. It regards the cash from the reserve as the new increased investment funds and allocates the funds into each asset according to the investment strategies (asset allocation strategies). Most of the assets will proportionally increase in this point.

On the asset side, there are two different situations about asset allocation. In the situation 1, the total fund has additional capital from the shareholders in the first year of the company. Otherwise we just keep ourselves staying in the situation 2. The situation 2 also has two cases about the time whether making the strategies or not.

Situation 1: $t = 0$

The quantity of each asset we will buy as following.

$$q_{0,x_i} = (Cptl + Fund_t) \times \theta_{0,x_i} / p_{0,x_i} \qquad i = 1,3,4,5$$

$$q_{0,x_2,k} = (Cptl + Fund_t) \times \theta_{0,x_2} \times bp_k / P_{0,x_2,k}$$

Situation 2: $t = 2, \cdots, T_n - 1$

Situation 2.1: $t = \zeta$

$$q_{t,x_i} = q^a_{t-1,x_i} + Fund_t \times \theta_{t,x_i} / p_{t,x_i} \qquad i = 1,3,4,5$$

$$q_{t,x_2,k} = q^a_{t-1,x_2,k} + Fund_t \times \theta_{t,x_2} \times bp_k / p_{t,x_2,k}$$

Situation 2.2: otherwise,

$$q_{t,x_i} = (A^a_{t-1} + Fund_t) \times \theta'_{t,x_i} / p_{t,x_i} \qquad i = 1,3,4,5$$

$$q_{t,x_2,k} = (A^a_{t-1} + Fund_t) \times \theta'_{t,x_2} \times bp_k / p_{t,x_2,k}$$

$$A_t = \sum_{i=1}^{5} v_{t,x_i} = q_{t,x_i} \times p_{t,x_i}$$

On the liability side, the value increases due to the new insurance businesses.

$$L_t = L_{t-1} + \mathrm{Re}serve_t$$

Since the total asset value equals the sum of liability and surplus, we can get the surplus form the following formulation.

$$S_t = A_t - L_t$$

2. At the End of Period *t* — Mark-to-the Markets for Assets

At the end of the period, the company has to pay for the loss of the customers. However the loss would be paid by each asset according to the proportion of each asset present value to the total assets present value. We have to complete mark to the markets for assets before the claims realized because of the price fluctuations in the financial markets. Regarding the bonds, the rest years of maturity of each type would be shorten at this time. For simplification, we use the one-year maturity bond, which would become $1 face value on the due day to purchase the new fifteen-year maturity bond. In the following formulations, the superscript *b* means the special time before claim realized. For example, A_t^b denotes the total asset value before claim realized at the time *t*.

On the asset side, the prices of assets are changed at the end of period *t*. The asset value would be recalculated with price p_{t+1}.

$$\begin{aligned} A_t^b &= \sum_{i=1}^{5} v_{t,x_i}^b \\ &= \sum_{i=1}^{4} q_{t,x_i} \times p_{t+1,x_i} + \sum_{k=2}^{k_n} (q_{t,x_2,k} \times p_{t+1,x_2,k-1}) + q_{t,x_2,1} \end{aligned}.$$

On the liability side, the value is still the same.

$$L_t^b = L_t .$$

We then deduct the liability from the asset value to get the equity value.

$$S_t^b = A_t^b - L_t^b$$

3. At the End of Period *t* — Deduct Claim Payments from Assets

Insurance company pays the claims by selling the assets. When the insurer sells the assets, each asset class would be sold the amount of loss with proportion of this asset value to the total asset value. We could see the positions of assets are already changed after claim realized. In the following formulations, the superscript *a* means the special time after claim realized. For example, A_t^a denotes the total asset value after claim realized at the time *t*.

On the asset side, the asset value would decrease because of paying for the claim loss.

$$A_t^a = \sum_{i=1}^{5} v_{t,x_i}^a$$

$$= (v_{t,x_i}^b - Loss_t \times v_{t,x_i}^b / A_t^b) + \sum_{k=1}^{k_n} (v_{t,x_2,k}^b - Loss_t \times v_{t,x_2,k}^b / A_t^b)$$

$$q_{t,x_i}^a = v_{t,x_i}^a / p_{t+1,x_i}$$

$$q_{t,x_2,k}^a = v_{t,x_2,k}^a / p_{t+1,x_2,k}$$

On the liability side, the original preparation of reserve for the insurance loss payments at the beginning of period *t* would decrease because the claim realized.

$$L_t^a = L_t^b - Loss_t .$$

The equity value of shareholders is again obtained by deducting the liability from the asset:

$$S_t^a = A_t^a - L_t^a .$$

IV. ASSET ALLOCATION STRATEGIES

The company must make proper proportion adjustment for the assets at the certain time to gain the greatest expected wealth in such uncertain investment markets. The principles of asset allocation strategies contain the adjusting rules and the adjusting ways. In this study we divided the adjusting rules as periodical and interval. As to the adjusting ways, we distinguish them into reallocation and rebalance. The following are brief accountings:

A. Adjusting Rules

1. Periodical Adjustment

The length of time for adjusting the asset allocation is fixed. For example, the simulated horizon is a total of twenty-five years and we readjust the proportions every 5 years.

2. Interval Adjustment

The adjusting timing is determined by if the proportion of any asset exceeds the predefined limitation. We assumed the limitation of increasing or decreasing degrees is allowed below r^b $(0 \le r^b \le 1)$. Once the amount of asset value is changed beyond r^b of its proportion θ_{t,x_i} suggested in the investment strategy, the company must undertake new adjustments. It means that the changeable range of the asset proportion in each period is permitted by $[\theta_{t,x_i} - r^b, \theta_{t,x_i} + r^b]$.

B. Adjusting Methods

1. Rebalancing

When the company encounters the time for making new strategies, every asset adjusts the present proportion according the old proportion in the first strategy. The investment portfolios $\tilde{\theta}$ are always still the same, whatever how many times the insurer has made the strategies.

2. Reallocation

The company has to create new allocations when it encounters the time for making strategies. This new allocations would adjust the assets to the new positions.

V. OPTIMIZATION AND EXPERIMENT DESIGN

We used two ways to evaluate the performance of alternative investment strategies. One is efficient frontier and the other is utility function. The prior can show us how the surplus changes across risk levels and the posterior directly display a single value, which integrates both the surplus and the risk.

A. Efficient Frontier

The main purpose that we employed the efficient frontier is to understand the performance of different investment strategies with an overview. We use an efficient frontier curve to express the entire performance of a specific strategy, because efficient frontier curve is made up of points representing several portfolios with differing levels of risk and return. When the position of the cure is located at more upper left, it signifies that the performance of these portfolios on this curve is better.

The expect return is the average discounted surplus as below. To calculate this item, we calculate the average of discounted surplus of all periods for each path and than average the total results of all φ simulated paths. For simplicity, we set and observe ten points under the specific and different risk levels r_p as 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.04, 0.05, 0.07, and 0.1 during using the MPES[23]. We set the greatest observed risk is at 0.1 because we believe this level still stays in the general tolerant range.

$$Max \quad \frac{\sum_{i=1}^{\varphi}\left[\frac{1}{n}\left(\sum_{t=1}^{n}\frac{S_t}{(1+r_s)^t}\right)\right]}{\varphi}$$

$$\begin{aligned} s.t. \quad & ruin\ probability \leq r_p \\ & \sum_{i=1}^{5}\theta_{t,x_i} = 1 \\ & \theta_{t,x_i} \geq 0 \end{aligned}$$

B. Utility Function

In contrast to the efficient frontier, the utility function just produce only one point (value). We usually use the utility function of the company as the objective function. The utility function has two parts that express the return and tolerable ruin probability at the same time. In this study, the first term is average discounted surplus (see below). The second term is a punished item to denote that the ruin probability $prob_{ruin}$ of the simulated result has outrun the tolerable ruin probability τ. This deduction (punishment) of the utility is implying the company's dissatisfaction. The constant k is chosen by the company to properly reflect the importance of excessive ruin probability.

[23] For the search technique ES, we can easily set the risk level as our constraint to search the return under this specific risk. However, the grid search method does not allow us to specify the risks.

$$\underset{\tilde{\theta}(t)}{Max}\left\{ \frac{\sum_{i=1}^{\varphi}\left[\frac{1}{n}\left(\sum_{t=1}^{n}\frac{S_t}{(1+r_s)^t}\right)\right]}{\varphi} - \max(k(prob_{ruin}-\tau),0) \right\}$$

$$s.t. \qquad \sum_{i=1}^{5}\theta_{t,x_i} = 1$$

$$\theta_{t,x_i} \geq 0$$

The entire utility function can convey the practical situation. That is when the company pursues the high profit and naturally lifts up the risk and return, it will result in the higher ruin probability and force the value of the utility lower. However, when the company avoids the high ruin probability of the risky assets, it usually gets lower returns. We use MPES to find the appropriate investment portfolio $\tilde{\theta}$ to make the utility reach the maximum. This means that the company can gain the greatest profit and lower the ruin probability at the same time.

C. Experiment Design

The experiments simulated investment of the insurance company for twenty-five years and the number of simulated paths is 10,000, which are the various situations of the return and loss of the markets. We preceded two main parts of experiments to compare the differences between the all sorts of searching methods and adjustment rules (see Table 1). The first one uses the grid search method to solve the problems and the other uses MPES. The parameters of the two main experiments are identical and the detail settings are listed in the appendix.

Table 1: Experiment design for the asset allocation strategies and search methods

Search Method	Asset Allocation Strategies		
	Adjusting Rule	Adjusting Method	Simplified Form
Grid search method	periodical	rebalance	Grid-PR
	interval	rebalance	Grid-IR

MPES	periodical	rebalance	MPES-PR
	interval	rebalance	MPES-IR
	periodical	reallocation	MPES-Reallocation

We used the grid search method as the benchmark to compare with the MPES because people usually adopt it to solve the searching problems when they do not have efficient methods. The grid search method used the combination of some specific figures to determine the allocation of investment portfolio. The number of the asset classes is five and each figure in the combination varies by 20%. All asset allocations are non-negative and sum to one, so it will produce 126 combinations[24]. Some examples of the combinations are such as (1, 0, 0, 0, 0), (0.8, 0.2, 0, 0, 0), (0.2, 0.6, 0, 0, 0.2), etc.

VI. MULTI-PHASE EVOLUTION STRATEGIES

In this section, we introduce the proposed evolutionary strategies algorithm, multi-phase evolutionary strategies (MPES). We will first introduce a basic evolutionary strategies algorithm and its application to generate the optimal asset allocation for the simulation models. Then, we will discuss the ideas and the procedure of MPES and explain how MPES is able to improve the performance of the basic evolutionary strategies. Furthermore, we will discuss the performance of MPES for five complicated benchmark functions..

A. Evolution Strategies

The evolution strategies algorithm (ES) has been presented since 1970s (Rechenberg, 1973, Schwefel, 1981). It is a randomized search method that incorporates the nature of evolution into its processes. Evolutionary algorithms, unlike traditional optimization

[24] The number of combinations is $3\times\frac{5!}{3!1!1!}+2\times\frac{5!}{2!2!1!}+5+1=126$ (The first addition term is the sum of permutations and combination in [0.8, 0.2, 0, 0, 0], [0.4, 0.2, 0.2, 0.2, 0], and [0.4, 0.4, 0.2, 0, 0]. The second addition term is the result from [0.6, 0.2, 0.2, 0, 0] and [0.4, 0.4, 0.2, 0, 0]. The last two terms are the results from [1, 0, 0, 0, 0, 0] and [0.2, 0.2, 0.2, 0.2, 0.2].)

techniques, use "population" instead of single points to search and solve complex optimization problems. The population for the initial generation is usually generated randomly. From the members (parents) in the population, genetic operators are then used to produce offspring, and the favorable offspring, corresponding to the "survival of the fittest theory" in the biological world, are chosen to constitute the population for the next generation. The process continues for generations until a termination criterion is satisfied and a superior solution is acquired. A concise pseudo code of a basic evolution strategies algorithm, denoted as (μ,λ) ES, is presented as follows, where μ is the number of parents in the current population and λ is the number of offspring produced by the parents for the next generation, and λ is about seven times of μ.:

(μ,λ) ES Pseudo Code:

1. *Initialize μ parents*
2. *For generation :=1 to n do*
3.   *Recombination & Mutation from μ parents to produce λ offspring*
4.   *Parent selection: Evaluate λ offspring and choose the best μ offspring to be new parent*
5. *End Do*
6. *Output the first best solution.*

A more detailed description of the steps of the (μ,λ) ES algorithm is presented below (Nissen and Biethahn, 1995, and Back, 1996).

Step 1: Generate a population for the initial generation.

A population of μ solutions (members) is generated. Each solution is usually represented by a row vector consisting of two parts. The elements in the first part are the values of the decision variables (x_j) considered in a given application, and the elements in the second part are the mutation step sizes (σ_j) corresponding to the decision variables in the first part. The decision variables in our application are dependent on the asset allocation strategies used. If the periodical reallocation strategy is used, the decision variables will be the proportions of the asset allocations for the five types of risky asset in five periods, so there will be twenty-five decision variables. We randomly generate the proportions of the five asset allocations for each period from a uniform distribution with a range of [0, 1]. Note that the sum of the proportions of the five asset allocations in a period has to be equal to one, so the proportions for each period are simply normalized by summing up the proportions and dividing each proportion by the sum. Additionally, in each solution, all the mutation step sizes are set to 3.0 (Back, 1996).

Step 2: Apply recombination and mutation to the parents to produce λ offspring.

A pair of parents, A and B, is randomly chosen from the population, and recombination and mutation are applied to A and B to produce a child C. Discrete recombination is used to determine the first part, the decision variable values of child C. The value of each decision variable in C is randomly and equally chosen from the value of the same variable in A and B. Intermediate recombination is used to determine the second part, the mutation step sizes of C. The j-th mutation step size in C is simply determined by the average of the j-th mutation step size in A and B ($\sigma_j(C) = 0.5\,(\sigma_j(A) + \sigma_j(B))$). The generated child C is then mutated by first modifying its mutation step sizes and then adding these step sizes to mutate the corresponding decision variables. Each mutation step size $\sigma_j(C)$ is modified by the following equation:

$\sigma_j'(C) = \sigma_j(C)\exp(\tau' N(0,1) + \tau N_j(0,1))$, where N(0,1) is a standard-normally distributed random variable, and the values of τ and τ' are set to 1.0 (Back, 1996) . And, each decision variable x_j is mutated by the following equation:

$$x_j'(C) = x_j(C) + N_j(0, \sigma_j'(C)).$$

When child C is generated, the simple normalization method is applied to the proportions of the asset allocations of each period to maintain its feasibility. The reproduction procedure is repeated until λ offspring are produced.

Step 3: Evaluate the λ offspring and choose the best μ offspring to constitute the population for the next generation.

The decision variables of a child are submitted to the simulation model, and the computational result is the fitness value of the child.

Step 4: Check the termination criterion. If the termination criterion is satisfied, stop; otherwise go to Step 2.

The most commonly used termination criterion is to set the fix generation for the entire process. This criterion is also used in this research.

B. Multi-Phase Evolution Strategies

Intensification and diversification are two principles in developing an optimization method. When we applied the basic ES algorithm to solve five benchmark functions, it was found that the algorithm usually converged within 100 iterations; the converged solutions were affected by randomly generated initial solutions, and the converged solutions were not always the optimal solutions. This may conclude that the basic ES algorithm was able to intensively converge to a local optimal solution; however, it was not able to diversely search large enough solution space. The idea behind the multi-phase evolution strategies is to improve the diversification capability of the basic ES algorithm. The basic ES algorithm is implemented *m* times (phases); however, it is not implemented from scratch. The best φ solutions in the last generation in a phase will automatically become the member in the initial generation in the next phase, and the other members will be generated randomly. This design will not only improve the diversification capability of the ES algorithm, but also guide the search to good solution regions. The pseudo code of the multi-phase ES algorithm is presented below.

Multi-Phase ES Pseudo Code:

1. *For multi-phase=1 to m do*

2. *if multi-phase =1*

3. *Initialize μ parents*

4. *else*

5. *Reset all parameters to initial conditions*

6. *parents = best φ solutions from previous phase + randomly generated (μ-φ) solutions*

7. *For generation :=1 to gen do*

8. *Recombination & Mutation from μ parents to produce λ offspring*

9. *Evaluate λ offspring and save the best φ offspring*

10. *End Do*

11. *EndDo*

12. *Output the first best solution.*

C. Evaluation of the Multi-Phase Evolution Strategies Algorithm

Since there are 25 independent variables in some of our asset allocation models, to ensure the performance of MPES for the candidate models, five non-linear and multi-modal functions, each with 50 variables, are used to evaluate the effectiveness of the algorithm. These benchmark functions are selected from Schwefel (1981), Yao and Liu (1996) and Vesterstrom and Thomsen (2004). Table 2 presents these benchmark functions, and Figure 3 to Figure 4 display the geometric figures of the last three benchmark functions with two variables. From these figures, one can see that there are lots of local optima and un-differentiable areas. It is hard to image how complicated these figures will become if 50 variables are considered. We applied MPES to each of the benchmark functions with five different randomly generated initial populations. Each decision variable in a solution in an initial population was randomly generated from a uniform distribution with a range constraining the variable in the benchmark functions. For instance, the constraints for the variables in the first benchmark function are $-10 \le x_i \le 10$, so the range for generating the decision variables was [-10, 10]. The parameters of MPES were determined by trial-and error in solving these five functions. The population size (μ) was set to be ten times of the decision variables considered in a solution (μ = n × 10); the number of offspring (λ) was set to be seven time of μ. Since, as mentioned above, the basic ES algorithm converged within 100 generations when applied to the benchmark functions, we set the number of generations (*gen*) for convergence in every phase to be 100. Also, both of the number of phases, m, and the number of best solutions retained for the next phase, φ, were set to be five. Table 3 presents the computational results. The results show that, for each benchmark function, since the standard deviation approaches 0.0, MPES always converged to the optimal solution regardless of the initial populations. Therefore, we are confident that MPES is an effective tool for generating the optimal asset allocation for the simulation models.

Table 2: High dimension benchmark functions

Function list (n=50)	Constrains	Minimal value	Remark
$f_1(\vec{x}) = \sum_{i=0}^{n-1} \lvert x_i \rvert + \prod_{i=0}^{n-1} x_i$	$-10 \le x_i \le 10$	$f_1(\vec{0}) = 0$	Schwefel's problem (2.22)
$f_2(\vec{x}) = \sum_{i=0}^{n-1} (\lfloor x_i + \frac{1}{2} \rfloor)^2$	$-100 \le x_i \le 100$	$f_2(\vec{p}) = 0$ $-0.5 \le p < 0.5$	Step function
$f_3(\vec{x}) = \sum_{i=0}^{n-1} -x_i \sin(\sqrt{\lvert x_i \rvert})$	$-500 \le x_i \le 500$	$f_3(\overrightarrow{420.97}) =$ -20949.14	Schewefel's problem (2.26)
$f_4(\vec{x}) = \sum_{i=0}^{n-1} (x_i^2 - 10\cos(2\pi x_i) + 10)$	$-5.12 \le x_i \le 5.12$	$f_4(\vec{0}) = 0$	Rastrigin's Function
$f_5(\vec{x}) = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=0}^{n-1} x_i^2}) - exp(\frac{1}{n}\sum_{i=0}^{n-1}\cos(2\pi x_i)) + 20 + e$	$-32 \le x_i \le 32$	$f_5(\vec{0}) = 0$	Ackley's Function

Table 3: Computational results of MPES for the five benchmark functions

	Mean Best Value	Std Dev
f_1	2.80914e-21	1.15020064249678e-21
f_2	1.0e-26	1.0e-26
f_3	-20949.1443636216	0.0
f_4	1.0e-26	1.0e-26
f_5	8.9706020390e-15	1.9459014222e-15

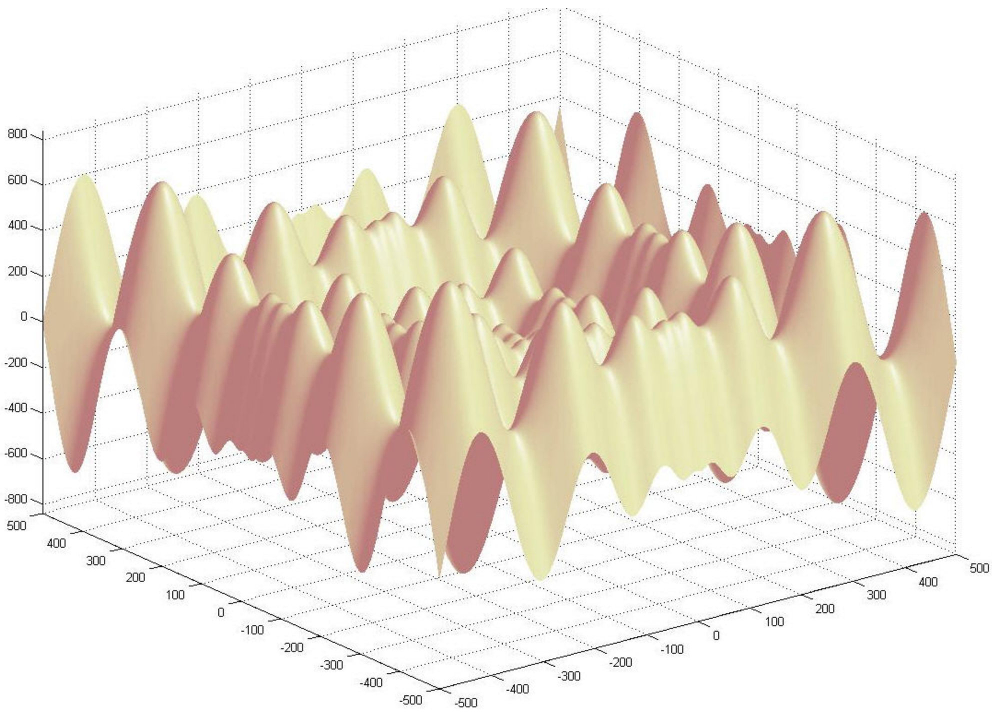


Figure 2: Two dimensional sketch of f_3.

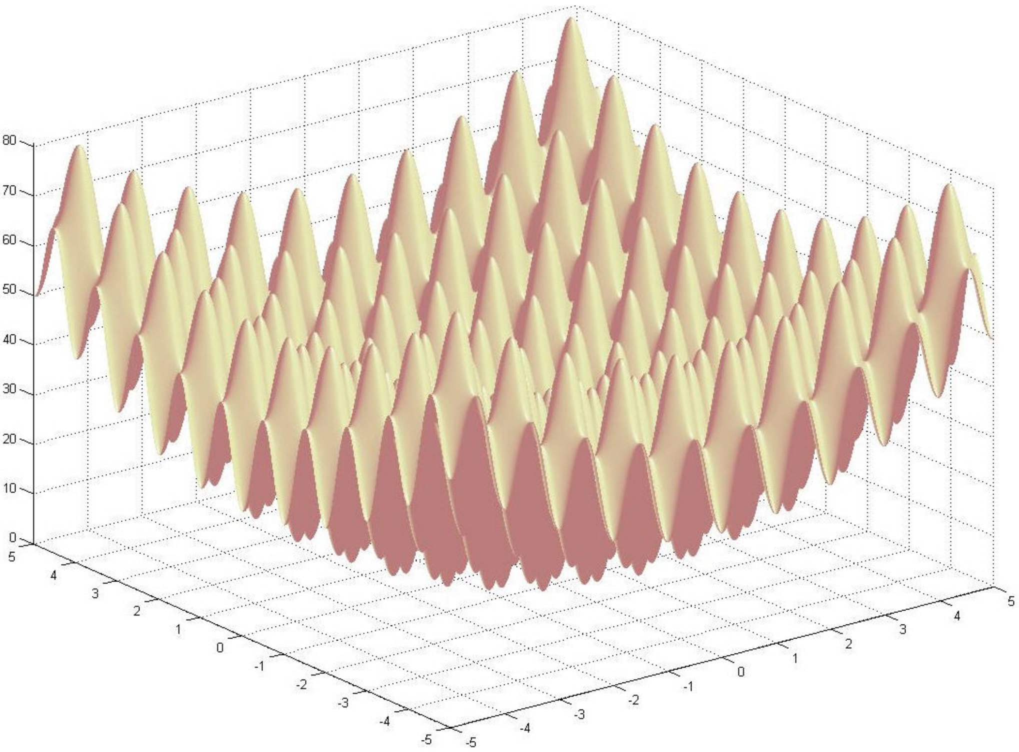


Figure 3: Two dimensional sketch of f_4.

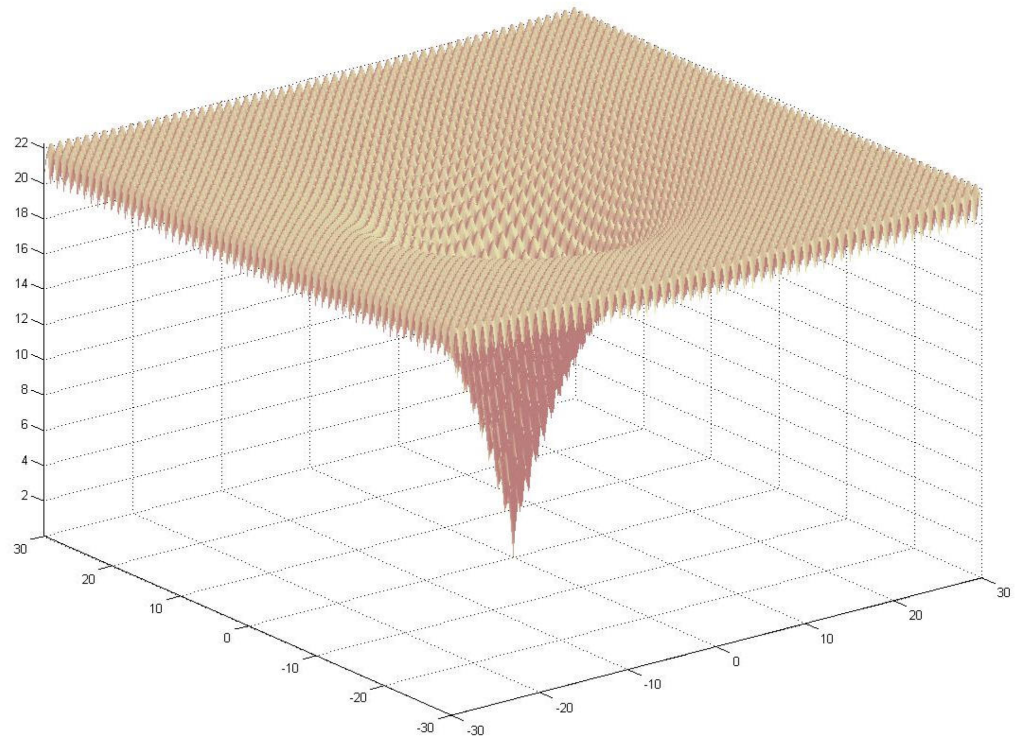


Figure 4: Two dimensional sketch of f_5.

VII. RESULTS ANALYSES

To analyze the performance and asset allocation under different investment strategies and search methods, we describe them in three aspects. First, we use the efficient frontier to compare the performance with an overview, the comparisons include (1) two rebalance strategies with the grid search method (Grid-PR and Grid-IR), (2) one reallocation and two rebalance strategies with MPES (MPES-PR, MPES-IR and MPES-Reallocation), and (3) all the strategies and search methods. Second, for further analysis, we particularly sought the single point of the optimized objective value that considered both of the return and risk together. We are not only interested in the improvement on the objective values of the reallocation with MPES, but also the asset allocation of all methods. Finally, we still examine the asset allocation but especially their changes across risk levels.

A. Analyses on Efficient Frontier

The result shows the performance of the interval rebalance is better than that of the periodical rebalance during using the grid search method (see Figure 5). The lower points of the efficient frontier of periodical rebalance are quite closed to that of the interval rebalance until arriving at the risk 0.04. This tells us that the return and the risk of the two methods are very similar when the company tends toward conservative. When the company could tolerant higher risk, the performance of interval rebalance is more significant.

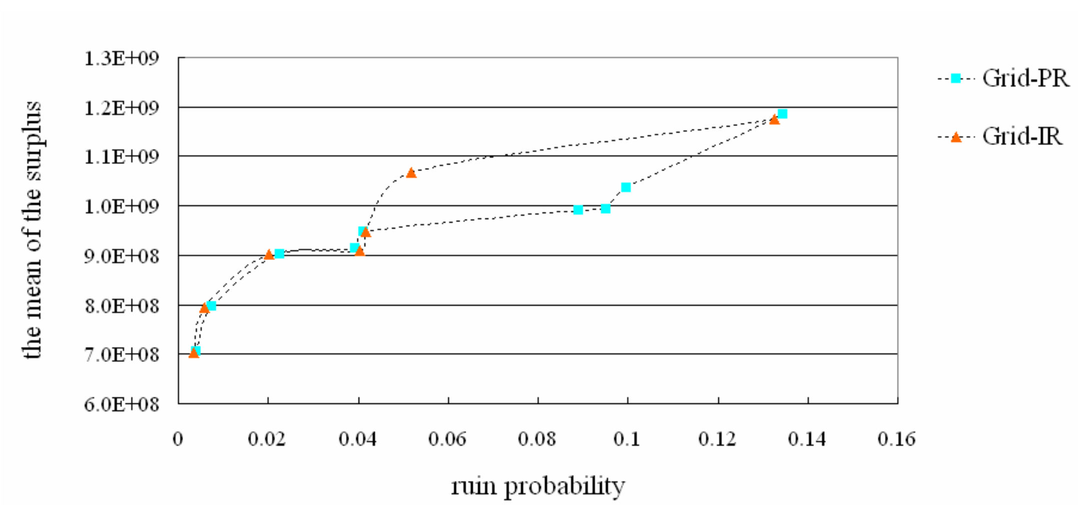


Figure 5: The efficient frontiers of all strategies using grid search method

The results of the MPES experiments showed that the reallocation strategy is better than other two types of rebalance strategies (see Figure 6). The performance of the MPES-IR seems a little higher than the MPES-PR's. In this figure we are interested in the performance improvement of the reallocation. We could understand it through the improvement of the return or of the risk. Let us see the part of the return first. There are ten different gaps between the returns of MPES-PR and MPES-Reallocation at ten risk levels. The improvements (gaps) of the ten points of MPES-Reallocation are 13.04%, 10.04%, 9.74%, 8.70%, 7.39%, 5.82%, 4.82%, 4.49%, 4.35% and 4.24%. The average is 7.26%. The gaps between MPES-IR and MPES-Reallocation are smaller and therefore the performance of MPES-IR is better than MPES-PR's. The gaps are 9.63%, 8.97%, 8.3%, 7.21%, 6.35%, 5.15%, 4.59%, 4.25%, 4% and 4.4% and the average is 6.28%. Corresponding to the improvement of the return, we also find any two points on the separate curves with similar return level but a gap of risk between them. We found five pairs of points and each pair has the similar return that the difference can be within 1.5%. Under almost the same return level, the risk can averagely decrease 0.02 when the company replaces MPES-PR method with MPES-Reallocation. We will omit the comparison between MPES-IR and MPES-Reallocation because the results are almost the same as above.

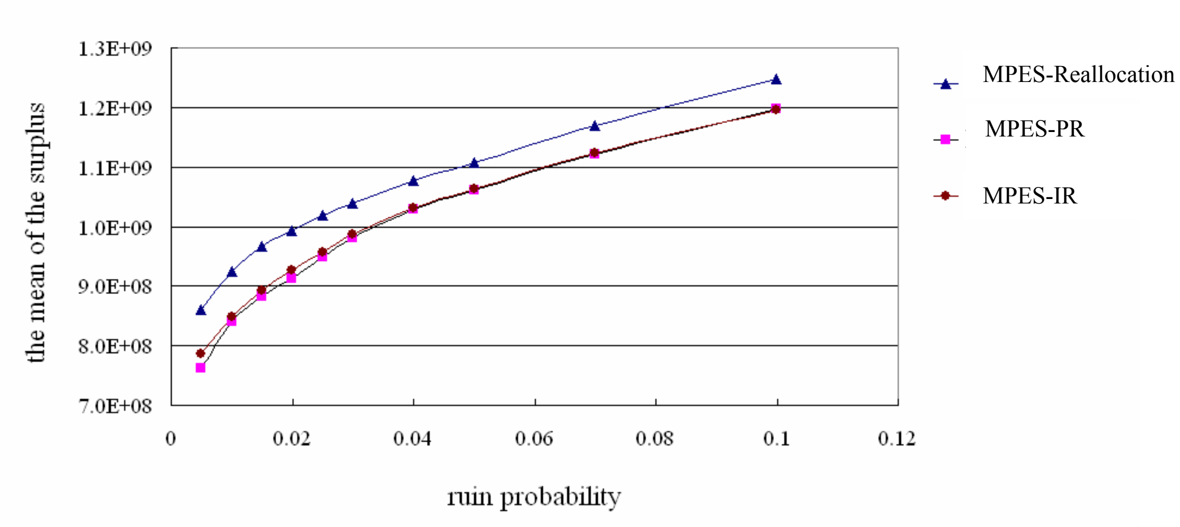


Figure 6: The efficient frontiers of all strategies using MPES

Now let us put all the curves together and see the relationship of all performance for all methods and strategies (see Figure 7). We found three interesting things. First, the figure shows the results from the best to the worst as ES-Reallocation, MPES-IR, MPES-PR, Grid-IR and Grid-PR. Therefore the MPES-Reallocation has the best result in all methods and strategies. Second, whether the periodical rebalance or the interval rebalance method, MPES has better results than the grid search method. Finally, we also compare the difference of the performance between the grid search method and the MPES. Comparing to the MPES-Reallocation, the risk of the Grid-IR averagely increases 0.037 under the similar returns and the Grid-PR averagely increase 0.053.

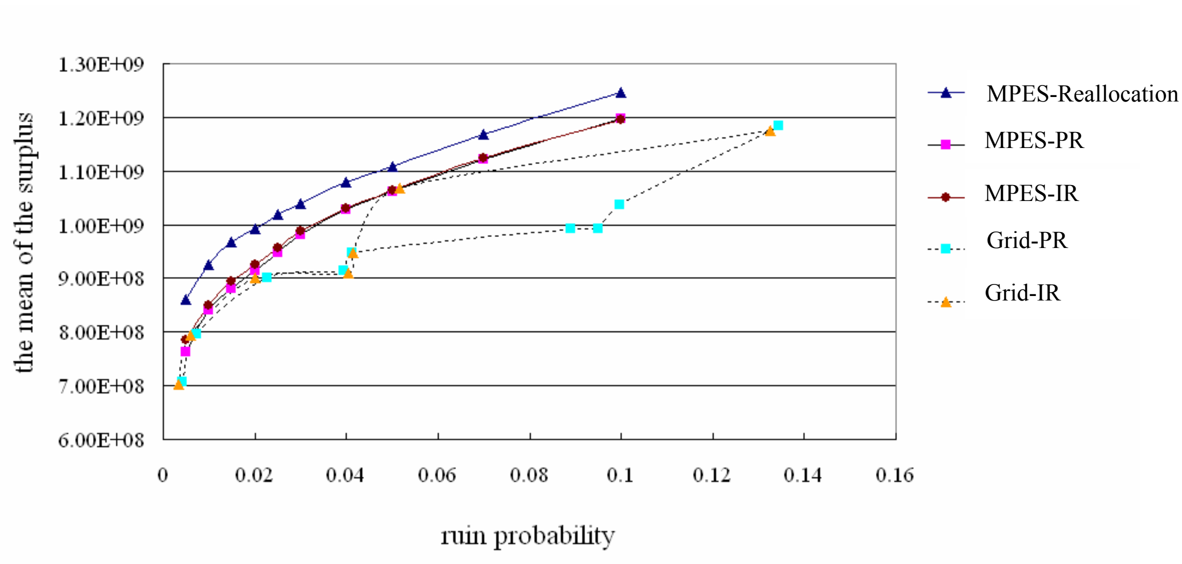


Figure 7: The efficient frontiers of all strategies and methods

B. Analyses on Utilities

The MPES-Reallocation still got the best result when it sought the value of objective function. It has improvement of 16.16%, 16.49%, 10.04% and 8.97% as it compares with the Grid-PR, Grid-IR, MPES-PR and MPES-IR (see Table 4). Under the same periodical rebalance, the objective value of the MPES is better then the grid search method by 5.56%. Similarly, under the same interval rebalance, the objective value of the MPES is better then the grid search method by 6.90%.

The MPES-Reallocation is the only one, which has different asset allocation across the strategy times. We can see the stocks are the largest investment item and the high-risk-high-return asset is the second largest. Bonds get zero allocation, on the other hand. During these strategy times, the largest change between the highest and lowest asset allocation happened in AI^{ll} and the second one is in AI^{hh}. The difference is that the proportion of AI^{hh} insists to increase; nevertheless it always decreases in AI^{ll}.

Table 4: Comparisons of optimized utility using different strategies and methods

Strateties and methods	Value of the Utility	Avg. Discounted Surplus	Imp. of MPES-Reallocation	Ruin Prob.	Asset Allocation				
					stock	bond	AI^{hh}	AI^{ll}	AI^{lh}
Grid-PR	796,883,205	796,883,205	16.16%	0.0074	0.4	0	0.2	0.2	0.2
Grid-IR	794,616,436	794,616,436	16.49%	0.0059	0.4	0	0.2	0.2	0.2
ES-PR	841,203,931	841,203,931	10.04%	0.01	0.4441	0	0.2284	0.1623	0.1652
ES-IR	849,424,629	849,424,629	8.97%	0.01	0.4751	0	0.2157	0.1130	0.1962
ES-Reallocation	925,653,245	925,653,245	—	0.01					
			Reallocations of five periods						
				t=0	0.3721	0	0.2043	0.2748	0.1488
				t=5	0.4262	0	0.2664	0.1137	0.1937
				t=10	0.4328	0	0.3543	0.0158	0.1971
				t=15	0.4249	0	0.3852	0	0.1899
				t=20	0.2967	0	0.5254	0.0002	0.1777
			avg. of the five reallocations		0.3905	0	0.3471	0.0809	0.1814

C. Asset Allocations

The proportions of assets allocation will change when the ruin probability are changed, as expected. The data in Table 5-1 and 5-2 indicates that the allocations of stocks are quite similar across all the risk levels. However, the bonds seldom appear and just have tiny proportions under all risk levels. Regarding three types of the alternative investment, the one with high risk and high return gets much more apparently extended proportion when the risk gets higher. The difference of the AI^{hh} allocation between its lowest point and highest point is 0.5106. The allocation proportion of AI^{ll} is quite smaller than other assets in any panel. When it compares with itself across the panels, we can find that it becomes smaller as soon as the risk gets higher. The last assets AI^{lh} with low return and high risk distributed quite stably at proportion 0.2 in all panels.

We also draw the average proportions of assets at each risk level in Figure 8. Although the curve of stock seems descendant, as the risk gets higher, it still maintains the proportions at the range 0.3 to 0.4. The curves of bonds, AI^{ll} and AI^{lh} allocations are quite smooth but bonds and AI^{ll} always keep at the low levels that almost reach zero. The curve of AI^{hh} is the only one that insists as ascendant as the risk and its gradient is the greatest. At the beginning the proportion of stocks is higher than AI^{hh} but the AI^{hh} starts to go beyond the stocks at risk 0.02. The proportion of AI^{hh} even could reach to 0.6 when risk is at 0.1.

We get some implications from the observations that the stocks and AI^{hh} always take the most part of the allocation but the bonds and AI^{ll} take quite small ones. This reveals that the returns of the stocks and AI^{hh} is relative high to other assets and even could weaken the threat of the risks. Unfortunately, the conservation of the bond and AI^{ll} makes themselves disappear immediately. Although AI^{lh} is associated with low return and high risk, it still could be used to moderately regulate the risks of the stocks and AI^{hh}. That makes AI^{lh} keep the stable position.

Table 5-1: The asset allocations across risk levels using MPES-Reallocation for finding the optimal utility (ruin probabilities are from 0.005 to 0.03)

	ruin prob.=0.005					*ruin prob.=0.01*					*ruin prob.=0.015*				
	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}
t=0	0.3236	0.0000	0.1757	0.3766	0.1241	0.3721	0.0000	0.2043	0.2748	0.1488	0.4097	0.0000	0.1925	0.2143	0.1835
t=5	0.3598	0.1773	0.2135	0.0926	0.1568	0.4262	0.0000	0.2664	0.1137	0.1937	0.5064	0.0000	0.2659	0.0502	0.1775
t=10	0.4562	0.0000	0.3497	0.0241	0.1700	0.4328	0.0000	0.3543	0.0158	0.1971	0.4350	0.0000	0.4116	0.0000	0.1534
t=15	0.5520	0.0000	0.3449	0.0000	0.1031	0.4249	0.0000	0.3852	0	0.1899	0.3692	0.0000	0.4891	0.0041	0.1376
t=20	0.4205	0.0000	0.3666	0.0000	0.2129	0.2967	0.0000	0.5254	0.0002	0.1777	0.2349	0.0000	0.5262	0.0000	0.2389
Avg.	0.4224	0.0355	0.2901	0.0987	0.1534	0.39054	0.0000	0.3471	0.0809	0.1814	0.3910	0.0000	0.3771	0.0537	0.1782

	ruin prob.=0.02					*ruin prob.=0.025*					*ruin prob.=0.03*				
	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}
t=0	0.3728	0.0023	0.2388	0.2164	0.1697	0.4180	0.0000	0.2470	0.1701	0.1649	0.4357	0.0021	0.2435	0.1679	0.1508
t=5	0.5310	0.0000	0.2719	0.0075	0.1896	0.4761	0.0000	0.3130	0.0039	0.2070	0.4702	0.0000	0.3301	0.0000	0.1997
t=10	0.4433	0.0000	0.4120	0.0000	0.1447	0.4256	0.0006	0.4222	0.0004	0.1512	0.4648	0.0000	0.4289	0.0000	0.1063
t=15	0.3902	0.0000	0.4867	0.0003	0.1228	0.2947	0.0000	0.5405	0.0000	0.1648	0.2645	0.0000	0.5830	0.0000	0.1525
t=20	0.2377	0.0000	0.5834	0.0000	0.1789	0.3138	0.0000	0.5823	0.0082	0.0957	0.2812	0.0000	0.6089	0.0000	0.1099
Avg.	0.3950	0.0005	0.3986	0.0448	0.1611	0.3856	0.0001	0.4210	0.0365	0.1567	0.3833	0.0004	0.4389	0.0336	0.1438

Table 5-2: The asset allocations of MPES-Reallocation across risk levels for finding the optimal utility (ruin probabilities are from 0.04 to 0.1)

	ruin prob. =0.04					*ruin prob. =0.05*					*ruin prob. =0.07*				
	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}
t=0	0.3762	0.0000	0.3076	0.1297	0.1865	0.3752	0.0000	0.3356	0.1050	0.1842	0.4172	0.0000	0.3661	0.0087	0.2080
t=5	0.4823	0.0000	0.3545	0.0000	0.1632	0.4371	0.0000	0.3991	0.0000	0.1638	0.4083	0.0000	0.4281	0.0000	0.1636
t=10	0.3601	0.0000	0.5006	0.0000	0.1393	0.4001	0.0000	0.4823	0.0000	0.1176	0.3395	0.0000	0.5587	0.0000	0.1018
t=15	0.3281	0.0000	0.5724	0.0000	0.0995	0.2983	0.0000	0.5875	0.0000	0.1142	0.2876	0.0000	0.6183	0.0000	0.0941
t=20	0.2775	0.0000	0.5833	0.0000	0.1392	0.2484	0.0000	0.6456	0.0000	0.1060	0.2144	0.0000	0.6677	0.0000	0.1179
Avg.	0.3648	0.0000	0.4637	0.0259	0.1455	0.3518	0.0000	0.4900	0.0210	0.1372	0.3334	0.0000	0.5278	0.0017	0.1371

	ruin prob. =0.1				
	stock	bond	AI^{hh}	AI^{ll}	AI^{lh}
t=0	0.4146	0.0000	0.3888	0.0021	0.1945
t=5	0.3956	0.0000	0.4908	0.0000	0.1136
t=10	0.2174	0.0000	0.6511	0.0000	0.1315
t=15	0.2356	0.0000	0.6632	0.0000	0.1012
t=20	0.2422	0.0000	0.6863	0.0000	0.0715
Avg.	0.3011	0.0000	0.5760	0.0004	0.1225

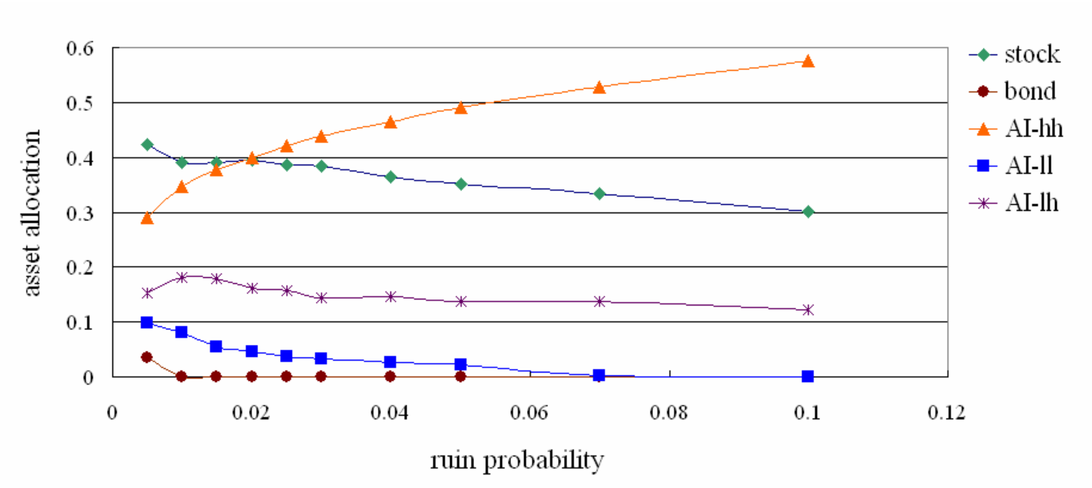


Figure 8: The avg. of five-times asset allocations across ten risk levels using MPES-Reallocation for finding the optimal utility

VIII. SUMMARIES AND CONCLUSIONS

We used the MPES to solve the hardship of the multi-period asset allocation for property-casualty insurer. This issue is based on the views of asset and liability management and therefore when the company pursuits the great wealth, it could consider not only the assets return and risk but also the business liabilities. For the goal of this study, all the experiments have to deal with more than a dozen controllable variables over real intervals. The feasible region is therefore large and seldom tools can solve it in effective ways. To verify the robust of our methodology MPES, we design some problems for solving by MPES and compare the results with other tools. The results of the MPES indicated that this method has sufficient superiority and reliability.

The main efforts of our study are constructing the models of five financial markets and two insurance markets, simulating 10,000 paths of scenarios, making five strategies during twenty-five years. To understand the performance of our strategies, we designed two major experiments. One is the grid search method the benchmark and the other is the MPES. Both of them have to adopt the periodical and interval rebalance and even reallocation method. These results of experiments were expressed and compared to each other under the objectives of efficient frontier and utility value.

The order of the performance that we got regarding the surplus of the company is MPES-Reallocation, MPES-IR, MPES-PR, Grid-IR and then Grid-PR. Therefore the MPES-Reallocation is the best search method and strategy and the interval rebalance seemed more proper than the periodical rebalance. As to the assets allocation in our experiments, the high-return and high-risk objectives are more popular. These objectives such as stocks and alternative investment with high-return and high-risk always take the most portion of the allocation. It means that its return is high enough to compensate the punishment of the risk. That successfully makes them outstanding among the assets. In the contrary, the conservative ones disappeared immediately.

After all, the results demonstrate that the performance of the reallocation using MPES is better than other methods in this study. The experiments of MPES also revealed some interesting implications to us. In addition to the verification of the robust of MPES we have made. Therefore we expect to apply it to study further applications and management issues of the property-casualty insurers in the future.

REFERENCES

Back, T.H. (1996). Evolutionary Algorithms in Theory and Practice, Oxford University Press, New York.

Brennan, M. J., Schwartz, E. S., and Lagnado, R. (1997). Strategic Asset Allocation, *Journal of Economic Dynamics and Control*, 21: 1377-1403.

Campbell, J. Y. (2000). Asset Pricing at the Millennium, *Journal of Finance*, 55: 1515-1567.

Chiu, M. C. and Li, D. (2006). Asset and liability management under a continuous-time mean–variance optimization framework, Insurance: Mathematics and Economics, 39: 330-355.

Cox, J. C. and Huang, C. F. (1989). Optimal Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process, *Journal of Economic Theory*, 39: 33-83.

Cox, J. C., Ingersoll, J. E., Ross, S. A. (1985). A theory of the term structure of interest rates. Econometrica, 53: 385-407.

Craft, T. M. (2005). Impact of Pension Plan Liabilities on Real Estate Investment, *Journal of Portfolio Management*, 23-28.

Harrington, S. E., Niehaus, G. R. (1999). Risk Management and Insurance. Irwin/ McGraw-Hill.

Markowitz, H. M. (1952). Portfolio Selection, *Journal of Finance*, 7: 77-91.

Merton, R. C. (1971). Optimal Consumption and Portfolio Rules in a Continuous-Time

Model, *Journal of Economic Theory*, 3: 373-413.

Merton, R. C. (1990). Continuous Time Finance, Chapters 4-6. Basil Blackwell, Cambridge, MA.

Harrington, S. E., Niehaus, G. R. (1999). Risk Management and Insurance. Irwin/ McGraw-Hill.

Nissen, V. and Biethahn, J. (1995), "An introduction to evolutionary algorithms in Management Applications", J. Biethahn, V. Nissen (Eds.), Springer, Berlin, pp. 3-43.

Rechenberg, I., 1973, Evolution sstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution. Frommann-Holzboog Verlag, Stuttgart.

Sharpe, W. F., Tint, L. G. (1990). Liabilities-A New Approach, *Journal of Portfolio Management*, 16: 5-10.

Schwefel, H.-P., (1981), Numerical Optimization for Computer Models, John Wiley, Chichester, UK.

Tekin, E. and Sabuncuoglu, I. (2004). Simulation optimization: a comprehensive review on theory and applications. IIE Transactions 36: 1067-1081.

Vesterstrom, J. and Thomsen, R. (2004), "A comparative study of differential evolution, particle swarm optimization, and evolutionary algorithms on numerical benchmark problems", Congress on Evolutionary Computation, 2004. CEC2004. 19-23.

Yao, X. and Liu, Y. (1996), "Fast evolutionary programming," *Proc. of the Fifth Annual*

Conference on Evolutionary Programming (EP'96), San Diego, CA, USA.

APPENDIX

Items	Value
System parameters	
φ	10,000
T_n	25
Model parameters in the investment markets	
(1) r	a= 0.07, b= 0.2, v= 0.017
(2) Stock	ℓ_{St} = 0.05, σ_{St} = 0.2
(3) Bond	
kn	15
bp_k $(k=1,\cdots,kn)$	[1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15, 1/15]
(4) AI^{hh}	$\mu_{AI^{hh}}$ =0.16; $\sigma_{AI^{hh}}$ =0.4
(5) AI^{ll}	$\mu_{AI^{ll}}$ =0.08; $\sigma_{AI^{ll}}$ =0.11
(6) AI^{lh}	$\mu_{AI^{lh}}$ =0.10; $\sigma_{AI^{hh}}$ =0.4
Model parameters of the insurance businesses	
$\Pr m_0$	2E+8
λ_0	0.5
(1) long-tail business	
t^l	10
f^l	0.4
g^l	0.03
d_i^l $(i=1,\cdots,t^l)$	[0.4, 0.2, 0.1, 0.05, 0.05, 0.05, 0.05, 0.04, 0.03, 0.03]
(1) short-tail business	
t^s	3
f^s	0.25
g^s	0.03
d_i^s $(i=1,\cdots,t^s)$	[0.8, 0.1, 0.1]

rules of asset adjustment

(1) periodical — Adjusting every five years

(2) interval — The changeable proportion of any asset is limited within ± 0.3. This result in $[\theta_a \times 0.7, \theta_a \times 1.3]$ to be the boundary of the proportions of asset allocation interval, where θ_a is the proportion of an asset.

others

r_s	0.05
k	2.5E+10
τ	0.01

correlation matrix R

	dW_r	dW_{St}	$dW_{AI^{hh}}$	$dW_{AI^{ll}}$	$dW_{AI^{lh}}$	dW_{lr^l}
dW_r	1	-0.3	-0.2	-0.2	0.4	0.4
dW_{St}	-0.3	1	0.2	0.1	-0.3	-0.2
$dW_{AI^{hh}}$	-0.2	0.2	1	0.3	-0.3	-0.3
$dW_{AI^{ll}}$	-0.2	0.1	0.3	1	-0.3	-0.3
$dW_{AI^{lh}}$	0.4	-0.3	-0.3	-0.3	1	0.2
dW_{lr^l}	0.4	-0.2	-0.3	-0.3	0.2	1

第三節

I. INTRODUCTION

The purpose of the following experiments is to understand whether the performance of the reallocation is better then the rebalance. Furthermore, we are curious about how the asset allocations would vary with different strategies and solution methods. The strategies of this study mean the change frequency of asset allocations made by the insurers. They include single-period, rebalance, and reallocation. The strategy single-period means the insurers make only one asset allocation at the beginning of the investment and do not to interrupt the allocation till to the end of the investment. The strategy rebalance means the insurers make only one asset allocation at the beginning of the investment and periodically adjust the assets back to the initial allocations they previous made. The strategy reallocation means the insurers have to make new asset allocations periodically. The solution methods include GA and Grid search method.

In these experiments, we use GA to solve the asset allocations problems and also use Grid search method as a benchmark to solve the same problems. Through comparing these results we can indirectly evaluate the performances of strategies and methods. We adopted the Grid search method as a benchmark because it is a common and basic method that researchers usually use it to compare with other methods. The other reason is that when people can not get an efficient method to solve their problems, the Grid search method is a basic and easily executed one. However, the Grid search method can not solve the difficult reallocation problem, so we just use it to solve the easier ones, rebalance and single-period.

We executed two categories of optimization problems in GA experiments. The problems of first category are to find the optimized value of our utility function without any constraints. The problems of the second category are also to find out the optimized value but with some constraints. The constraints are some certain values that we set on the ruin probability and the sigma value of the capital. The constrained values of the ruin probability could be chosen from the seven default values 0.01, 0.02, 0.03, 0.04, 0.05 and 0.07, and the sigma default values of the capital are 0.5E+09, 1E+09, 2E+09, 4E+09, 7E+09, 10E+09, and 15E+09. Therefore, we can generate 49 combinations at most from the two sets of default vales. We used GA solve the problems of rebalance and reallocation strategies. Therefore, this study has one hundred GA experiments. In addition, we executed other 126 experiments[25] for each of the rebalance and single-period strategies using Grid search method. The experiments using Grid search method are easier than these using GA because we can simply get the values of the asset allocations from figure combinations. The following table is concise

[25] The 126 experiments are executed by the diverse combinations of the asset allocations. This study has five kinds of assets and the sum of the allocations of all assets must be one. Each value of the asset allocation can be 0, 0.2, 0.4, 0.6, and 0.8 as we set. Therefore, the number of combinations is $3\times\frac{5!}{3!1!1!}+2\times\frac{5!}{2!2!1!}+5+1=126$ (The first addition term is the sum of permutations and combination in [0.8, 0.2, 0, 0, 0], [0.4, 0.2, 0.2, 0.2, 0], and [0.4, 0.4, 0.2, 0, 0]. The second addition term is the result from [0.6, 0.2, 0.2, 0, 0] and [0.4, 0.4, 0.2, 0, 0]. The last two terms are the results from [1, 0, 0, 0, 0, 0] and [0.2, 0.2, 0.2, 0.2, 0.2].)

summary about the total experiments.

Table 1: Experiment design

Search method	Asset Allocation Strategies	Number of exp.	Simplified Form
Grid search method	single-period	126	Grid_SP
	rebalance	126	Grid_Reb
	reallocation	50	GA_Rea
GA	rebalance	50	GA_Reb

In the following sections are the experiment results and analyses. The analyses include (1) the comparisons of the efficient frontiers between different asset allocation strategies and solution methods, (2) the comparisons of the optimized utility function values and the portfolios, and (3) the changes of each asset allocation across different risks. The related experiments parameters are set in the appendix.

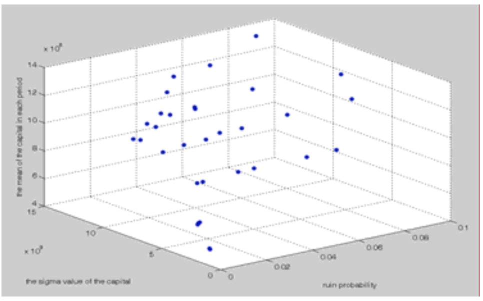
II. ANALYSES ON EFFICIENT FRONTIER

Utilizing the efficient frontiers could provide us a quick look at the performance of which method and which strategies is the best. The efficient frontier was first defined by Markowitz (1952). He considers that an optimal portfolio should be one of the following two situations— For any level of volatility, select one portfolio which has the highest expected return from all portfolios with that same level of volatility. For any level of expected return, select one portfolio which has the lowest volatility from all portfolios with that same expected return. When the investors undertake the different risks, there should be a portfolio which can reach the greatest expected return at the risk level that they accept. The various highest expected returns across the different risks would form a curve that is called the efficient frontier.

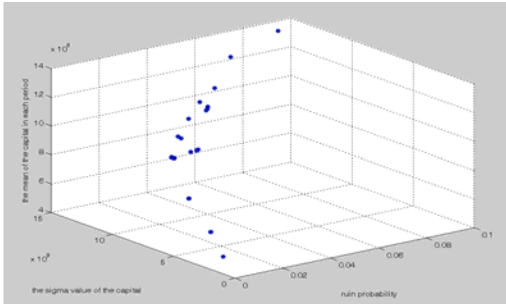
The efficient frontier in Figure 2 is from the results of optimization with the certain constraints in our 49 experiments. Actually, both of GA_Rea and GA_Reb have 7 experiments that we cannot find out the optimized value in all levels of ruin probability associated with the value 0.5E+09 of the sigma of capital[26]. Besides, we have omitted the inefficient points from the figure. In the three-dimensional (3D) plot, the x axis and y axis express two kinds of risks- ruin probability and sigma of the capital. The z axis is the mean of the capital.

The whole distributions of the points in two plots of Grid search method are quite similar (see Figure 2 (c) and (d)). The points of GA_Rea are spread over the wider areas in the axis x, y and z. Nevertheless, the values of the ruin probabilities of the points are much closed to each other under the same risk levels of sigma of capital in GA_Reb plot. Although the whole distributions of the points could be viewed briefly in 3D plots, the efficient frontier is too hard to compare the performances of strategies and methods in this style.

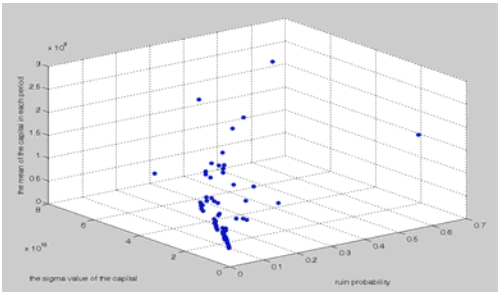
[26] The value 0.5E+09 of the sigma of the capital we set is too small to find out the optimized value.



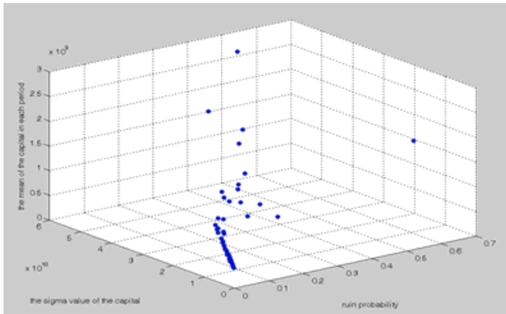
(a) GA-Reallocation



(b) GA_Reb



(c) Grid-SP



(d) Grid_Reb

Figure 2: The efficient frontier of various strategies and methods in three-dimension plots

The efficient frontier in two-dimension (2D) style is a valid way to understand the performance relationship between the strategies and methods. We could choose one of the risks to observe the efficient frontier. The risk we present here is the sigma of capital at a certain ruin probability. However, this study cannot search out the optimized portfolio under some risk levels we set. The interpolation is a popular method to create the points that we need. After creating the new points, all the following 2D plots would be sketched at certain value 0.015 of the ruin probability, and we would first discuss the results of single-period and rebalance in Grid search method. Latter, we would compare the performances of reallocation and rebalance in GA. Finally, a comprehensive analysis of the relationships between strategies and all solution methods would be described together.

In Figure 3, the performance of Grid_Reb is better than Grid_SP's. The expected return of the Grid_Reb are greater 8.17%, 4.4%, 14.17%, 17.13%, 13.4% and 8.68% than Grid_SP as the risks 3.00E+09, 3.50E+09, 4.00E+09, 5.00E+09, 6.50E+09, and 7.00E+09. Two curves have positive growth, yet the growth rate of Grid_Reb tends decrease and Grid_SP always increases. However, the performance of the Grid_Reb is still better than Grid_SP. Even though the Grid_SP grows beyond the Grid_Reb after risk 7.00E+09, the Grid_Reb would be more suitable the general and conservative insurers.

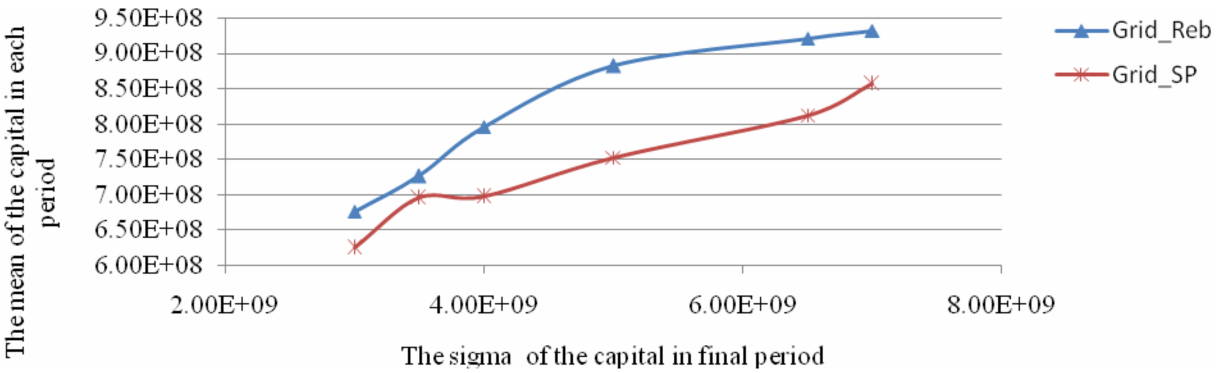


Figure 3: The efficient frontier of Grid_Reb and Grid_SP

In the GA part, the efficient frontier of reallocation is higher than that of rebalance, so the performance of GA_Rea is greater than GA_Reb (see Figure 4). The differences between all corresponding points from the left of two curves are 14.10%, 9.81%, 5.48%, 2.21%, 1.79%, 1.37%, and 0.69% under individual risk 4.50E+09, 5.00E+09, 5.50E+09, 6.00E+09, 6.50E+09, 7.00E+09, and 7.50E+09. The growths of both curves are positive and their growth speeds tend to decline. It look similar as Grid part, two curves gradually meet together after the risk 7.00E+09.

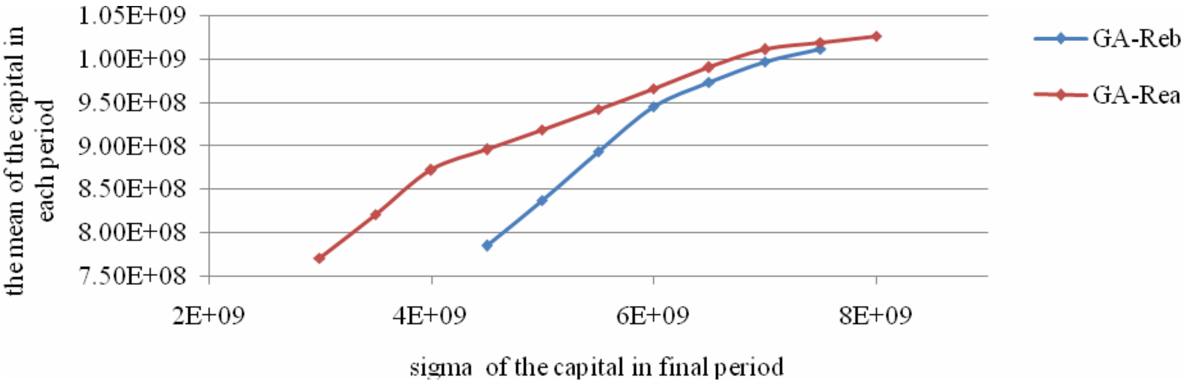


Figure 4: The efficient frontier of the GA_Rea and GA_Reb

The performances of all strategies and solution methods arranged from high to low are GA_Rea, GA_Reb, Grid_Reb, and Grid_SP (GA_Rea, Grid_Reb, GA_Reb, and Grid_SP when the risk is before 5.50E+09) in Figure 5. The curves of GA_Reb and Grid_Reb intersect at risk about 5.50E+09. It denotes that Grid_Reb is better than GA_Reb when the risk is not so high. As the risk is getting higher, the expected return of GA_Reb is preferable to Grid_Reb. For the GA and the Grid search method, the GA_Rea and the Grid_Reb have the best performance individually. The difference between GA_Rea and Grid_Reb are 14.02%, 13.02%, 9.57%, 4.26%, 4.42%, and 8.61% at the corresponding points under the same risk and from the lower left of the curves.

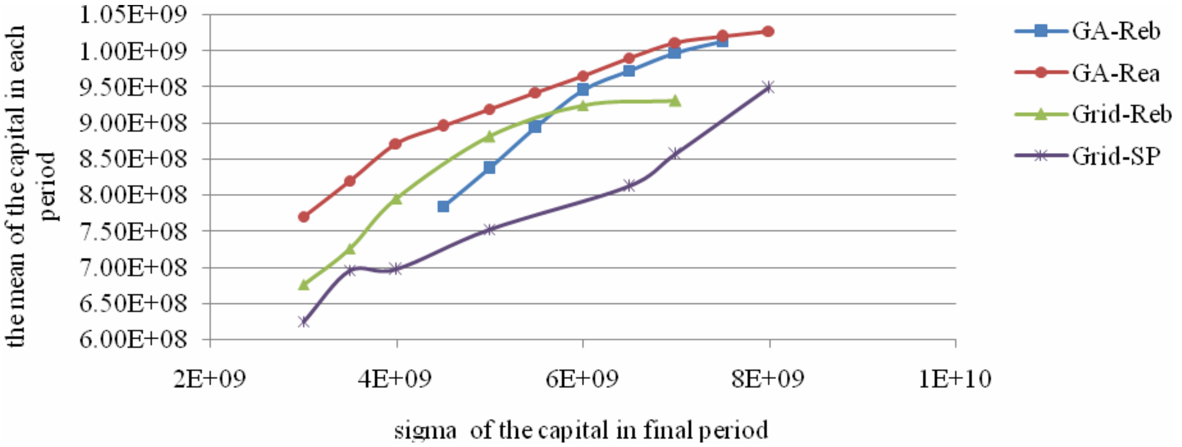


Figure 5: The efficient frontier of all strategies and methods

III.ANALYSES ON UTILITIES

When optimization problems are without any constraints, the values of the objective function arranged from the greatest to the worst in Table 2 are GA_Rea, GA_Reb, Grid_Reb, and Grid_SP. The utility value of GA_Rea is better 2.5%, 12.1% and 12.7% than GA_Reb, Grid_Reb, and Grid_SP. The optimization problems are without any constraints, so the sigma of the capital and the ruin probability are not limited within some ranges. However, relative to the Grid search method, the ruin probabilities of GA_Rea and GA_Reb are much closed to the value 0.01 (not exceed yet). The value 0.01 is our punitive boundary that the optimized value would be deducted once the ruin probability is beyond this default value.

As to the asset allocations of all strategies and methods are shown in Figure 6. The AI-hh takes the greatest portion of the asset allocation in GA_Rea and in GA_Reb when the stock takes the biggest part in Grid_Reb and in Grid_SP. The bond takes the smallest portion of the asset allocations in all strategies and methods. The whole distribution of assets of

Grid_Reb and Grid_SP are the same.

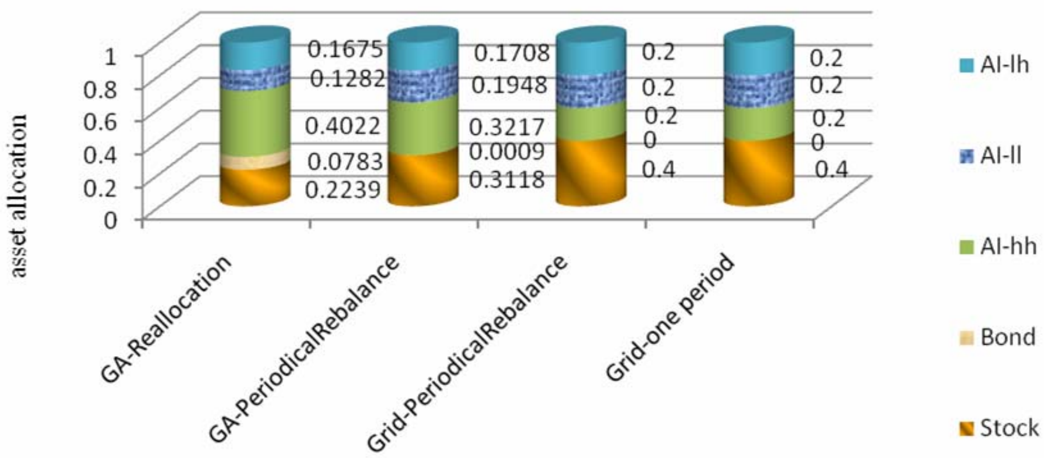


Figure 6: The different asset allocations between various strategies and methods

IV. ANALYSES ON ASSET ALLOCATIONS

We choose the GA_Reallocaiton which has the best performance as the object for analyzing the changes of asset allocation across different risks. The data is from the optimization results of the objective function with constraints. For simplify, we adopt the so-called integrated risk as the observed risk. The integrated risk is the sum of the two parts of risk in our objective function. Finally, we averaged the allocations of all experiments for each kind of assets across different integrated risks.

Table 2: The results of find the optimized objective vale using various strategies and methods

The value of utility function	The degree of the utility value improved by GA _Reallocation	Mean: The mean of the capital	Risk: Sigma of the capital in the final period	Risk: Ruin prob.	Asset allocation	Stock	Bond	AI-hh	AI-ll	AI-lh
GA_Rea										
927,699,346	-	1,032,838,326	8,761,581,673	0.0098	t =1	0.1810	0.0183	0.3055	0.2745	0.2207
					t=7	0.2389	0.0526	0.4050	0.1593	0.1442
					t=13	0.2519	0.0721	0.5009	0.0121	0.1630
					t=19	0.2237	0.1700	0.3973	0.0668	0.1422
					Avg.	0.2239	0.0783	0.4022	0.1282	0.1675
GA – Periodical Rebalance										
905,175,255	2.5%	989,624,799	6,929,128,680	0.01		0.3118	0.0009	0.3217	0.1948	0.1708
Grid – Periodical Rebalance										
827,707,091	12.1%	886,285,207	4,881,509,679	0.0038		0.4	0	0.2	0.2	0.2
Grid-one period										
822,971,133	12.7%	912,802,369	7,485,936,406	0.0038		0.4	0	0.2	0.2	0.2

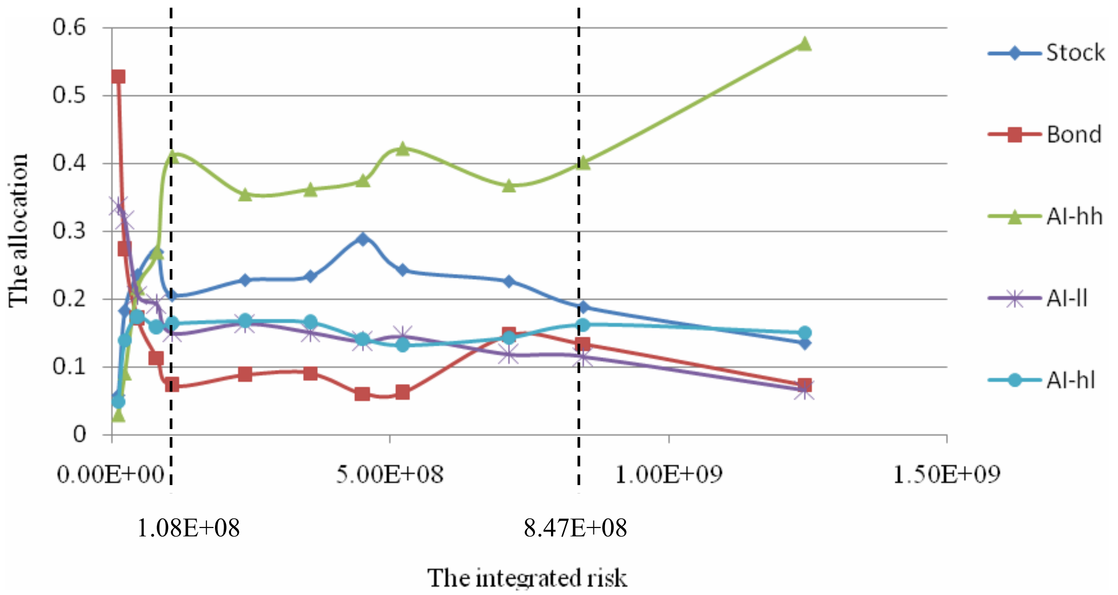


Figure 7: The changes of asset allocation across different risks in GA_Rea

The asset allocations of optimization for the objective function with constraints across risks are shown in Figure 7. The bond and AI-ll have the more portions in the asset allocation at the lowest integrated risk but they take the small parts at the highest risk. On the contrary, AI-hh has the opposite situation to the bond and AI-ll. According to the relationship between the asset allocation and risks in Figure 7, the insurer may allocate more funds on the bond and AI-ll when he plans a conservative investment. The insurer may also allocate more money on the AI-hh and some funds on the stock and AI-hl but little on the bond and AI-ll if he tries to get more expected return under the risky markets.

Regarding the view on the whole tendency of the lines, most assets change their allocations greatly at the risk 1.08E+08 and gets steady after that risk. The AI-hh almost obtains the greatest proportion about 0.4. However, the risk 8.47E+08 is the second turning point and the AI-hh even can reach the allocation 0.5762. The stock almost changes its allocation between 0.2 and 0.3. The changes of AI-hl and AI-ll are quite similar and their fluctuations occur within the range 0.1 and 0.2. The bond nearly stays under the position 0.1.

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

計畫期間參加的第一次會議

報告人姓名	蔡政憲	服務機構及職稱	政治大學風險管理與保險學系專任副教授
時間 會議 地點	2006 / 7 / 30 – 8 / 2 日本東京	本會核定補助文號	95-2416-H-004 -009
會議名稱	(中文) (英文) The Tenth Annual Conference of Asia-Pacific Risk and Insurance Association		
發表論文題目	(中文) (英文) 1. The Effectiveness of the Asset Allocation Using the Technique of Simulation Optimization 2. The Application of KMV's Private Firm Model to the Solvency/Insolvency Predictions on Life Insurers		

報告內容應包括下列各項：

一、參加會議經過

個人先是於 7/30 下午參加 board meeting，接著參加 welcome reception。星期一早上是開幕演講以及 Plenary Sessions。星期二早上與下午則各有一篇文章的 presentation，星期二早上還同時主持了一場。

二、與會心得

在會議的 reception and breaks 中，和一些舊識與新認識的學者有不錯的互動，很好。也有機會看到其他學者正在研究的東西，一方面 update 最新資訊，二方面激勵自己。

三、考察參觀活動(無是項活動者省略)

無

四、建議

五、攜回資料名稱及內容

大會製作之發表人論文集光碟片一份

六、其他

無

計畫期間參加的第二次會議

第二次會議是由蔽系在台北所主辦的第十一屆亞太風險與保險協會的年會。會議前我們邀請了聖約翰大學的 Dr. Jean Kown 來台互動。會議期間個人和許多的學者有相當的互動，一方面由於個人是此協會的理事，二方面我們是地主，比較瞭解各種狀況，心態上也比較主動。這一次的會議對蔽系在國際上的知名度有相當的提升，也提升了個人和這個學術社群的關係，相當值得。