行政院國家科學委員會專題研究計畫　成果報告

在利率敏感型解約行為下之壽險責任準備金的實質存續期間

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中文摘要

壽險責任準備金的存續期間對壽險公司的利率風險管理是很重要的。由於責任準備金是由在不同時販賣的個別保單準備金所組成，本計畫先估計不同保單年度保單準備金的存續期間，然後再估計總準備金的存續期間。

我們發現保單存續期間和保單的到期日之間有一個有趣的關係，這個關係我們稱之為保單存續期間的期間結構。這個期間結構有一個漸進線與兩條曲線。位於漸進線左手邊的曲線是在正值的範圍內，另外一條則是在負的值域。準備金的存續期間可能小於0，也可能有個很大的數字。這些都是新的發現。

不過，這些數值詭異的存續期間，並不會使總準備金的存續期間有特殊的數值，因為有特殊數值存續期間的個別準備金，其金額都很小，因此在和其他有正常數值的準備金加權平均之後，影響力很小。我們的結果顯示總準備金的存續期間都落在正常的範圍，讓保險公司可以進行資產負債管理。此外，我們還發現成長（衰退）的公司需要存續期間比較大（小）的資產來降低公司的利率風險。

關鍵詞：實質存續期間，解約選擇權，壽險責任準備金

Abstract

Knowing the duration of the policy reserve liability is important for a life insurer to manage its interest rate risk. Since the reserve liability results from policies sold in different times, this paper calculates the reserve durations for individual policies that have different maturities and then estimate the duration of aggregate reserves. The results show that the duration of the policy reserve may be negative and/or have a large figure. They further reveal an interesting pattern of the reserve duration with respect to the policy's time to maturity. The term structure consists of two curves separated by an asymptote. The curve on the left hand side of the asymptote is in the positive domain while the right-hand-side one is in the negative domain. Such a term structure with abnormal durations, however, does not result in an abnormal duration of aggregate reserves. This is because the policies with abnormal durations always have small reserves and play insignificant roles in determining the duration of aggregate reserves. In addition, a growing/declining company should seek assets with larger/smaller durations to match its policy reserve liability.

JEL Classification: G22

Keywords: effective durations, policy reserves, life insurance

Managing the company's interest rate risk is vital to a life insurer. Life insurance policies are long-term contracts. Small changes in interest rates thus can cause large changes in the policy reserve liability that usually constitutes more than 90 percent of a company's total liabilities. To offset the resulted fluctuations in the value of the reserve liability, the company looks for an asset portfolio that can produce matched changes in values. If the match is not perfect, the high liability-to-surplus ratio prevalent in the life insurance industry would make the mismatch large relative to the insurer's surplus. Movements in interest rates, therefore, can have a significant adverse impact on the solvency of a life insurance company.

A common measure of a life insurer's exposure to the interest rate risk is surplus duration. Surplus duration represents the sensitivity of an insurer's surplus to changes in interest rates. Since surplus duration equals a weighted average of asset and liability durations, the building blocks to measure a life insurer's interest rate risk are the durations of individual assets and liabilities. The durations of the important assets held by life insurers such as bonds, mortgages, and stocks have been investigated extensively in the finance literature (e.g., Boquist, Racette, and Schlarbaum, 1975; Bierwag, Kaufman, and Toevs, 1983; Bierwag, 1987; Bierwag, Corrado, and Kaufman, 1992; Babbel, Merrill, and Panning, 1997). In contrast, the durations of insurance liabilities receive limited attention. Babbel (1995) estimates the option-adjusted durations of the liabilities associated with a dozen life insurance products using the PTS Shane Chalke software. Santomero and Babbel (1997) list the effective durations of the liabilities of several life insurance products, drawing from their on-site investigation on the risk management practices of insurance companies. Briys and Varenne (1997; 2001) calculate the effective duration of the liability associated with the single-premium participating contract that has a minimum guaranteed rate of return.

This paper contributes to the literature in two aspects. Firstly, we calculate the effective durations of reserves for policies that have different maturities. These duratons are important because the policy reserve liability comprises the reserves of policies that are sold in different times and its duration is a weighted average of individual reserve durations. No one has reported such calculations yet. Secondly, we take some weighted averages to estimate the durations of aggregate reserves. Analyzing the estimated durations is helpful to understand the exposure of the policy reserve liability to changes in interest rates and provides the insurer guidance when constructing its asset portfolio. Babbel (1995) and Santomero and Babbel (1997) did estimate/report the effective durations of reserves for some products, but they disclose only the final results without the policy specification, pool composition, interest rate model, surrender behavior, and other assumptions. Due to the missing details, the characteristics of the duration of the policy reserve liability are still obscure in the literature.

We employ effective duration as the measures of interest rate sensitivity. Effective duration is the better measure since the insurance literature pinpoints the importance of the interest rate sensitivity of cash flows when estimating the durations of life insurance liabilities. We adopt the vector autoregression (VAR) model of the interest rate and surrender rate established in Tsai, Kuo, and Chen (2002) to generate interest-dependent surrender rates and thus interest-sensitive cash flows. A 20-year endowment product issued to 30-year-old males in different years serves as an illustrative example in this study.

We find that the effective duration of the policy reserve might be negative and might have a figure far exceeding the policy's maturity. We further discover an interesting pattern of reserve durations with respect to maturities as Figure 1. Figure 1 is named the term structure of reserve durations. It demonstrates that the reserve duration is a function of the policy's time to maturity that has a vertical asymptote at a break-even maturity. The break-even maturity is the time to maturity when the policy's reserve equals to zero. The vertical asymptote is thus called the zero-reserve line. The modified duration increases with the maturity and approaches infinity when the maturity approaches the break-even maturity from the left. The modified duration turns into negative infinity when the maturity crosses over the break-even maturity and then increases with the maturity until approaching zero.

This newly identified term structure of reserve durations originates from the bi-directional cash flows embedded in the policy reserve. The policy reserve equals the present value of expected cash outflows minus the present value of expected cash inflows. It has the opposite sign to the net present value (NPV) of a policy that is the difference between the present value of future cash inflows and that of future cash outflows. A policy that is profitable to the life insurer has a positive NPV and thus a negative reserve. Newly-sold policies should be profitable as long as they are priced correctly. The profit of a two- or three-years-old policy would be smaller than that of a brand-new policy since fewer premiums are to be collected while the mortality rate increases. A correctly-priced policy would become break-even some time after it is sold. Older policies would be a liability to the insurer since the present value of expected cash inflows is smaller than that of expected cash outflows. In short, the NPV/reserve of a policy is an increasing/decreasing function with respect to the policy's time to maturity. New/old policies tend to have negative/positive reserves, and a correctly-priced policy has a reserve of zero sometime in its term. Since the policy reserve is in the denominator when calculating duration, the modified duration is negative, negative/positive infinite, and positive for young policies, break-even policies, and old policies respectively.

The above findings seem to imply that the duration of the policy reserve liability on a life insurer's balance sheet might have an abnormal value because it is a weighted average of individual reserve durations. This inference ignores a feature of the reserve duration: abnormal duration values are coupled with small reserves. Since the underlying reserves of abnormal durations account for only a small percentage of the aggregate reserves, these abnormal values are immaterial in calculating the aggregate reserve duration. The duration of aggregate reserves therefore has a normal value even when the component reserves have abnormal values. For instance, the duration of the aggregate reserves resulted from equal numbers of endowment policies sold in different years could have a duration less than 8 even though some policies have reserve durations larger than 80. Further analyses show that growing/declining businesses lead to larger/smaller aggregate reserve durations because younger/older policies tend to have larger/smaller figures of reserve duration.

We summarize all our results in the following table and figures.

Table 4: The effective durations of aggregate reserves with various initial interest rates

Initial Surrender Rate		0%			2%			4%			6%			8%		
Years to Maturity	Number of Policies	Effective Duration	Reserve	Weight	Effective Duration	Reserve	Weight	Effective Duration	Reserve	Weight	Effective Duration	Reserve	Weight	Effective Duration	Reserve	Weight
1	1,000	0.97	965,353	11.16%	0.96	961,601	11.60%	0.95	952,827	12.47%	0.94	939,849	13.49%	0.92	924,525	14.39%
2	1,000	1.93	893,934	10.34%	1.92	886,936	10.70%	1.89	870,855	11.40%	1.84	847,760	12.17%	1.79	821,463	12.79%
3	1,000	2.90	825,896	9.55%	2.87	816,104	9.85%	2.81	794,002	10.39%	2.72	763,153	10.95%	2.61	729,334	11.35%
4	1,000	3.88	760,850	8.80%	3.84	748,668	9.03%	3.73	721,649	9.45%	3.57	685,047	9.83%	3.39	646,441	10.06%
5	1,000	4.90	698,754	8.08%	4.83	684,510	8.26%	4.66	653,519	8.55%	4.42	612,830	8.80%	4.13	571,577	8.90%
6	1,000	5.96	639,522	7.40%	5.86	623,516	7.52%	5.61	589,382	7.71%	5.26	545,988	7.84%	4.85	503,737	7.84%
7	1,000	7.09	582,658	6.74%	6.94	565,161	6.82%	6.60	528,608	6.92%	6.12	483,690	6.94%	5.56	441,731	6.88%
8	1,000	8.30	527,889	6.10%	8.10	509,134	6.14%	7.65	470,811	6.16%	7.01	425,363	6.11%	6.28	384,691	5.99%
9	1,000	9.64	475,201	5.50%	9.38	455,401	5.49%	8.79	415,879	5.44%	7.96	370,728	5.32%	7.02	332,089	5.17%
10	1,000	11.14	424,558	4.91%	10.81	403,903	4.87%	10.06	363,681	4.76%	9.01	319,513	4.59%	7.83	283,453	4.41%
11	1,000	12.87	375,711	4.34%	12.46	354,373	4.28%	11.53	313,932	4.11%	10.21	271,337	3.89%	8.74	238,251	3.71%
12	1,000	14.89	329,567	3.81%	14.38	307,666	3.71%	13.25	267,344	3.50%	11.63	226,698	3.25%	9.81	196,735	3.06%
13	1,000	17.36	284,768	3.29%	16.76	262,416	3.17%	15.42	222,563	2.91%	13.46	184,208	2.64%	11.22	157,474	2.45%
14	1,000	20.52	241,236	2.79%	19.84	218,562	2.64%	18.33	179,526	2.35%	16.03	143,783	2.06%	13.26	120,344	1.87%
15	1,000	24.77	198,903	2.30%	24.09	176,056	2.12%	22.61	138,210	1.81%	20.08	105,365	1.51%	16.68	85,247	1.33%
16	1,000	30.73	158,517	1.83%	30.33	135,683	1.64%	29.54	99,403	1.30%	27.54	69,680	1.00%	23.60	52,830	0.82%
17	1,000	40.03	119,477	1.38%	40.82	96,885	1.17%	43.57	62,576	0.82%	47.43	36,150	0.52%	47.80	22,460	0.35%
18	1,000	56.93	81,644	0.94%	62.99	59,598	0.72%	90.12	27,700	0.36%	321.64	4,682	0.07%	-150.05	-6,020	-0.09%
19	1,000	94.23	47,316	0.55%	134.10	26,193	0.32%	-770.17	-2,922	-0.04%	-57.47	-22,584	-0.32%	-24.17	-30,608	-0.48%
20	1,000	260.76	16,120	0.19%	-897.89	-3,596	-0.04%	-67.29	-29,608	-0.39%	-23.74	-46,072	-0.66%	-11.29	-51,786	-0.81%
Portfolio Duration		**9.32**			**8.67**			**7.66**			**6.34**			**5.29**		

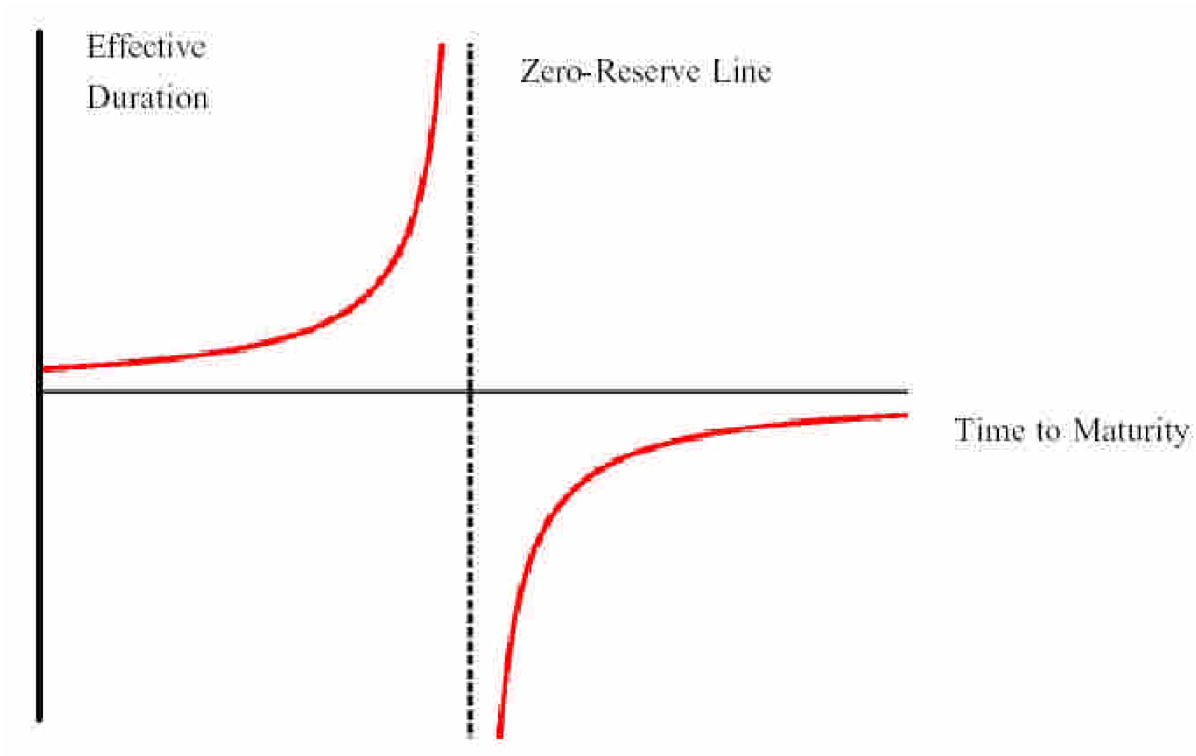

Figure 1: General pattern of the reserve's duration with respect to the policy's time to maturity

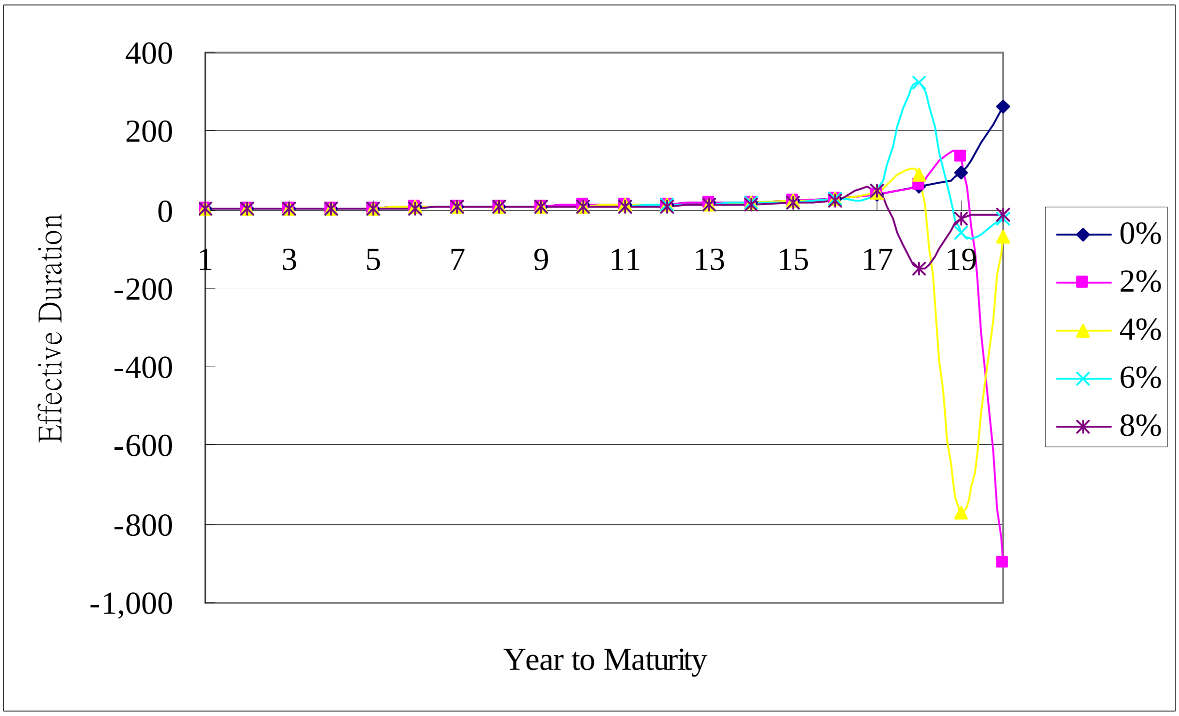

Figure 2: Term structures of effective durations with various initial interest rates

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