Milevsky　Posner (2001)

Follmer　Sondermann (1986)

Abstract

The commencement of financial holding company act has accelerated the integration and innovation for the banking, securities and insurance industries. The creation of equity-linked life insurance policy plays a crucial role in delineating these innovations. Equity-linked life insurance with guarantees integrates the attributes from the mutual fund and life insurance. Since the execution of the implied options depends on policyholder's survival, Milevsky and Posner (2001) defined this kind of innovation as "titanic option". In this study, actuarial equivalent principal and non-arbitrage pricing theory are used in performing the valuation for the unit-linked life insurance policies. Since their values cannot be replicated uniquely through the self-financing strategies due to market incompleteness, the theoretical setup in Follmer and Sondermann (1986) are adopted to

develop the valuation and hedging strategy. Counting process is employed to characterize the underwriting experience of the insurer. Since the payoffs depend on the market values of the underlying portfolios and the health status of the policyholder, mean-variance minimization criterion is employed to evaluate the intrinsic risk. Finally, risk-minimization hedging strategies are examined through several simplified examples. Based on our simulations, we find that the selected asset portfolio, the frequency in hedging and underwriting volume significantly affect the insurer's intrinsic risks.

Keywords: titanic option; valuation; guarantees; underwriting risk; mean-variance minimization criterion.

Milevsky   Posner (2001)

(titanic option)

(death-protected mutual funds)

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1.

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2.

3.

4.

(cash value)

5.

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[1]   136

guarantees

(Brownian motion)W

(Counting process)N

(   )

(sub-account)

(general account)

(

)   ;

(complete market)

(self-financing strategy)

;

;

Thiele   1875

Follmer   and

Sondermann (1986)   Follmer   and

Schweizer (1988)

B-S

;

$(\sqrt{l_x}/l_x)$

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