

行政院國家科學委員會專題研究計畫成果報告

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計畫名稱: 不完全市場下權益連結保險之計價及風險極小化避險策略

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摘要

金融控股公司法的頒佈加速銀行，證券與保險產業的整合與創新，其中以投資連結人壽保險具有顯著的代表性，附保證之投資連結壽險乃結合共同基金與傳統壽險特性之投資型態保險，隱含以死亡時間為條件之選擇權價值，Milevsky 與 Posner (2001)因此定義為鐵達尼選擇權。傳統壽險應用精算等價原理評價，給定利率與死亡率假設進行評價，而附保證之投資連結壽險除包含傳統壽險之利率與核保風險外，因加入指定投資標的之市場風險，避險策略成敗攸關經理人之整體獲利表現。以往研究主要以等價平賭測度理論定價及避險，然就投資連結壽險而言，因引入核保風險，故保險市場不具備完全特性。本研究回顧 Follmer 與 Sondermann (1986)之均變異數避險策略，計算風險測度以評估保險人之內在風險，並舉例分析附有給付限制及連結投資組合之保險，由模擬結果發現保險人可藉由指定投資標的之資產配置、增加避險頻率及擴大承保同質風險之經濟規模以降低自身之內在

財務風險。

關鍵字：鐵達尼選擇權、評價、保證、核保風險、均變異數避險。

Abstract

The commencement of financial holding company act has accelerated the integration and innovation for the banking, securities and insurance industries. The creation of equity-linked life insurance policy plays a crucial role in delineating these innovations. Equity-linked life insurance with guarantees integrates the attributes from the mutual fund and life insurance. Since the execution of the implied options depends on policyholder's survival, Milevsky and Posner (2001) defined this kind of innovation as "titanic option". In this study, actuarial equivalent principal and non-arbitrage pricing theory are used in performing the valuation for the unit-linked life insurance policies. Since their values cannot be replicated uniquely through the self-financing strategies due to market incompleteness, the theoretical setup in Follmer and Sondermann (1986) are adopted to

develop the valuation and hedging strategy. Counting process is employed to characterize the underwriting experience of the insurer. Since the payoffs depend on the market values of the underlying portfolios and the health status of the policyholder, mean-variance minimization criterion is employed to evaluate the intrinsic risk. Finally, risk-minimization hedging strategies are examined through several simplified examples. Based on our simulations, we find that the selected asset portfolio, the frequency in hedging and underwriting volume significantly affect the insurer's intrinsic risks.

Keywords: titanic option; valuation; guarantees; underwriting risk; mean-variance minimization criterion.

一、計畫緣由及目的

順應當前金融環境的劇變與創新，就保險領域而言，為滿足投保人享有投資標的之資本利得及保障生活需求之雙重目標，並規避因通貨膨脹對保險給付的侵蝕，許多新型態的金融保險商品因應而生，其中以整合傳統壽險與共同基金之投資連結壽險最具代表性。投資連結壽險為隱含選擇權之投資型態保險，保險契約之價值與到期死亡給付決定於被保險人指定之投資標的市場價值與附帶之保證條件，因為到期給付介於歐式

與美式選擇權，且與死亡時間有關，因此 Milevsky 與 Posner (2001) 定義此類型商品為鐵達尼選擇權(titanic option)或是提供死亡給付之共同基金 (death-protected mutual funds)，依現行保險法規，僅能由保險公司¹銷售。

保險公司為金融市場中重要的財務仲介機構，從事分散風險與移轉風險的交易行為，其中保險人以投保人繳交之保費進行投資，執行類似基金經理人之信託行為，此時保險人多半提供保證收益及保單分紅，因投保人無法參與投資標的之選擇，僅可獲得有限度之保單紅利，而投資連結壽險除提供投保人投資選擇的權利並附有最低保障外，保險人必須承擔加入投資標的之市場風險，投資連結壽險與一般基金(或證券投資)之差異如下：

1. 依精算等價原理計算死亡風險，提供死亡及附約給付(如醫療住院等)，
2. 資本利得自動轉投資至指定投資帳戶，
3. 保險給付具有稅賦減免及遞延效果，
4. 允許被保險人利用保單現金價值(cash value)貸款，
5. 被保險人於契約期間，可彈性

¹ 依現行保險法第 136 條第二項之規定「非保險業不得兼營保險或類似保險之業務」。

調整投資風險標的與一般帳戶之比例，
投保人享有之給付可分成兩項，第一項為保證給付 (guarantees)，即保險人承諾被保險人符合給付條件之金額，第二項為指定投資標的高於保證給付之價值，可視成執行價為保證給付之買權，依賣權與買權相依理論，保險人利用專設帳戶 (sub-account) 持有指定投資標的，而將執行價為保證給付之賣權置於一般帳戶 (general account)，此時投資連結壽險可視為專設帳戶價值與賣權之組合。

附有最低保證給付的投資型保險商品使得保險公司除面臨核保風險外，也面臨財務風險，依投資型保險商品管理規則第三條第五款規定，若具有最低保證給付，專設帳簿之資產所產生的投資損益風險保險人需部分承擔，也就是保險人必須將投資損失部分認列於一般帳簿之負債項目，如此將損及股東權益，對保險人營運影響甚鉅，所以對於因最低保證給付所產生的保證成本，保險人應予以估算規避。本研究回顧選擇權及精算方法，評估鐵達尼選擇權(即附保證之投資連結壽險)之避險與內在風險，選擇權評價方法基於完全市場 (complete market) 假設，藉由自我籌資策略 (self-financing strategy) 計算請求權價

值，投資連結保險與一般選擇權之差異在於核保風險，請求權之履行與價值決定於投資標的市值與保險人之健康狀態，鐵達尼選擇權因受市場存在資產變化之布朗運動 (Brownian motion) W 與承保狀態之計數過程 (Counting process) N 兩隨機變項(見第三節)之影響，不滿足市場的完全性假設，於是避險策略將無法完全規避風險，本研究比較連結單一與組合型態之投資連結保險，依風險極小化條件比較避險策略與計算不可規避之風險。

全文共分六節，第二節敘述相關研究成果，第三節探討避險理論，市場不完全特性下之避險策略，第四節回顧鐵達尼選擇權之避險策略與內在風險，第五節進行模擬分析與數值計算，第六節結論，歸納所得結果與後續研究。

二、結果與討論

投資連結壽險之給付連結至投資組合，因此增加保險人於評價與避險上之複雜度，本文所探討之風險極小化避險策略建立於利率與股票波動度皆為常數，歸納結果如下，

避險策略: 附有鐵達尼選擇權之生存保險因加入核保風險，使得實際到期履行契約之人數較一般選擇權少，故相對最適避險比率也較低。

指定投資標的物; 由於可運用資產配置降低收益之波動程度，連結單一資產所承受之風險比連結多資產組合為高，保險人可利用標的物之相關性降低風險。

避險頻率; 避險頻率必須增加才能達到風險最小化目標，連續避險時保險人承受內在風險，當保險人改變避險頻率時，財務風險將隨避險次數多寡變動，保險人需視其所能承擔之風險擬定避險頻率。

保單發行量; 隨著投保人數增加，相對風險將乘上調整因子($\sqrt{l_x}/l_x$)予以修正，投保人數愈多則因子愈小，反應大數法則之效果將顯著影響保險人之財務風險。

同時對於長年期之人壽保險契約而言，無風險利率與指定資產隨時間之波動度變化對於風險之衡量具有顯著的影響，保險人發行附有鐵達尼選擇權之投資保險時，應特別加強財務風險的控管，此議題將於後續研究中探討。

三、計畫成果自評

本研究考量到期日及時間因素之傳統型式，歐式權益連結變額保險之計價及避險，於不完全市場假設下，建立準備金隨機微分方程表示式，建立完整理論的推導過程；推廣財務經濟的選擇權理

論至保險精算的傳統型式，歐式權益連結變額保險之計價及避險模型。

研究所得結果，可提供政府監理投資型變額保險的部門、國內學術及研究單位參考，進一步了解投資型保險。

保險精算同時跨理論與應用科學，本研究延續 Thiele 於 1875 所建立之準備金模型，並參考 Follmer and Sondermann (1986) 與 Follmer and Schweizer (1988) 之隨機財務模型的概念，利用 B-S 選擇權計價理論，建立完整的數學理論架構於保險精算及財務領域。

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