行政院國家科學委員會專題研究計畫 成果報告

多階製程下動態 EWMA 管制圖之研究(第 2 年)
研究成果報告(完整版)

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計 畫 主 持 人 ： 楊素芬

報 告 附 件 ： 出席國際會議研究心得報告及發表論文

公 開 資 訊 ： 本計畫涉及專利或其他智慧財產權，2 年後可公開查詢

中 華 民 國 98 年 09 月 23 日

附件一

行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告 / 期中進度報告

多階製程下動態 EWMA 管制圖之研究

計畫類別：■ 個別型計畫　　□ 整合型計畫
計畫編號：　NSC 96-2118-M-004-001-MY2－
執行期間：96　年　8　月　1　日 至　98　年　7　月　31　日

計畫主持人：楊素芬教授
共同主持人：
計畫參與人員：　陳琬昀 余翌寧 林政憲 林亮予　許伊萱　葉金田

成果報告類型(依經費核定清單規定繳交)：□精簡報告　■完整報告

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
■出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢
　　□涉及專利或其他智慧財產權，□一年■二年後可公開查詢

執行單位：

中　華　民　國　98　年　7　月　31　日

I. 中、英文摘要及關鍵詞(keywords)

1. 中文摘要及關鍵詞(keywords)

在兩年期的計畫裡,已經完成所有有關的研究主題. 二篇論文已發表於SCI 期刊,二篇論文已被 SCI 期刊接受, 一篇已投搞, 目前尚正在整理完成之成果及撰寫論文以投稿. 有關的主題是分別考慮過度調整及不過度調整下對相依之製程同時追踪平均值和變異數是否偏移之調適性管制技術. 提出的管制圖包含舒華特型及 EWMA 管制圖. 由數值分析結果發現,這些提出的調適性追踪技術在偵測力上優於固定的追踪技術. 因此這些調適性管制技術被建議使用.

2. 英文摘要及關鍵詞(keywords)

In the two years' project, I have finished all related topics. Two papers have published in journals, two are accepted, one is under-reviewing and some are going to write. All the papers propose the adaptive scheme to monitor the shifts in means and variances in the dependent process steps including the situations of correct adjustments and the incorrect adjustment situations. A single control chart to monitor mean and variance on a single process are also proposed. The control charts including the Shewhart-type charts and EWMA charts.

The detection ability of the proposed adaptive monitoring schemes performs the fixed monitoring schemes based on the results of some data analyses. Hence, the adaptive control charts are recommended.

Keywords: Control charts; adaptive control scheme; Dependent process steps; Incorrect adjustment; Markov chain

II. 報告內容：

Some papers of the project have been published or accepted in journals. Some reports are writing, some results will be summarized. Those will be submitted to SCI journals in the near future.

Journals:

1.Yang, S. and Yu,Y. (2009),"Monitoring cascade processes using VSI EWMA control charts," Journal of Chemometrics, accepted.

2.Yang, S and Yu,Y. (2009),"Using VSI EWMA charts to monitor dependent process steps with incorrect adjustment," Expert Systems with Applications 36 (2009) 442–454.

3.Yang , S. and Chen, W. (2009),” Controlling over-adjusted process means and variances using VSI cause selecting control charts,” Expert Systems with Applications 36 (2009) 7170–7182.
4.Yang, S. (2009),”Variable control scheme in the cascade processes”. Expert Systems with Applications. Accepted.
5.Yang , S. and Chen, W. (2009),”On-Line Monitoring Using VSI Cause Selecting Control Charts”, Submit to Journal of Statistical Planning and Inference.

Conference

1. Yang, S and Lin, J. (2009), **“***Variable Parameters Loss Function Control Chart*,” SSS2009, Cape town, South Africa.
2. Yang, S., Chen, Y and Yang, C., 2009,” Controlling Means and Variances Using VSSI Cause Selecting Control Charts,” ISI, Durban, South Africa.
3. Yang, C. and Yang, S., 2009,” *Adaptive Control Scheme for a Process with Incorrect Adjustment*”,1st Proceeding of Asian Conference on Intelligent Information and Database Systems, 404-409. Published by IEEE Computer Society. (EI)
4. Yang, S and Yu, Y., 2008, “*Monitoring Process Steps Using VSSI EWMA Control Charts*, “ International conference on Engineering Optimization, Rio, Brasil. ISBN 978-85-7650-152-7.
5. Yang, S and Yu, Y., 2008, “*Monitoring Cascade Process Using VSI EWMA Control Charts*, “ISBIS 7, Prague, Czech.
6. Yang, S. and Yeh, J.,2008,” *Using Cause Selecting Control Charts to Monitor Dependent Process Stages* with Attribute Data,” ISBIS 7, Prague, Czech.
7. Yang, S. and Chen, Y., 2008,” Design of VSI Cause Selecting Control Charts for Monitoring process m and Variances,” ISBIS 7, Prague, Czech.
8. Yang, S. and Chen, W. ,2007, “*Controlling Incorrect Adjustment Processes Using Optimum VSI Control Charts*”, ISI 56, Lisbon, Portugal.
9. Yang, S. ,2007, “*Controlling a Process with Incorrect Adjustment Using VSI Control Charts*”, ISBIS 6, Azores, Portugal.
10. Yang, S. and Chen, W. ,2007, “*Variable sampling interval control charts,*” International Conference Multiple Decision Theory , Taiwan. (in honor of Prof. Der-Yun Hwang)
11. Yang, S., 2007, “V*ariable Control Scheme for a Process with Incorrect Adjustment*”, Association of Chinese probability and Statistics, NCCU, Taiwan.

III. **計畫成果自評部份**，請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等.

自評部份:研究內容與原計畫內容相符, 目前發表於期刊之論文篇數已超過預期. 現在尚有文章在整理及撰寫中. 未來, 預期尚會有論文被期刊接受.

參加第一屆亞洲智慧資訊和資料庫系統研討會(ACIIDS)報告

楊素芬教授

政治大學統計系

一、內容摘要

第一屆亞洲智慧資訊和資料庫系統研討會(ACIIDS)在越南中部洞海的廣明大學舉行，開會時間是 2009 年 4 月 1 日至 4 月 3 日為期 3 天。由於研討會的論文屬於 EI 而且論文需經過評審後錄取率只有 30%，所以去年底我很好奇的投稿一篇論文，結果大會通知錄取，所以就準備前往未曾到過的越南看看。

3/30 日，我搭長榮的飛機由桃園先到越南首都河內(Honoi)，等到晚上再搭越南航空到 Hue(順化)。抵達順化已近 9 點，大會派車接我們到洞海，路途遙遠，抵達旅館已近凌晨 1 點，越南交通不方便，對旅客來說真是辛苦。

4/1 日，到廣明大學(Quang Binh University)報到並領取開會資料,之後即為 keynote speech，主要說明如何應月 IT 在綠色環保上。環保議題是近年地球暖化後的重要議題。

這次會議報名的有 100 人,但實際參加之學者約 60 人,來自十幾個國家，其中台灣來的有 15 人左右，是陣容最大的。參加此會議後，才知道此會議偏重在資訊 IT 的應用，而與會者也大多是資訊相關領域的，對工業統計專長的我，好像不太適合。

二、研討會議程

4/1 日下午，大會安排我擔任[資訊系統和資料庫管理系統]場次之主持人，3 位演講者只出席 2 位，第一位是來自何志明科大的 DR. Anh，介紹如何以 TA 方法自數據中萃取有意義的訊息，以支持決策應用，並將此方法實際應用於分析皮膚癌治療的病人資料庫。和其他方法比較起來，此方法效果不錯。

第二位是來自南韓的 Dr. Kim 報告如何建立環境追踪系統，以使人們生活於安全環境下以得有高品質的生命。在許多研究團隊下，大多選擇網路為研究環境追踪系統之工具。數據管理是使用網路相關方法的最挑戰議題。作者探討如何以數據管理系統追踪每天空氣污染變化情形。

我發現他們探討的問題一樣是可以以統計方法解決的。倘若可以進一步比較統計方法和資訊方法之績效，則會更有趣。但實際上，要資訊系或統計系的老師一起合作研究，其實也是不容易的。

晚上廣明大學動員所有的師生準備一頓越南傳統晚宴和歌舞表演，表現出熱忱的歡迎。

4/2 選擇聽模糊控制與分析場次。來自台灣的林老師對非線性系統問題，提出以基因演算法先決定參數的初始值，再根據 Lyapunov 穩定理論訂定適應方法來控制非線性系統，接著再以界限函數保證誤差在設定的範圍內，最後一個適應性的神經網路控制器(ANNC)被推導來同時穩定和控制系統。

接著仍由林老師報告時間延誤模糊系統的穩定分析方法。他們用模糊 Lyapunov 方法提出 H 無限控制設計，並以模糊二次函數表示。時間延誤的穩健和穩定標準可以模糊 Lyapunov 方法保證干擾下的時間延誤的模糊系統之穩定性。

這些二次函數干擾及穩健性的問題對設計人也常發生，模糊(Fuzzy)和 GA 基因演算法應該也可以做為統計人的分析工具。

下午，5:20 分的「人工智慧和系統科學」場次也有 2 人出席，我的演講被排在這場上。我的報告是流程不正確調整的適應性控制技術。主要提出有效偵測流程穩定的方法以維持品質之穩定。由於是品管上的專業，故無人發問。其實我覺得這個問題可以考慮用基金演算法(GA)和模糊方法(Fuzzy)解決，只是我對 GA 和 Fuzzy 並不十分了解。倘若他們(資訊領域者)有興趣倒是可以一起合作，好像也很有趣。

晚上，大會又招待我們晚宴，這次是在飯店舉行，不像星期三的晚上是在學校禮堂的歡迎宴。10 人一圓桌，吃一道道的菜，類似中國人的餐宴。而食物內容和我們臺灣人吃的也沒什麼大差異，只是不像台灣那麼講究和精緻。

4/3 會議最後的半天，我聽了資料採礦的應用場次。來自波蘭的學者對在實驗室收集的 DNA 數據以資料採礦的方法對數據做分類預測，基因選擇和型態的辨識。接著來自台灣瀧華科大的吳老師也提出以資料採礦方法對問題做

分類。由於神經網路方法在分類上有使用之限制，如訓練時間慢，複雜的解釋和不易以最適網路表示。但是以 CMACNN 則可克服這些問題。數據分析顯示 CMACNN 分類法對試驗的數據是有效的，因此 CMACNN 分類法可以被考慮為資料採礦的分類工具。

二天半的會議到此結束，大會主辦人 Dr. Nguyen 為感恩廣明大學的栽培而有機會以公費至波蘭大學念博士，特選擇廣明大學舉辦這第一次的國際會議。「飲水思源」Dr. Nguyen 是有情有義的學者。

三、結論及建議

與會人士中，越南人佔一半以上，其次以台灣最多。這個資訊會議明年將移駕至越南順化大學舉辦。Dr. Nguyen 回饋祖國的心令人敬佩。希望台灣也多一些這樣的學者，定可大大提升台灣在國際的能見度。

越南早期是中國管轄，獨立後因民主檔和共產主義長期戰爭而國力衰退，經濟成長落後而生活也貧困。又由於知識普遍低落，交通亂且人民品質也低，相當於台灣的五十和六十年代。然而這幾年越南政府引進外資，經濟已明顯比過去好。這幾年國際研討會在越南舉辦的次數不少，國際能見度不輸台灣。由舉辦單位的執行力來看，他們很努力在做事，由發表的論文來看他們也很上進。目前他們也努力廣泛培育博士生。看來似乎是具潛力的國家。另外，在觀光據點外國人遊客也不少，看得出政府有在為國家經濟打拼。反觀我們，一切似乎都在原地踏步。政府和百姓都應該在經濟和新知發展上更加努力。

此次會議攜回大會手冊及論文集。

Adaptive Control Scheme for a Process with Incorrect Adjustment

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Abstract

This article considers an adaptive process control scheme for a process with incorrect adjustment. Incorrect adjustment of a process may result in shifts in both the process mean and variance, ultimately affecting the quality of products. We construct control charts with adaptive sampling intervals (ASI) to control the quality variable produced by the process with incorrect adjustment. The performance of the proposed control charts is measured by the adjusted average time to signal derived by a Markov chain approach. An example of process control for automobile braking system shows the application and the performance of the proposed control charts in detecting small and medium shifts in mean and variance. Furthermore, some numerical analysis results show that the performance of the ASI control charts is much better than the FSI control charts in detecting small and medium shifts in mean and variance.

1. Introduction

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control. A common problem in statistical process control is unnecessary adjustment of a process (or over-adjustment of a process) (see Deming, 1982) due to incorrect use of control charts by the operator or since the only information about the state of the process is available through sampling. A process requires adjustment, when a control chart indicates that it is out of control. However, the process may be adjusted unnecessarily, when a false alarm occurs. Woodall (1986) noted that the effect of incorrect adjustment is a quite significant increase in variability of the quality characteristic. It is thus important to effectively control the significant increase in variability of a process with incorrect adjustment. Collani and Saniga (1994) proposed an economic adjustment model for the $\overline{X}$ control chart with a single special cause that considers the effects of a process with incorrect adjustment. Their model determines the optimal design parameters of the $\overline{X}$ control chart which maximize the profitability of the process with incorrect adjustment. Yang and Yang (2004) derived economic adjustment model for the $\overline{X}$ control chart with multiple special causes.

However, the above papers, even Shewhart $\overline{X}$ control charts, always monitor a process by taking equal samples of size at a fixed sampling interval (FSI), they are usually slow in signaling small to medium shifts in the process mean. Consequently, several alternatives have been developed to improve the performance of $\overline{X}$ control charts in recent years. One of the useful approaches to improve the detecting ability is to use a control chart with ASIs (Adaptive Sampling Intervals) instead of the traditional FSI. Whenever there is some indication that a process parameter may have changed, the next sampling interval should be shorter. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer.

The properties of the $\overline{X}$ chart with adaptive sampling intervals were studied by Reynolds et al. (1988). Their paper has been extended by several others: Reynolds and Arnold (1989); Runger and Pignatiello (1991); Saccucci, Amin, and Lucas (1992); Runger and Montgomery (1993); Amin and Miller (1993); Baxley (1996); Reynolds, and Arnold, and Baik (1996). Tagaras (1998) reviewed the literature on adaptive control charts. Very little work has been down on ASI control charts for monitoring process mean and variance simultaneously. Chengular, Arnold and Reynolds (1989) detected process mean and variance using ASI $\overline{X}$ and R control charts. Reynolds and Stoumbos (2001) discussed the properties of ASI X and MR control charts. These papers show that most work on developing ASI control charts has been down for the problem of monitoring process mean without considering the effects of a process with incorrect adjustment. The ASI control charts used to control the mean and variance of a process with incorrect adjustment has not yet been addressed. Therefore, to study the performance of the ASI control charts on the whole process with incorrect adjustment has arisen. In this paper, the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts are proposed to control the whole process with incorrect adjustment. In next section, the performance of the proposed $Z_{\overline{X}}$ and Z_{S^2} control charts is measured by the Adjusted Average Time to Signal (AATS), which is derived by a Markov chain approach. Finally, we illustrate the application of the proposed control charts using a real example and compare the performance between the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts and FSI $Z_{\overline{X}}$ and Z_{S^2} control charts.

2. Description of the ASI $Z_{\overline{X}}$ and Z_{S^2} Charts for Controlling a Process with Incorrect Adjustment

Consider a process controlled by the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts. Let X be the measurable quality variable on the process. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X.

In our study rational samples of size (n) are taken from the process; the standardized sample mean and variance, $Z_{\overline{X}}$ and Z_{S^2} are

$$Z_{\overline{X}_i} = \frac{\overline{X}_i - \mu_X}{\sigma_X/\sqrt{n}} \sim N(0,1) \quad \text{and}$$

$$Z_{s_i^2} = \frac{(n\text{-}1)S_i^2}{\sigma_x^2} \sim \chi^2(\text{n - 1}) \qquad \text{(2-1)}$$

where $\overline{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}$ and

$$S_i^2 = \frac{\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2}{n\text{-}1}, \quad i = 1,2,3,....$$

Also assume that a special cause or incorrect adjustment may occur in the in-control process. The process is subject to the special cause or incorrect adjustment such that the mean of X shifts from μ_X to $\mu_X + \delta_1\sigma_X$ ($\delta_1 \neq 0$) and the variance shifts from σ_X to $\delta_2\sigma_X$ ($\delta_2 > 1$). The out-of-control distribution of X will be corrected to in-control state, once at least one true signal is obtained from the proposed control charts. Let T_{sc} be the time until the occurrence of a special cause, and follow an exponential distribution of the form

$$f(t) = \lambda \exp(-\lambda t) \quad t > 0, \qquad \text{(2-2)}$$

where $1/\lambda$ is the mean time that the process remains in a state of statistical control.

An in-control state analysis for the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts is performed since the shifts in the process mean and variance do not occur when the process is just starting, but occur at some time in the future. The samples $z_{\overline{X}_i}$ and z_{S^2} are plotted on the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts with warning limits of the form $\pm w_1$ and w_2 and control limits of the form $\pm k_1$ and k_2, respectively, where $0 < w_1 < k_1$ and $0 < w_2 < k_2$.

The search for the special cause and adjustment in the process is undertaken when the sample $z_{\overline{X}_i}$ falls outside the interval ($-k_1, k_1$) and/or when the sample z_{S^2} falls outside the interval (0, k_2), that is when the $Z_{\overline{X}}$ and/or Z_{S^2} charts produce a signal. For a discontinuous process, the process is stopped to search for and eliminate the special cause and correction after a signal is obtained from a control chart. The process adjustment may be incorrect when the signal is false, but the adjustment may be corrected when the signal is true and then the process is brought back to an in-control state.

The positions of the current samples in the joint $Z_{\overline{X}}$ and/or Z_{S^2} charts define the sampling interval of the next sample.

We divide the proposed ASI $Z_{\overline{X}}$ and/or Z_{S^2} control charts into the following three regions (2-3), respectively.

$$I_{Z_{\bar{X}1}} = [-w_1, w_1] \quad \text{(central region)}$$
$$I_{Z_{\bar{X}2}} = (\text{-}k_1, \text{-}w_1) \cup (w_1, k_1) \quad \text{(warning region)}$$
$$I_{Z_{\bar{X}3}} = [-k_1, k_1] \quad \text{(control region)}$$
$$I_{Z_{s_{\bar{x}}^2 1}} = (0, w_2) \quad \text{(central region)} \qquad \text{(2-3)}$$
$$I_{Z_{s_{\bar{x}}^2 2}} = (w_2, k_2) \quad \text{(warning region)}$$
$$I_{Z_{s_{\bar{x}}^2 3}} = (0, k_2) \quad \text{(control region)}$$

Three ASIs are adopted, $0 < t_1 < t_2 < t_3 < \infty$. If both the samples, $z_{\overline{X}_i}$ and z_{S^2}, fall within the central regions, $I_{\overline{X}1}$ and $I_{Z_{s_{\bar{x}}^2 1}}$, then the next sampling interval should be long (t_3). If any one of the samples fall within the central region but the other falls within the warning region, then the next sampling interval should be medium (t_2). If both the samples fall within the warning regions, then the next sampling interval should be short (t_1).

The relationship between the next sampling interval $(t_m, m = 1,2,3)$ and the position of the current samples is expressed as equation (2-4).

$$t_m = \begin{cases} t_3 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}1}} \\ t_2 & \text{if } (Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}1}}) \text{or} \\ & \quad (Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}2}}) \\ t_1 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}2}} \end{cases} \qquad (2\text{-}4)$$

Assume that the first sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sampling intervals, including the first one, should have a probability of p_{01} of being long, a probability of p_{02} of being medium, a probability of p_{03} of being short, where $\sum_{i=1}^{3} p_{0i} = 1$, p_{01}, p_{02} and p_{03} are given by

$$p_{01} = \mathrm{P}(Z_{\overline{X}} \in I_{Z_{\overline{X}1}},\ Z_{S^2} \in I_{Z_{S^2 1}} \left| \overline{X} \sim N(\mu, \frac{\sigma^2}{n}), |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2 \right.)$$

$$= \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}$$

$$p_{02} = \mathrm{P}(Z_{\overline{X}} \in I_{Z_{\overline{X}1}},\ Z_{S^2} \in I_{Z_{S^2 2}} \left| \overline{X} \sim N(\mu, \frac{\sigma^2}{n}), |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2 \right.) +$$

$$\mathrm{P}(Z_{\overline{X}} \in I_{Z_{\overline{X}2}},\ Z_{S^2} \in I_{Z_{S^2 1}} \left| \overline{X} \sim N(\mu, \frac{\sigma^2}{n}), |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2 \right.)$$

$$= \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]} + \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}$$

$$p_{03} = \mathrm{P}(Z_{\overline{X}} \in I_{Z_{\overline{X}2}},\ Z_{S^2} \in I_{Z_{S^2 2}} \left| \overline{X} \sim N(\mu, \frac{\sigma^2}{n}), |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2 \right.)$$

$$= 1 - p_{01} - p_{02} = \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}.$$

$\Phi(\cdot)$ and $F_{\chi^2}(\cdot)$ are the cumulative probabilities of the standard normal and $\chi^2_{(n\text{-}1)}$ distributions, respectively.

If both w_1=0, w_2 =0 and $t_1 = t_2 = t_3 = t_0$, then the ASI $Z_{\overline{X}}$ and/or Z_{S^2} charts reduce to $Z_{\overline{X}}$ and/or Z_{S^2} charts with FSI t_0 .

3. Control Charts Comparison

Sampling schemes should be compared under equal conditions; that is, ASI and FSI schemes should demand the same average sampling interval under the in-control period. That is,

$E\left[t_m \middle| \delta_1 = 0, \delta_2 = 1, |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2\right] = t_0$, m=1, 2, 3.

or $p_{01}t_3 + p_{02}t_2 + p_{03}t_1 = t_0$ (3-1)

Based on the equation (3-1), the warning limit w_1 of the ASI $Z_{\overline{X}}$ control chart is derived as follows.

$$w_1 = \Phi^{-1}\left\{\frac{F_{\chi^2}(w_2)[t_3\text{-}t_2\text{-}2\Phi(k_1)(t_2\text{-}t_1)] + F_{\chi^2}(k_2)[t_2\text{-}t_0 + 2\Phi(k_1)(t_0\text{-}t_1)]}{2[F_{\chi^2}(w_2)](t_1\text{-}2t_2 + t_3) + 2[F_{\chi^2}(k_2)](t_2\text{-}t_1)}\right\} \qquad (3\text{-}2)$$

However, to obtain w_1 and let $0 < w_1 < k_1$, the constraints $0 < w_2 < k_2$ and $0 < t_1 < t_2 < t_3 < \infty$ are required. Thus, the warning limit, w_1 , can be obtained by using equation (3-2) and choosing a combination of the ASIs, (t_1 , t_2 , t_3), the fixed sampling interval, t_0 , and the constraint $0 < w_2 < k_2$.

In this paper, the ASI scheme is compared with the FSI scheme and sampling scheme was considered to be better than another when it allowed the $Z_{\overline{X}}$ and Z_{S^2} charts to detect changes in the process mean and variance faster.

4. Measurement of Performance

The speed with which a control chart detects process shifts measures its statistical efficiency. For a ASI, the detection speed is measured by the average time from either mean or variance or both shifting until either $Z_{\overline{X}}$ or Z_{S^2} chart or both signals, which is known as the Adjusted Average Time to Signal (AATS). That is, the AATS is the mean time that the process remains out of control without signaling.

Since $T_{SC} \sim \exp(-\lambda t)$, $t > 0$, the occurrence time until the special cause occurs.

Hence,

$$AATS = ATC - \frac{1}{\lambda} \qquad (4\text{-}1)$$

The average time of the cycle (ATC) is the average time from the start of a process until a true signal obtained from one of the proposed charts (see Duncan (1956)). The ATC is the sum of the average in-control time and average out-of-control time (AATS). The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 14 states is assigned based on whether the process step is in or out of control and the position of samples. The status of the process when the $(i+1)^{th}$ sample is taken, and the position of the i^{th} sample on the joint $Z_{\overline{X}}$ and Z_{S^2} charts define the transition states of the Markov chain. The joint ASI $Z_{\overline{X}}$ and Z_{S^2} charts produce a signal when at least one of the samples falls outside the control limits. If the current state is any one of the States 1, 2, 4, 5, 10~13, then there is no signal. If the current state is State 3, it indicates one false signal comes from the Z_{S^2} chart then the process is adjusted unnecessarily and State 3 instantly becomes any one of the States 10~13 with probability $P_{3=j}$, and $\sum_{j=10}^{13} P_{3=j} = 1$, j = 10 ~ 13. Any one of the States 10~13 thus transits to any other state after a sampling time interval. States 6, 7, 8 and 9 are same to State 3. If the current state is any one of the States 1~13, then it may transit to any other state, hence States 1~13 are transient states. The absorbing state (State 14) is reached when at least one true signal occurs.

Let $\mathbf{P}$ be the transition probability matrix, and $\mathbf{P}$ is a square matrix of order 14. Let $P_{i,j}(t_m)$ to be the transition probability from prior state i to the current state j with sampling interval t_m, where t_m is determined by the prior state i, $i = 1,2,...,14$, $j = 1,2,...,14$, $m = 1,2,3$. The transition probability, for example, from state 1 to state 4 with sampling interval t_3 and fixed sample size n is calculated as

$$P_{1,4}(t_3) = P(T_{sc} > t_3)P(Z_{\overline{X}} \in I_{Z_{\overline{X}2}}, Z_{S^2} \in I_{Z_{S^2 1}} | \delta_1 = 0, \delta_2 = 1)$$

$$= (e^{-\lambda t_3})[2\Phi(k_1) - 2\Phi(w_1)][F_{\chi^2}(w_2)]$$

The calculation of all transition probabilities can thus be obtained

From the elementary properties of Markov chains (see, Cinlar (1975)) and following Yang (1993), the ATC is derived as

$$ATC = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}t + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}M_f + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(A)T_r \quad (4\text{-}2)$$

where $b' = (p_{01}, \frac{p_{02}}{2}, 0, \frac{p_{02}}{2}, p_{03}, 0, 0, 0, 0, 0, 0, 0, 0)$ is a (1×13) is the vector of starting probabilities for state 1, 2, 3,…,13, and the first sampling interval has probability p_{01} of being long (state 1 with probability p_{01}) , the probability p_{02} of being medium (state 2 or stat 4 with probability $\frac{p_{02}}{2}$), and the probability p_{03} of being short (state 5 with probability p_{03}); $\mathbf{I}$ is the identity matrix of order 13; $\mathbf{Q}$ is the transition probability matrix where each element represents the transition probability, $P_{i,j}(t_m)$, from transient state i to transient state j with sampling interval t_m, where i=1,…,13, j=1,…,13, m=1, 2, 3; $\mathbf{t}' = (t_3, t_2, t^*, t_2, t_1, t^*, t^*, t^*, t^*, t_3, t_2, t_2, t_1)$ is the vector of the sampling intervals for State 1~State 13, t^* is the average sampling interval for State 3, 6, 7, 8, and 9; and $M_f' = (0 \quad 0 \quad T_{ad} \quad 0 \quad 0 \quad T_{ad} \quad T_{ad} \quad T_{ad} \quad T_{ad} \quad 0 \quad 0 \quad 0 \quad 0)$ is the vector of the time of incorrect adjustment for States 1, 2, … and 13, T_{ad} is the time interval for making an incorrect adjustment in the process for States 3, 6, 7, 8 and 9. A is the vector of transition probability, $P_{i,j}(t_m)$, form transient state i, i=1,…,13, to an absorbing state j, j=14; T_r is the time interval for making a correction in the process.

5. An Example

An example of process control for automobile braking system is presented, and the data of the process are measurements of bake weight. Let X = bake weight be measured from the end of the second process step. One machine could fail in the process, and shift the mean and variance on bake weight. Presently, the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts are used to monitor the process per hour (t_0 =1h). Information about the state of the process is available only through sampling. When the proposed control charts indicate that the process is out of control, it requires correction. Sometimes, the process may be corrected unnecessarily when at least one false signal occurs. To construct the control charts, thirty samples of size five are collected from historical data under the stable process. The estimated mean and standard deviation of X are ($\hat{\mu}$=210.1, $\hat{\sigma}$ =1.23).

One machine could fail in the process. From historical data, the estimated failure frequency for the machine is 2 times per hour. The failure machine or the incorrect adjustment of a process would shift the mean and variance of X to ($\hat{\mu} + \delta_1\hat{\sigma}, \delta_2^2\hat{\sigma}^2$), where $\delta_1 = 0.5$ and $\delta_2 = 2.0$ are estimated from previous out-of-control process. Hence, for out-of-control process, $\overline{X} \sim N(210.1 + 1.23 \bullet 0.5,\ 4\left(\frac{1.23}{\sqrt{5}}\right)^2)$. The FSI $Z_{\overline{X}}$ and Z_{S^2} charts have control limits placed at ± 3 and 16.25, respectively. Thus, approximately 5.4 false alarms are expected per 1,000 samples have in-control average run length (ARL) of 185 hours and AATS=19.482 hours calculated from equation (4-1) and (4-2) by letting $t_1 = t_2 = t_3 = t_0$ =1. The slowness with which the FSI $Z_{\overline{X}}$ and Z_{S^2} charts detect shifts in the process (δ_1=0.5 and δ_2 =2.0) has led the quality manager to propose building the $Z_{\overline{X}}$ and Z_{S^2} charts with ASIs, (t_1 , t_2 , t_3). The construction and application of the proposed ASI $Z_{\overline{X}}$ and Z_{S^2} charts is illustrated. The following are the guidelines for using the proposed control charts:

Step 1. Let the control limits $k_1 = 3$ and $k_2 = 16.25$, to maintain the average false alarm rate at around 2.7 per 1000 samples for $Z_{\overline{X}}$ and Z_{S^2} charts, respectively. The reciprocal of 2.7 false alarms is also the ARL, but for the in-control case of $\delta_1 = 0, \delta_2 = 1$.

Step 2. The incorrect adjustment time of the process is 0.5 hours ($T_{ad} = 0.5$).

Step 3. The constraint $0 < t_1 < t_2 < t_0 = 1 < t_3 < \infty$ is required for obtaining a better performance, but the combination (n =5, t_1 =0.1 hours, t_2 =0.5 hours, t_3 =2) is adopted by engineers.

Step 4. Letting t_1 =0.1, t_2 =0.5, t_3 =2, n =5, $T_{ad} = 0.5$, $k_1 = 3$, $k_2 = 16.25$, $\lambda = 2$ and $0 < w_2 < 16.25$ in the equation (3-2) leads to $w_2 = 8.500$ and w_1 =0.483 with calculated AATS=3.450 using equations (4-1) and (4-2).

Consequently, the structures of the proposed ASI $Z_{\overline{X}}$ and Z_{S^2} charts are as follows.

$$\mathrm{UCL}_{z_{\overline{X}}} = 3$$
$$UWL_{z_{\overline{X}}} = 0.483$$
$$\mathrm{CL}_{z_{\overline{X}}} = 0$$
$$\mathrm{LWL}_{z_{\overline{X}}} = -0.483$$
$$\mathrm{LCL}_{z_{\overline{X}}} = -3$$
$$\mathrm{UCL}_{z_{S^2}} = 16.25$$
$$UWL_{z_{S^2}} = 8.5$$
$$\mathrm{LCL}_{z_{S^2}} = 0$$

With the design parameters determined, the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts can be used for controlling the bake weight of the automobile braking system with incorrect adjustment. According to the ASI scheme, if all samples, ($z_{\overline{X}}, z_{s^2}$), fall within warning limits, then the long sampling interval t_3 =2.0h should be taken. If one of the samples fall within warning limits but another falls between warning and control limits, then a sampling interval t_2 =0.5h should be taken. If both the samples fall between warning and control limits, then a sampling interval t_1 =0.1h should be taken. If at least one signal is obtained from the control charts, then the process is stopped and corrected.

An example using the ASI is introduced. When the process starts, a random procedure decides the first sampling interval t_3 =2.0h with sample of size five. The first sample mean and variance are ($\bar{x}_1 = 210$, $s_1^2 = 0.625$) and the standardized values are ($z_{\overline{X}}$ =0.182, z_{S^2} = 1.652). Since both the samples fall within warning limits, the second samples will be observed adopting a sample of size five after t_3 =2.0h. The second sample, ($z_{\overline{X}}, z_{S^2}$), is (1.64, 10.576). Since both samples fall within warning and control limits, the third sample will be observed after t_1=0.1h. The process is stopped and corrected when one signal is obtained.

The AATS is used to measure the performance of the proposed ASI $Z_{\overline{X}}$ and Z_{S^2} control charts. The proposed Markov chain approach is used to obtain the ATC and calculate the AATS. There are 14 possible process states, as presented in Section 4. Hence, the AATS is 3.450h according to equations (4-1) and (4-2).

The ASI scheme improves the sensitivity of the FSI $Z_{\overline{X}}$ and Z_{S^2} charts. From the example, in order to detect a shift in the mean and variance of the process, the AATS for the ASI $Z_{\overline{X}}$ and Z_{S^2} charts has been reduced from 19.482 hours to only 3.450 hours, and the saving rate of detecting time is 82.291%.

6. Performance Comparison Between ASI and FSI Schemes

Twelve AATSs of ASI and FSI schemes, which are obtained under twelve combinations of parameters based on orthogonal array $L_{12}(2^{11})$ table with $\lambda = 2.0, 4.0$, $\delta_1 = 0.5, 1.5$, $\delta_2 = 1.0, 2.0$, $t_1 = 0.01, 0.1$, $t_2 = 0.1, 0.5$, $t_3 = 2.0, 4.0$, $t_0 = 1.0$, Tad=0.5,1.0, $k_1 = 3$, $k_2 = 16.25$, and $n = 5$.

The results show that the performance of the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts is much better for detecting small and medium shifts ($0.5 \le \delta_1, \delta_2 \le 2.0$) in process mean and variance. The ASI $Z_{\overline{X}}$ and Z_{S^2} control charts save detection time from 20.336% to 82.291% compared to the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts. To examine the effects of various parameters on the ASI AATS, the main effects plots show the significant parameters are t_1, t_2, δ_1 and δ_2. The proposed ASI control scheme is thus recommended.

7. Conclusions

The proposed ASI scheme controlling a process with incorrect adjustment substantially improves the performance of the FSI scheme by increasing the speed with which small and medium shifts in the mean and variance of a process are detected. We have found that the ASI $Z_{\overline{X}}$ and Z_{S^2} control charts always work better (in the cases examined) than the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts for small and medium δ_1 and δ_2 values.

This paper considers a process with incorrect adjustment. However, a study of the adaptive sample size (ASS), adaptive sample size and sampling interval (ASSI) or adaptive parameters (AP) $Z_{\overline{X}}$ and Z_{S^2} control charts under a process with incorrect adjustment is an interesting topic for future research. Other important extensions of the proposed model can also be developed. The extension of the proposed model to study AP control charts on multiple process steps or other control charts, such as attributes charts, CUSUM charts or multivariate cases, is straightforward.

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