行政院國家科學委員會專題研究計畫 成果報告

相依製程上平均值和變異數過度調整之適應性管制(2/2)
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行政院國家科學委員會補助專題研究計畫 □成 果 報 告 ■期中進度報告

相依製程上平均值和變異數過度調整之適應性管制

計畫類別：■ 個別型計畫 □ 整合型計畫
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計畫主持人：楊素芬教授
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計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：□精簡報告 ■完整報告

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中 華 民 國 97 年 3 月 6 日

I have finished about 6papers written for the 2 years project. It is impossible to put all the papers in the file., so just put 3 papers in.

USING VSI EWMA CHARTS TO MONITOR DEPENDENT PROCESS STEPS WITH INCORRECT ADJUSTMENT

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ABSTRACT

The article considers the variable process control scheme for two dependent process steps with incorrect adjustment. Incorrect adjustment of a process may result in shifts in process mean, ultimately affecting the quality of products. We construct variable sampling interval (VSI) $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts to effectively monitor the quality variable produced by the first process step with incorrect adjustment and the quality variable produced by the second process step with incorrect adjustment. The performance of the proposed VSI control charts is measured by the adjusted average time to signal (AATS) derived using a Markov chain approach. An example of the automobile braking system with incorrect adjustment shows the application and performance of the proposed VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts in detecting shifts in process mean. Furthermore, the performance of the proposed VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts and the fixed sampling interval (FSI) $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts are compared by numerical analysis results. These demonstrate that the former is much faster in detecting small and median shifts in mean. The optimum VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts are also proposed using optimization technique when quality engineers cannot specify the values of variable sampling intervals. It has been found that the optimum VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts always work better than the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts.

Key words: Control charts; dependent process steps; incorrect adjustment; Markov chain.

1. INTRODUCTION

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control. A common problem in statistical process control is process adjusted unnecessarily (or overadjustment of a process) (see Deming, 1982) due to incorrect use of control chart by the operator or since the only information about the state of the process is available through sampling. A process requires adjustment, when a control chart indicates that it is out of control. However, the process may be adjusted unnecessarily, when a false alarm occurs. Woodall (1986) noted that the effect of overadjustment is a quite significant increase in variability of the quality characteristic. Collani and Saniga (1994) proposed an adjustment model for the $\overline{X}$ control chart with a single special cause that considers the effects of process with incorrect adjustment from economic viewpoint. Their model determines the optimal design parameters of the $\overline{X}$ control chart which maximize the profitability of the process. Yang and Yang (2004) addressed the economic adjustment model for $\overline{X}$ control chart with multiple special causes.

However, the above papers, even Shewhart $\overline{X}$ control charts, always monitor a process by taking equal samples of size at a fixed sampling interval (FSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, several alternatives have been developed to improve the performance of $\overline{X}$ control charts in recent years. One of the useful approaches to improve the detecting ability is to use a variable sampling interval (VSI) control chart instead of the traditional FSI. Whenever there is some indication that a process parameter may have changed, the next sampling interval should be shorter. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer. There have been several alternatives developed to improve this problem in recent years (see Tagaras (1998)).

The exponential weighted moving average (EWMA) control chart is a very effective alternative to the Shewhart control chart when small process shifts are of interest. The properties of EWMA charts with variable sampling intervals were studied by Amin and Letsinger (1991), Saccucci, Amin, and Lucas (1992), Saccucci, Amin, and Lucas (1992), Reynolds (1995), Reynolds (1996a and 1996b) and Capizzi and Masarotto (2003). Tagaras (1998) reviewed the literature on adaptive control charts. These papers show that most work on developing VSI control charts has been down for the problem of monitoring process mean without considering the effects of incorrect adjustment of process.

However, these articles assume that there is only a single process step, whereas many products are currently produced in several dependent process steps. Consequently, it is not appropriate to monitor these process steps by utilizing a control chart for each individual process step. Zhang (1984) proposes the simple cause-selecting control chart to control the specific quality in the current process by adjusting the effect of in-coming quality variable (X) on out-going quality variable (Y), since the in-coming quality variable on the first process step and the out-going quality variable on the second process step are dependent. The cause-selecting values (e) are Y minus the effect of X, and the cause-selecting control chart is constructed accordingly. Wade and Woodall (1993) review and analyze the cause-selecting control chart and examine the relationship between the cause-selecting control chart and the Hotelling T^2 control chart. In their opinion the cause-selecting control chart outperforms Hotelling T^2 control chart, since it is easy to distinguish whether the second step of the process is out-of-control. Therefore, it seems reasonable to develop variable control schemes to control dependent process steps. However, the properties of the VSI control charts used to control the process means on two dependent steps with incorrect adjustment have not yet been addressed. Therefore, to study the performance of the joint VSI EWMA control charts on two dependent process steps with incorrect adjustment is reasonable. In this paper, the joint VSI EWMA control charts are proposed to control the process means on dependent steps with incorrect adjustment. In next section, the performance of the proposed EWMA control charts is measured by the adjusted average time to signal (AATS), which is derived using a Markov chain approach. Finally, we illustrate the application of the proposed joint EWMA control charts using the example of automobile braking system with incorrect adjustment, and compare the performance between the joint VSI EWMA control charts and joint FSI EWMA control charts. In case the variable sampling intervals cannot be specified by the engineers, the optimum VSI EWMA control charts are suggested.

2. DESCRIPTION OF THE JOINT VSI $EWMA_{Z_X}$ AND $EWMA_{Z_e}$ CONTROL CHARTS

Consider a process with two dependent process steps controlled by the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts. Let X be the measurable in-coming quality variable on the first process step. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X; let Y be the measurable out-going quality characteristic of interest for the second process step, and follow a normal distribution conditional on X. Since the two process steps are dependent, and the second process step is affected by the first process step, then following Wade and Woodall (1993), the relationship between X and Y is generally expressed as

$$Y_i|X_i = f(X_i) + \varepsilon_i, i = 1,2,3,...m \qquad (2\text{-}1)$$

where, it is assumed that $\varepsilon_i \sim NID(0,\sigma^2)$. Let Y instead of $Y \mid X$. If the function $f(X_i)$ is known, the values of the standardized error term $\varepsilon_i^* = \dfrac{Y_i - f(X_i)}{\sigma}$ are called the cause-selecting values since they are the values of Y_i adjusted for the effects of X_i. In practice, the true function $f(X_i)$ is usually unknown and thus must be estimated using the m observations obtained from the initial m samples of size one, and thus the estimate for $f(X_i)$ will be $\hat{Y}_i$. The residuals, $e_i = Y_i - \hat{Y}_i$, are generated by the model used. Hence, e_i ~NID(0, σ_e^2). Consequently, the standardized residuals $e_i^* = \frac{e_i}{\sigma_e}$ are called the cause-selecting values.

In our study the chosen sample size is one and taken from the end of the two dependent process steps; when the process steps are all in control, the standardized samples, Z_X and Z_e, are

$$Z_{X_i} = \frac{X_i - \mu_X}{\sigma_X} \sim \text{N}(0,1) \text{ and } Z_{e_i} = \frac{e_i - 0}{\sigma_e} \sim \text{N}(0,1) \qquad (2\text{-}2)$$

Also assume that the first step is only subject to the special cause 1 or incorrect adjustment such that the mean of X_i shifts from μ_X to $\mu_X + \delta_1 \sigma_X$ ($\delta_1 \neq 0$) and the variance is unchanged; the second step is only subject to the special cause 2 or incorrect adjustment such that the mean of e_i shifts from 0 to δ_2 ($\delta_2 \neq 0$) and the variance is unchanged. That is $Z_{X_i} = \frac{X_i - \mu_X}{\sigma_X} \sim \text{N}(\delta_1,1)$ and/or $Z_{e_i} = \frac{e_i - 0}{\sigma_e} \sim \text{N}(\delta_2,1)$ for out-of-control process step1 and/or step 2. The out-of-control distribution of Z_{X_i} or Z_{e_i} will be adjusted to in-control state, once at least one true signal is obtained from the Z_X or Z_e control chart. Let T_{sci} be the time until the occurrence of special cause i, where i=1,2, and follow an exponential distribution of the form

$$f(t_{sci}) = \gamma_i \exp(-\gamma_i t) \quad t_{sci} > 0,\ \text{i} = 1,2. \qquad (2\text{-}3)$$

where $1/\gamma_i$ is the mean time that the process step i remains in a state of statistical control.

To detect the small shifts in process means faster, the $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts are constructed. Thus, The distributions of the statistics $EWMA_{Z_X}$ and $EWMA_{Z_e}$ should be derived. The statistics and distributions are as follows.

$$EWMA_{Zx,i} = \lambda_1 Z_{X_i} + (1-\lambda_1) EWMA_{Z_{X,i-1}}, i = 1,2,...., \text{where } EWMA_{Zx,i} \sim N(0, \frac{\lambda_1}{2-\lambda_1})$$

$$EWMA_{Ze,i} = \lambda_2 Z_{e_i} + (1-\lambda_2) EWMA_{Ze,i-1}, i = 1,2,...., \text{where } EWMA_{Ze,i} \sim N(0, \frac{\lambda_2}{2-\lambda_2})$$

An in-control state analysis for the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts is performed since the shifts in the mean on process step 1 and step 2 do not occur when the process is just starting, but occur at some time in the future. The samples $EWMA_{Z_X}$ and $EWMA_{Z_e}$ are plotted on the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts with warning limits of the form $\pm w_X$ and $\pm w_e$, and control limits of the form $\pm k_X$ and $\pm k_e$, respectively, where $0 \le w_X < k_X$ and $0 \le w_e < k_e$ (see figure 2.1).

$$UCL_{EWMA_{Z_X}} = k_X \sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad UCL_{EWMA_{Z_e}} = k_e \sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$UWL_{EWMA_{Z_X}} = w_X \sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad UWL_{EWMA_{Z_e}} = w_e \sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$CL_{EWMA_{Z_X}} = 0 \qquad CL_{EWMA_{Z_e}} = 0$$

$$LWL_{EWMA_{Z_X}} = -w_X \sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad LWL_{EWMA_{Z_e}} = -w_e \sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$LCL_{EWMA_{Z_X}} = -k_X \sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad LCL_{EWMA_{Z_e}} = -k_e \sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

(1) $EWMA_{Z_X}$ chart (2) $EWMA_{Z_e}$ chart

Figure 2.1 The control limits of VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts

The search for the special cause 1 and adjustment in the first process step is undertaken when the sample $EWMA_{Z_X}$ falls outside the interval ($-k_x, k_x$), that is when the $EWMA_{Z_X}$ chart produces a signal. The search for the special cause 2 and adjustment in the second process step is undertaken when the sample $EWMA_{Z_e}$ falls outside the interval ($-k_e$, k_e), that is when the $EWMA_{Z_e}$ chart produces a signal. For a discontinuous process, the two process steps are stopped to search for the special causes and adjustment after at least one signal is obtained from the proposed control charts. The process adjustment is incorrect when the signal is false, but the adjustment is correct when the signal is true and then the process steps are brought back to an in-control state.

The position of the current sample in each control chart constructs the sampling interval of the next sample.

We divide the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts into the following three regions (2-4).

$$\begin{aligned} I_{Z_{X1}} &= (-w_X, w_X) && \text{(central region)} \\ I_{Z_{X2}} &= (\text{-}k_X, \text{-}w_X) \cup (w_X, k_X) && \text{(warning region)} \\ I_{Z_{X3}} &= (\text{-}k_X, \text{-}k_X) && \text{(control region)} \end{aligned} \tag{2-4}$$

$$I_{Z_{e1}} = (\text{-}w_e, w_e) \qquad \text{(central region)}$$
$$I_{Z_{e2}} = (\text{-}k_e, \text{-}w_e) \cup (w_e, k_e) \qquad \text{(warning region)}$$
$$I_{Z_{e3}} = (\text{-}k_e, \text{-}k_e) \qquad \text{(control region)}$$

The first region, within two warning limits, is called the central region. The second region, within warning limit and control limit, is called the warning region. The third region, within control limits, is called the control region.

Three VSIs are adopted, $0 < t_1 < t_2 < t_3 < \infty$. If the samples, $EWMA_{Z_X}$ and $EWMA_{Z_e}$, all fall within the central regions, $I_{z_{X1}}$ and $I_{Z_{E1}}$, then the next sampling interval should be long (t_3). If one sample falls within the central region but another falls within the warning region, then the next sampling interval should be median (t_2). If all samples fall within the warning regions, then the next sampling interval should be short (t_1).

The relationship between the next sampling interval $(t_m, m = 1,2,3)$ and the position of the current samples is expressed as follows.

$$t_m = \begin{cases} t_3 \quad if \quad Z_{X_i} \in I_{Z_{X1}},\ Z_{e_i} \in I_{Z_{e1}}, \\ t_2 \quad if \quad Z_{X_i} \in I_{Z_{X2}},\ Z_{e_i} \in I_{Z_{e1}} \\ t_2 \quad if \quad Z_{X_i} \in I_{Z_{X1}},\ Z_{e_i} \in I_{Z_{e2}} \\ t_1 \quad if \quad Z_{X_i} \in I_{Z_{X2}},\ Z_{e_i} \in I_{Z_{e2}} \end{cases} \tag{2-5}$$

Following Costa (1997), the first sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sampling intervals, including the first one, should have a probability of p_{01} of being t_3, a probability of $p_{02} + p_{03}$ of being t_2, and a probability of p_{04} of being t_1, where $\sum_{i=1}^{4} p_{0i} = 1$, p_{01}, p_{02}, p_{03} and p_{04} are given by

$$p_{01}=P\left(\left|EWMA_{Z_X}\right|<w_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}\middle|\left|EWMA_{Z_X}\right|<k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}},\delta_1=0\right)\bullet P\left(\left|EWMA_{Z_e}\right|<w_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}\middle|\left|EWMA_{Z_e}\right|<k_e\sqrt{\frac{\lambda_2}{2-\lambda_2}},\delta_2=0\right)$$

$$=\left(\frac{2\Phi(w_X)-1}{2\Phi(k_X)-1}\right)\left(\frac{2\Phi(w_e)-1}{2\Phi(k_e)-1}\right) \quad (2\text{-}6)$$

$$p_{02}=P\left(\left|EWMA_{Z_X}\right|<w\sqrt{\frac{\lambda_1}{2-\lambda_1}}\middle|\left|EWMA_{Z_X}\right|<k\sqrt{\frac{\lambda_1}{2-\lambda_1}},\delta_1=0\right)$$

$$\bullet P\left(-k\sqrt{\frac{\lambda_2}{2-\lambda_2}}<EWMA_{Z_e}<-w\sqrt{\frac{\lambda_2}{2-\lambda_2}}\quad or\quad w\sqrt{\frac{\lambda_2}{2-\lambda_2}}<EWMA_{Z_e}<k\sqrt{\frac{\lambda_2}{2-\lambda_2}}\middle|\left|EWMA_{Z_e}\right|<k\sqrt{\frac{\lambda_2}{2-\lambda_2}},\delta_2=0\right)$$

$$=\frac{(2\Phi(w_X)-1)\bullet(2\Phi(k_e)-2\Phi(w_e))}{(2\Phi(k_X)-1)(2\Phi(k_e)-1)}$$

$$p_{03}=P\left(-k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}<EWMA_{Z_X}<-w_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}\quad or\quad w_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}<EWMA_{Z_X}<k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}\middle|\left|EWMA_{Z_X}\right|<k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}},\delta_1=0\right)$$

$$\bullet P\left(\left|EWMA_{Z_e}\right|<w_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}\middle|\left|EWMA_{Z_e}\right|<k_e\sqrt{\frac{\lambda_2}{2-\lambda_2}},\delta_2=0\right)$$

$$=\frac{(2\Phi(w_X)-1)\bullet(2\Phi(k_e)-2\Phi(w_e))}{(2\Phi(k_X)-1)(2\Phi(k_e)-1)}=P_{02}$$

$$p_{04}=P\left(-k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}<EWMA_{Z_X}<-w_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}\quad or\quad w_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}<EWMA_{Z_X}<k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}}\middle|\left|EWMA_{Z_X}\right|<k_X\sqrt{\frac{\lambda_1}{2-\lambda_1}},\delta_1=0\right)$$

$$\bullet P\left(-k_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}<EWMA_{Z_e}<-w_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}\quad or\quad w_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}<EWMA_{Z_e}<k_e\sqrt{\frac{\lambda_2}{2-\lambda_2}}\middle|\left|EWMA_{Z_e}\right|<k_e\sqrt{\frac{\lambda_2}{2-\lambda_2}},\delta_2=0\right)$$

$$=\left(\frac{2\Phi(k_X)-2\Phi(w_X)}{2\Phi(k_X)-1}\right)\left(\frac{2\Phi(k_e)-2(w_e)}{2\Phi(k_e)-1)}\right)$$

To facilitate the computation of the performance measures, w_X, k_X, w_e and k_e will be specified with the constraint that the probability of a sample falling in the central region is same for both the $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts when the process is in control. Thus,

$$P_r(|EWMA_{Z_X}|<w_X\big|\ |EWMA_{Z_X}|<k_X,\delta_1=0)\cdot=P_r(|EWMA_{Z_e}|<w_e\big|\ |EWMA_{Z_e}|<k_e,\delta_2=0) \quad (2\text{-}7)$$

implying, $w_X=w_e=w$, $k_X=k_e=k$ and $\lambda_1=\lambda_2=\lambda$.

If both $w_X=w_e$=0, and $t_1=t_2=t_3=t_0$, then the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts reduce to the joint $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts with FSI t_0.

3. COMPARISON OF CONTROL CHARTS

Sampling schemes should be compared under equal conditions; that is, VSI and FSI schemes should demand the same average sampling interval under the in-control period. That is,

$$E\left[t_m\ |\ EWMA_{Z_X}\ |<k,|\ EWMA_{Z_e}\ |<k,\big|\ \delta_1=0,\delta_2=0\right]=t_0 \quad (3\text{-}1)$$

Based on the equation (3-1), the following equation can be formulated as

$$
\begin{aligned}
& t_3 \bullet P(EWMA_{Z_{X,i-1}} \in I_{Z_{X1}} | \delta_1 = 0) \bullet P(EWMA_{Z_{e,i-1}} \in I_{Z_{e1}} | \delta_2 = 0) \\
& + t_2 \bullet P(EWMA_{Z_{X,i-1}} \in I_{Z_{X1}} | \delta_1 = 0) \bullet P(EWMA_{Z_{e,i-1}} \in I_{Z_{e2}} | \delta_2 = 0) \\
& + t_2 \bullet P(EWMA_{Z_{X,i-1}} \in I_{Z_{X2}} | \delta_1 = 0) \bullet P(EWMA_{Z_{e,i-1}} \in I_{Z_{e1}} | \delta_2 = 0) \\
& + t_1 \bullet P(EWMA_{Z_{X,i-1}} \in I_{Z_{X2}} | \delta_1 = 0) \bullet P(EWMA_{Z_{e,i-1}} \in I_{Z_{e2}} | \delta_2 = 0) \\
& = t_0 \bullet P(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{X,i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0) \bullet P(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{e,i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_2 = 0)
\end{aligned} \quad (3\text{-}2)
$$

Simplifying,

$$
\begin{aligned}
& 4\Phi(w)^2[t_3 - 2t_2 + t_1] + 4\Phi(w)[-t_3 + 2t_2\Phi(k) + t_2 - 2t_1\Phi(k)] \\
& - t_0(2\Phi(k)-1)^2 + t_3 - 4t_2\Phi(k) + 4t_1(\Phi(k))^2 = 0
\end{aligned} \quad (3\text{-}3)
$$

where $\Phi(\cdot)$ denotes the standard normal cumulative function.

The warning limit is derived as follows:

$$
w = \Phi^{-1}\left[\frac{-4B \pm \sqrt{16B^2 - 16AC}}{8A}\right] \quad (3\text{-}4)
$$

where

$$A = t_3 - 2t_2 + t_1$$
$$B = -t_3 + 2t_2\Phi(k) + t_2 - 2t_1\Phi(k)$$
$$C = -\left[t_0(2\Phi(k)-1)^2 - t_3 + 4t_2\Phi(k) - 4t_1(\Phi(k))^2\right]$$

However, to obtain w and let $0 < w < k$, the constraints $0 < t_1 < t_2 < t_3 < \infty$ is required. Thus, the warning limit can be obtained by using equation (3-4) and choosing a combination of the three VSIs, (t_1, t_2, t_3), and the FSI, t_0.

In this paper, the VSI scheme is compared with the FSI scheme and one adaptive scheme was considered to be better than another when it allows the joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts to detect changes in the process means on step 1 and step 2 faster.

4. PERFORMANCE MEASUREMENT

The speed with which a control chart detects process shifts measures the chart's statistical efficiency. For a VSI, the detection speed is measured by the average time from either mean shifting until either $EWMA_{Z_X}$ or $EWMA_{Z_e}$ chart or both signal, which is known as the AATS. That is, the AATS is the mean time that the process remains out of control.

Since $T_{SCi} \sim \exp(-\gamma_i t)$, $t > 0$, $i = 1, 2$, the occurrence time, $T_{(1)}$, until the first special cause occurs is

$$T_{(1)} \sim \exp(\gamma_1 + \lambda_2) \qquad \text{where } T_{(1)} = \min(T_{SC1}, T_{SC2})$$

Hence,

$$AATS = ATC - \frac{1}{\gamma_1 + \gamma_2} \tag{4-1}$$

The average time of the cycle (ATC) is defined as the average time from the start of process until at least one true signal obtained from one or both of the proposed charts and the out-of-control process step1 and/or step2 are correct adjusted. The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 32 states is assigned based on whether the process step is in or out of control and the position of samples (see Table 4.1 for the 32 states of the process). The status of the process when the $(i+1)^{th}$ sample is taken, and the position of the i^{th} sample on the joint $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts defines the transition states of the Markov chain. The joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts produce a signal when at least one of the samples falls outside its control limits. If the current state is any one of the States 1, 2, 4, 5, 10, 11, 12, 13,17, 18, 20, 21, 24, 25, 27, 28 and 29 then the process steps are not adjusted and the current state may transit to any other state after sampling time interval t_m, m=1,2,3. If the current state is State 3, it indicates one false signal comes from the second process step then the process is adjusted unnecessarily and State 3 instantly becomes any one of the States 10~13 with probability $P_{3=j}$, $\mathrm{j} = 10 \sim 13,$ and $\sum_{j=10}^{13} P_{3=j} = 1$. Any one of the States 10~13 thus transits to any one of the States 10, 11, 12, 13 and 32 after a sampling time interval t_m, with probability $P_{i,j}(t_m)$ and $\sum_j P_{i,j}(t_m) = 1,\ i = 10,\sim 13,\ \mathrm{j} = 10,11,12,13,32, m = 1,2,3.$ State 6, 7, 8, 9, 14, 15, 16, 19, 22, 23, 26, 29, 30 and 31 are similar to State 3, since they indicate at least one false signal. .

If the current state is any one of the States 1~31, then there is no true signal, hence States 1~31 are transient states. State 32 is reached when at least one true signal obtained from the out-of-control process step1 and/or step2. State 32 cannot transit to any other state, hence it is an absorbing state.

Insert Table 4.1

Denote $\mathbf{P}$ be the transition probability matrix, where $\mathbf{P}$ is a square matrix of order 32. Let $P_{i,j}(t_m)$ to be the transition probability from prior state i to the current state j with sampling interval t_m, where t_m is determined by the prior state i,

$i=1,2,...,32, j=1,2,...,32, m=1,2,3$. The transition probability, for example, from state 1 to state 5 with sampling interval t_3 and fixed sample size n is calculated as

$$P_{1,5}(t_3)=P(T_{SC1}>t_3)\bullet P(T_{SC2}>t_3)\bullet P(EWMA_{Z_X}\in I_{Z_{X2}}|\delta_1=0)\bullet P(EWMA_{Z_e}\in I_{Z_{e2}}|\delta_2=0)$$
$$=e^{-\gamma_1 t_3}\bullet e^{-\gamma_2 t_3}\bullet\left(2(\Phi(k)-\Phi(w))\right)^2$$

The calculation of all transition probabilities is shown in Appendix.

From the elementary properties of Markov chains (see, e.g., Cinlar (1975)), the ATC is derived as follows:

$$ATC=\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{t}+\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}M_f+\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(A)T_r \qquad (4\text{-}2)$$

where $\mathbf{b}'=(p_{01},p_{02},0,p_{03},p_{04},0,0,0,0,0,0,0,0,0,0,0,0,0,.....0)$ is the vector of starting probabilities for States 1, 2, …, 31, where the first sampling interval has probability p_{01} (see equation (2-6) for calculation) of being long (or State 1 with probability p_{01}), the probability $p_{02}+p_{03}$ of being median (or State 2 and State 4 with probability p_{02}, p_{03}, respectively) and the probability p_{04} of being short (or State 5 probability p_{04}); $\mathbf{I}$ is the identity matrix of order 31; $\mathbf{Q}$ is the transition probability matrix where elements represent the transition probability, $P_{i,j}(t_m)$, from transient state I, i=1,…,31, to transient state j, j=1,…,31; $M_f'=(0,0,T_f,0,0,T_f,T_f,T_f,T_f,0,0,0,0,T_f,T_f,T_f,0,0,T_f,0,0,T_f,T_f,0,0,T_f,0,0,T_f,T_f,T_f)$ is the vector of in-correct adjustment time for State1 ~ State 31; $\mathbf{t}'=(t_3,t_2,t_1^*,t_2,t_1,t_1^*,t_2^*,t_2^*,t_3^*,t_3,t_2,t_2,t_1,t_3^*,t_3^*,t_2^*,t_3,t_2,t_3^*,t_2,t_1,t_3^*,t_1^*,t_3,t_2,t_2^*,t_2,t_1,t_2^*,t_1^*,t_1^*)$ is the vector of the variable sampling intervals for state1-state 31, where t_1^* is the average time of sampling interval for State 3, 6, 23, 30 and 31, t_2^* is the average time of sampling interval for State 7, 8, 16, 26 and 29, t_3^* is the average time of sampling interval for State 9, 14, 15, 19 and 22. The calculations of t_1^*, t_2^* and t_3^* are shown in Appendix; A is the vector of transition probability, $P_{i,32}(t_m)$, form transition state i, i=1,…,31, to absorbing state 32; T_r is the time to adjust any process step correctly.

5. AN EXAMPLE

An example of process control for automobile braking system is presented, and the data of the process are measurements of roll weight and bake weight. Let variables X=roll weight and Y= bake weight be measured from the end of the second process step. The bake weight produced in the second step is influenced by the roll weight produced in the first step. Two machines are used in the process steps. One machine could only fail in the first process step and shift the mean of X distribution, and another machine could only fail in the second process step, and shift the mean of Y distribution. Presently, the joint FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts are used to monitor the shifts of mean on the two process steps every hour. Information about the state of the process steps is available only through sampling. When the control charts indicate that at least one of the process steps is out of control, it requires adjustment. Sometimes, the process steps may be adjusted unnecessarily when at least one false signal occurs. To construct the control charts, fifty-five samples of size one (X, Y) are taken from Wade and Woodall (1993) to analysis their statistical relationship. After delete 24 outliers, the QQ plot (see Johnson 1992) of the 31 samples indicates that the data follows bivariate normal distribution. The relationship of quality variables X and Y is expressed by a linear regression model. The fitted model is

$$\hat{Y}|X = 30.3 + 0.812X \qquad (5\text{-}1)$$

Thus, the residuals or specific quality (e) are obtained by $Y - \hat{Y}|X$. The estimated means and standard deviations of variables X and e are ($\hat{\mu}_X = 210.5$, $\hat{\sigma}_X = 1.435$), and ($\hat{\mu}_e = 0$, $\hat{\sigma}_e = 0.817$), respectively. That is, when both process steps are in-control, $X \sim N(210.5, 1.435^2)$ and $e \sim N(0, 0.817^2)$. From historical data, the estimated failure frequency is 0.167 time per hour (or $\gamma_1 = 0.167$) for machine 1 and 0.125 time per hour (or $\gamma_2 = 0.125$) for machine 2. The failure machine1 and 2 are independent and only influence the means of X and Y, but the standard deviations are unaffected. The failure machine 1 would shift the mean of X to $\hat{\mu}_X + \delta_1 \hat{\sigma}_X$ where $\delta_1 = 1.0$. The failure machine 2 would shift the mean of e to $\delta_2 \hat{\sigma}_e$ where $\delta_2 = 0.5$. Hence, for out-of-control process step1, $X \sim N(210.5 + 1.0 * 1.435, 1.435^2)$; for out-of-control step2, $e \sim N(0.5 * 0.817, 0.817^2)$.

The FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts have control limits placed at ± 2.492 and $\lambda = 0.05$ with average run length 370 (see Montgomery (2005)) when $T_f = T_r = 0$, respectively. The average incorrect adjustment time of any process step is 0.5h (or $T_f = 0.5$) when at least one false signal occurs. The average correct adjustment time of any process step is 6.0h (or $T_r = 6.0$) when at least one true signal occurs. The AATS of the FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts is 46.572h. The slowness with which the FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts detect shifts in the process ($\delta_1 = 1.0$, $\delta_2 = 0.5$) has led the quality manager to propose building the $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts with

VSIs. The construction and application of the proposed VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts is illustrated. The following are the guidelines for using the proposed charts:

Step 1. Let the factor of control limits, $k = 2.492$ and $\lambda = 0.05$, to maintain the in-control average run length is 370 for each $EWMA_{Z_X}$ or $EWMA_{Z_e}$ control chart.

Step 2. Since $0<t_1<t_2<t_0<t_3<\infty$ is required, and for performance of process control engineers adopt the combination (t_1=0.09h, t_2=0.1h, and t_3=3.5 h).

Step 3. Letting t_1=0.09h, t_2=0.1h, t_3=3.5h, $k = 2.492$ and $\lambda = 0.05$ in the equation (3-4) leads to w=0.688.

Consequently, the structures of the proposed VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts are as follows.

$UCL_{EWMA_{Z_X}} = 0.3990$ $\quad$ $UCL_{EWMA_{Z_e}} = 0.3990$

$UWL_{EWMA_{Z_X}} = 0.1102$ $\quad$ $UWL_{EWMA_{Z_e}} = 0.1102$

$CL_{EWMA_{Z_X}} = 0$ $\quad$ $CL_{EWMA_{Z_e}} = 0$

$LWL_{EWMA_{Z_X}} = -0.1102$ $\quad$ $LWL_{EWMA_{Z_e}} = -0.1102$

$LCL_{EWMA_{Z_X}} = -0.3990$ $\quad$ $LCL_{EWMA_{Z_e}} = -0.3990$

Figure 5.1 the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control limits

With the design parameters determined, the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts can be used for controlling the two dependent process steps for producing automobile braking. According to the VSI scheme, if both samples ($EWMA_{Z_X}$ and $EWMA_{Z_e}$), fall within warning limits, then the long sampling interval t_3=3.5h is taken. If one of the samples falls within the warning limits but the other falls between warning and control limits, then a middle sampling interval t_2=0.1h is taken. If both samples fall between warning and control limits, then the short sampling interval t_1=0.09h is taken. If at least one sample falls outside the control limits of any proposed control chart, then the process steps are stopped and adjusted. The AATS is used to measure the performance of the proposed VSI control charts. The proposed Markov chain approach is used to obtain the ATC and calculate the AATS. There are 32 possible states, as presented in Table 4.1. The AATS is 38.449h according to equation (4-1).

The VSI scheme improves the sensitivity of the joint FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts. From the example, in order to detect a shift in the process mean, the AATS for the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ charts has been reduced from 46.572 hours to only 38.449 hours. The percentage of saving time is 17.44%.

An example using the VSIs is introduced now. When the process starts, a random procedure decides the first sampling interval t_2 =0.1h with sample of size one, and the observation is (x=213, y=203). The first sample is (x=213, e=-0.3668) and the values of Z_X and Z_e are (1.7422, -0.4489). Thus, their $EWMA_{Z_X}$ and $EWMA_{Z_e}$ values are calculated as follows.

$$EWMA_{Zx,1} = \lambda Z_{X_1} + (1-\lambda)EWMA_{Z_{X,0}} = 0.05 \bullet 1.7422 + 0.95 \bullet 0 = 0.0871$$
$$EWMA_{Ze,1} = \lambda Z_{e_1} + (1-\lambda)EWMA_{Ze,0} = 0.05 \bullet (-0.4489) + 0.95 \bullet 0 = -0.0225$$

Since both samples fall within the warning limits, the second sample will be observed adopting a sample of size one after long sampling interval t_3=3.5h. The second sample is (x=211, y=203). Since $Z_X = 0.3484$ and $Z_e = 1.5399$, so $EWMA_{Z_X}$ and $EWMA_{Z_e}$ values are calculated as follows.

$$EWMA_{Zx,2} = \lambda Z_{X_2} + (1-\lambda)EWMA_{Z_{X,1}} = 0.05 \bullet 0.3484 + 0.95 \bullet 0.0871 = 0.1002$$
$$EWMA_{Ze,2} = \lambda Z_{e_2} + (1-\lambda)EWMA_{Ze,1} = 0.05 \bullet 1.5399 + 0.95 \bullet (-0.0255) = 0.0557$$

Both samples fall within the warning limits, the third sample will be observed adopting a sample of size one after the long sampling interval t_3 =3.5h. The $EWMA_{Z_X}$ and $EWMA_{Z_e}$ values for samples 1~35 are illustrated in the Table 5.1, and plotted on the constructed joint VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts (Fig. 5.2). We find that all $EWMA_{Z_X}$ values fall within the VSI $EWMA_{Z_X}$ chart, but the 35^{th} $EWMA_{Z_e}$ value falls outside the VSI $EWMA_{Z_X}$ chart. It indicates that the process step1 is in control, but the process step2 is out of control on the 35^{th} sample. Hence, the process step2 is stopped and machine 2 is adjusted.

Insert Table 5.1 and Figure 5.2

6. PERFORMANCE COMPARISON BETWEEN ASI AND FSI SCHEMES

Table 6.1 provides the AATS of the VSI and FSI schemes, which are obtained under various combinations of parameters based on orthogonal array $L_{27}(3^{13})$ table, $\gamma_1 = 0.03 \sim 0.167$, $\gamma_2 = 0.06 \sim 0.25$, $\delta_1 = 0.5 \sim 2.0$, $\delta_2 = 0.5 \sim 2.0$, $t_0 = 1.0$, $t_3 = 1.5 \sim 4.0$, $t_2 = 0.1 \sim 0.9$, $t_1 = 0.01 \sim 0.09$, $T_f = 0.5 \sim 3.0$, $T_r = 1.0 \sim 6.0$ and $n = 5$.

Comparing the AATS between the FSI and VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts, it can be seen that the performance of the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts is better for detecting small and median shifts in process means ($0.5 \le \delta_1 \le 2.0$, $1.5 \le \delta_2 \le 2.0$). The VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts save detection time from 3.88% to 30.49% compared to the FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts. To examine the effects of various parameters on the AATS, the main effects plots show the significant parameters are δ_1, δ_2 and T_f (see Fig. 6.1). As δ_1 or δ_2 increases, AATS decreases; as T_f increases, AATS decreases.

Insert Table 6.1 and Figure 6.1

Sometimes, quality engineers cannot specify the VSIs. The optimal VSIs of the proposed charts are thus suggested. The optimal VSI of the proposed charts are determined using optimization technique (Fortran IMSL BCONF subroutine) to minimum AATS under the same constraints and parameters as described before. The optimum VSI and AATS under various combinations of parameters are illustrated in Table 6.2. We find that the optimum VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts save detection time from 4.38% to 34.63% compared to the FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts, and the optimum VSI charts $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts also work better than the $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts with specific variable sampling intervals.

Insert Table 6.2

7. CONCLUSIONS

The proposed VSI scheme controlling two dependent process steps with incorrect adjustment substantially improves the performance of the FSI scheme by increasing the speed with which small and median shifts in the means of process steps are detected. We have found that the VSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts always work better (in the cases examined) than the FSI $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts for small and median δ_1 and δ_2 values. The optimum VSI scheme controlling two dependent process steps with incorrect adjustment is also suggested when quality engineers cannot specify the VSIs.

This paper considered two dependent process steps with incorrect adjustment. However, a study of the variable sample sizes (VSS), variable sample sizes and sampling intervals (VSSI) or variable parameters (VP) $EWMA_{Z_X}$ and $EWMA_{Z_e}$ control charts under two dependent process steps with incorrect adjustment is an interesting topic for future research. Other important extensions of the proposed model can also be developed. It is straight forward to extend the proposed model to study VP control charts or other control charts, such as attribute charts, CUSUM-charts or multivariate charts.

Table 4.1 Definition of 32 process states

State	Does AC1 occur?	$EWMA_{Z_X}$ Chart alarm?	Adjustment	Does AC2 occur?	$EWMA_{Z_e}$ chart alarm?	Adjustment	Transient state or absorbing state?
1.	No	No, $I_{Z_{X_1}}$	No	No	No, $I_{Z_{e1}}$	No	Transient state
2.	No	No, $I_{Z_{X_1}}$	No	No	No, $I_{Z_{e2}}$	No	
3.	No	No, $I_{Z_{X_1}}$	No	No	Yes, $I_{Z_{e3}}$	over-adjustment	
4.	No	No, $I_{Z_{X_2}}$	No	No	No, $I_{Z_{e1}}$	No	
5.	No	No, $I_{Z_{X_2}}$	No	No	No, $I_{Z_{e2}}$	No	
6.	No	No, $I_{Z_{X_2}}$	No	No	Yes, $I_{Z_{e3}}$	over-adjustment	
7.	No	Yes, $I_{Z_{x3}}$	over-adjustment	No	No, $I_{Z_{e1}}$	No	
8.	No	Yes, $I_{Z_{x3}}$	over-adjustment	No	No, $I_{Z_{e2}}$	No	
9.	No	Yes, $I_{Z_{x3}}$	over-adjustment	No	Yes, $I_{Z_{e3}}$	over-adjustment	
10.	No	No, $I_{Z_{X_1}}$	No	Yes	No, $I_{Z_{e1}}$		
11.	No	No, $I_{Z_{X_1}}$	No	Yes	No, $I_{Z_{e2}}$	No	
12.	No	No, $I_{Z_{X_2}}$	No	Yes	No, $I_{Z_{e1}}$	No	
13.	No	No, $I_{Z_{X_2}}$	No	Yes	No, $I_{Z_{e2}}$	No	
14.	No	Yes, $I_{Z_{x3}}$	over-adjustment	Yes	No, $I_{Z_{e1}}$	No	
15.	No	Yes, $I_{Z_{x3}}$	Over-adjustment	Yes	No, $I_{Z_{e2}}$	No	
16.	No	Yes, $I_{Z_{x3}}$	over-adjustment	Yes	Yes, $I_{Z_{e3}}$	correct adjustment	
17.	Yes	No, $I_{Z_{X_1}}$	No	No	No, $I_{Z_{e1}}$	No	
18.	Yes	No, $I_{Z_{X_1}}$	No	No	No, $I_{Z_{e2}}$	No	
19.	Yes	No, $I_{Z_{X_1}}$	No	No	Yes, $I_{Z_{e3}}$	over-adjustment	
20.	Yes	No, $I_{Z_{X_2}}$	No	No	No, $I_{Z_{e1}}$	No	
21.	Yes	No, $I_{Z_{X_2}}$	No	No	No, $I_{Z_{e2}}$	No	
22.	Yes	No, $I_{Z_{X_2}}$	No	No	Yes, $I_{Z_{e3}}$	over-adjustment	
23.	Yes	Yes, $I_{Z_{x3}}$	correct adjustment	No	Yes, $I_{Z_{e3}}$	over-adjustment	
24.	Yes	No, $I_{Z_{X_1}}$	No	Yes	No, $I_{Z_{e1}}$	No	
25.	Yes	No, $I_{Z_{X_1}}$	No	Yes	No, $I_{Z_{e2}}$	No	
26.	Yes	No, $I_{Z_{X_1}}$	No	Yes	Yes, $I_{Z_{e3}}$	correct adjustment	

27.	Yes	No, $I_{Z_{X_2}}$	No	Yes	No, $I_{Z_{e1}}$	No	
28.	Yes	No, $I_{Z_{X_2}}$	No	Yes	No, $I_{Z_{e2}}$	No	
29.	Yes	No, $I_{Z_{X_2}}$	No	Yes	Yes, $I_{Z_{e3}}$	correct adjustment	
30.	Yes	Yes, $I_{Z_{x3}}$	correct adjustment	Yes	No, $I_{Z_{e1}}$	No	
31.	Yes	Yes, $I_{Z_{x3}}$	correct adjustment	Yes	No, $I_{Z_{e2}}$	No	
32.	correct adjustment for process step1 and/or step2, given process step1 and/or process step2 are/is out-of-control						Absorbing state

Table 5.1 $EWMAZ_{X.i}$ and $EWMAZ_{e.i}$ values for the samples 1~35

sample (i)	x	y	e	$Z_{X,i}$	$EWMAZ_{X.i-1}$	$Z_{e,i}$	$EWMAZ_{e.i-1}$
1	213	203	-0.3668	1.7422	0.0871	-0.4489	-0.0225
2	211	203	1.2581	0.3484	0.1002	1.5399	0.0557
3	210	201	0.0705	-0.3484	0.0777	0.0863	0.0572
4	210	201	0.0705	-0.3484	0.0564	0.0863	0.0587
5	209	202	1.8830	-1.0453	0.0013	2.3047	0.1710
6	211	202	0.2581	0.3484	0.0187	0.3159	0.1782
7	211	201	-0.7419	0.3484	0.0352	-0.908	0.1239
8	211	202	0.2581	0.3484	0.0509	0.3159	0.1335
9	212	202	-0.5543	1.0453	0.1006	-0.6785	0.0929
10	208	200	0.6954	-1.7422	0.0084	0.8512	0.1308
11	212	202	-0.5543	1.0453	0.0603	-0.6785	0.0903
12	209	201	0.8830	-1.0453	0.0050	1.0807	0.1399
13	210	202	1.0705	-0.3484	-0.0127	1.3103	0.1984
14	210	201	0.0705	-0.3484	-0.0295	0.0863	0.1928
15	211	201	-0.7419	0.3484	-0.0106	-0.9081	0.1377
16	210	200	-0.9295	-0.3484	-0.0275	-1.1377	0.0740
17	210	200	-0.9295	-0.3484	-0.0435	-1.1377	0.0134
18	210	200	-0.9295	-0.3484	-0.0588	-1.1377	-0.0442
19	211	201	-0.7419	0.3484	-0.0384	-0.9081	-0.0874
20	211	202	0.2581	0.3484	-0.0191	0.3159	-0.0672
21	211	201	-0.7419	0.3484	-0.0007	-0.9081	-0.1092
22	210	201	0.0705	-0.3484	-0.0181	0.0863	-0.0995
23	212	203	0.4457	1.0453	0.0351	0.5455	-0.0672
24	209	200	-0.1170	-1.0453	-0.0189	-0.1432	-0.0710
25	209	199	-1.1170	-1.0453	-0.0702	-1.3672	-0.1358
26	210	202	1.0705	-0.3484	-0.0842	1.3103	-0.0635
27	212	203	0.44566	1.0453	-0.0277	0.5455	-0.0331
28	212	204	1.4457	1.0453	0.0260	1.7695	0.0572
29	208	199	-0.3046	-1.7422	-0.0624	-0.3728	0.0356
30	208	198	-1.3046	-1.7422	-0.1464	-1.5968	-0.0462
31	214	204	-0.1792	2.4390	-0.0172	-0.2194	-0.0547
32	212	203	0.4460	1.0453	0.0360	0.5459	-0.0247
33	209	200	-0.1170	-1.0453	-0.0181	-0.1432	-0.0306
34	209	204	3.8830	-1.0453	-0.0695	4.7528	0.2086
35	206	201	8.3200	-3.1359	-0.2228	10.1836	0.7073*

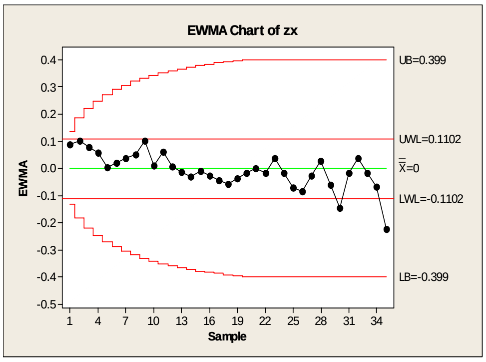

Fig. 5.2 (1) VSI $EWMA_{Z_X}$ control chart

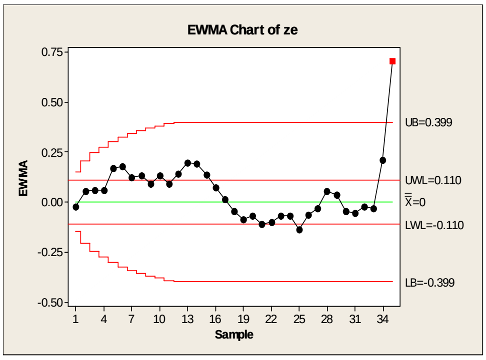

Fig. 5.2 (2) VSI $EWMA_{Z_X}$ control chart

Table 6.1 AATS of VSI and FSI charts under various combinations of parameters

Combination of various parameters									VSI		FSI	Save time percentage
No.	t_1	t_2	t_3	δ_1	δ_2	γ_1	γ_2	(T_f, T_{sr})	w^*	AATS	AATS	
1.	0.01	0.1	1.5	0.5	0.5	0.03	0.06	(0.5,1.0)	1.262	65.234	70.881	7.97
2.	0.01	0.1	1.5	0.5	1	0.083	0.125	(1.5,2.0)	1.262	36.954	43.211	14.48
3.	0.01	0.1	1.5	0.5	2	0.167	0.25	(3.0,6.0)	1.262	18.946	20.977	9.68
4.	0.01	0.5	2.5	1	0.5	0.03	0.06	(1.5,2.0)	0.740	36.787	42.919	14.29
5.	0.01	0.5	2.5	1	1	0.083	0.125	(3.0,6.0)	0.740	24.773	29.714	16.63
6.	0.01	0.5	2.5	1	2	0.167	0.25	(0.5,1.0)	0.740	6.987	9.224	24.25
7.	0.01	0.9	3.5	2	0.5	0.03	0.06	(3.0,6.0)	0.519	29.203	31.032	5.89
8.	0.01	0.9	3.5	2	1	0.083	0.125	(0.5,1.0)	0.519	8.763	11.141	21.34
9.	0.01	0.9	3.5	2	2	0.167	0.25	(1.5,2.0)	0.519	4.669	5.836	20.00
10.	0.05	0.1	2.5	2	0.5	0.083	0.25	(0.5,2.0)	0.854	25.331	30.398	16.67
11.	0.05	0.1	2.5	2	1	0.167	0.06	(1.5,6.0)	0.854	11.058	12.538	11.81
12.	0.05	0.1	2.5	2	2	0.03	0.125	(3.0,1.0)	0.854	2.709	3.898	30.49
13.	0.05	0.5	3.5	0.5	0.5	0.083	0.25	(1.5,6.0)	0.602	83.998	89.168	5.80
14.	0.05	0.5	3.5	0.5	1	0.167	0.06	(3.0,1.0)	0.602	46.523	55.772	16.58
15.	0.05	0.5	3.5	0.5	2	0.03	0.125	(0.5,2.0)	0.602	10.214	11.439	10.71
16.	0.05	0.9	1.5	1	0.5	0.083	0.25	(3.0,1.0)	0.853	49.716	53.161	6.48
17.	0.05	0.9	1.5	1	1	0.167	0.06	(0.5,2.0)	0.853	18.818	22.057	14.69
18.	0.05	0.9	1.5	1	2	0.03	0.125	(1.5,6.0)	0.853	10.240	11.148	8.14
19.	0.09	0.1	3.5	1	0.5	0.167	0.125	(0.5,6.0)	0.688	38.449	46.572	17.44
20.	0.09	0.1	3.5	1	1	0.03	0.25	(1.5,1.0)	0.688	15.576	21.511	27.59
21.	0.09	0.1	3.5	1	2	0.083	0.06	(3.0,2.0)	0.688	10.009	12.904	22.44
22.	0.09	0.5	1.5	2	0.5	0.167	0.125	(1.5,1.0)	1.078	15.858	17.923	11.52
23.	0.09	0.5	1.5	2	1	0.03	0.25	(3.0,2.0)	1.078	14.459	16.547	12.62
24.	0.09	0.5	1.5	2	2	0.083	0.06	(0.5,6.0)	1.078	6.903	7.715	10.53
25.	0.09	0.9	2.5	0.5	0.5	0.167	0.125	(3.0,2.0)	0.603	91.200	94.881	3.88
26.	0.09	0.9	2.5	0.5	1	0.03	0.25	(0.5,6.0)	0.603	27.962	30.463	8.21
27.	0.09	0.9	2.5	0.5	2	0.083	0.06	(1.5,1.0)	0.603	21.691	24.230	10.48

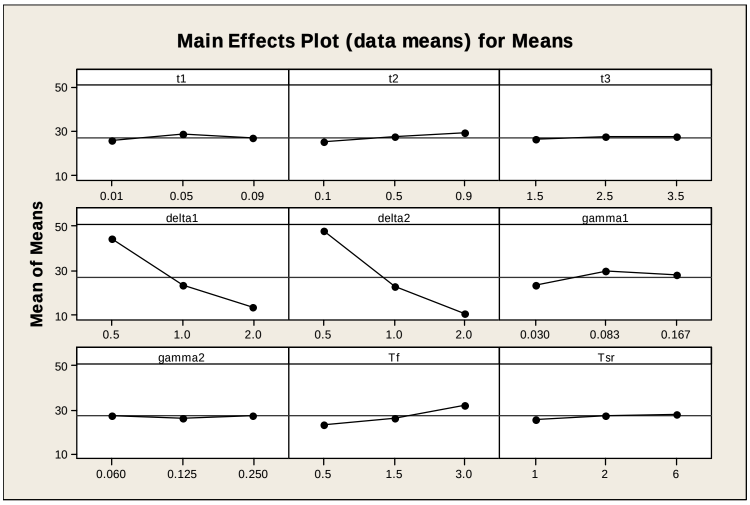

Fig. 6.1 The main effects for average AATS under various parameters

Table 6.2 AATS of optimum VSI and FSI charts under various combinations of parameters

Combination of various parameters						VSI					FSI	Save time
No.	δ_1	δ_2	γ_1	γ_2	(r_f, r_m)	w^*	t_1^*	t_2^*	t_3^*	AATS*	AATS	percentage
1.	0.5	0.5	0.03	0.06	(0.5,1.0)	0.645	0.001	0.1	4	62.205	70.881	12.24
2.	0.5	1	0.083	0.125	(1.5,2.0)	0.720	0.001	0.1	3.315	35.047	43.211	18.89
3.	0.5	2	0.167	0.25	(3.0,6.0)	1.456	0.001	0.1	1.304	18.836	20.977	10.21
4.	1	0.5	0.03	0.06	(1.5,2.0)	0.645	0.001	0.1	4	34.948	42.919	18.57
5.	1	1	0.083	0.125	(3.0,6.0)	0.674	0.001	0.1	3.708	23.647	29.714	20.42
6.	1	2	0.167	0.25	(0.5,1.0)	1.093	0.001	0.1	1.78	6.625	9.224	28.18
7.	2	0.5	0.03	0.06	(3.0,6.0)	0.990	0.001	0.1	2.029	28.496	31.031	8.17
8.	2	1	0.083	0.125	(0.5,1.0)	0.923	0.001	0.1	2.242	8.041	11.141	27.83
9.	2	2	0.167	0.25	(1.5,2.0)	1.181	0.001	0.1	1.619	4.114	5.836	29.50
10.	2	0.5	0.083	0.25	(0.5,2.0)	1.034	0.001	0.1	1.914	25.104	30.398	17.42
11.	2	1	0.167	0.06	(1.5,6.0)	1.224	0.001	0.1	1.553	10.775	12.538	14.06
12.	2	2	0.03	0.125	(3.0,1.0)	1.115	0.001	0.1	1.734	2.548	3.898	34.63
13.	0.5	0.5	0.083	0.25	(1.5,6.0)	1.589	0.001	0.1	1.214	82.719	89.168	7.23
14.	0.5	1	0.167	0.06	(3.0,1.0)	0.645	0.001	0.1	4	44.857	55.772	19.57
15.	0.5	2	0.03	0.125	(0.5,2.0)	1.137	0.001	0.1	1.693	9.380	11.439	18.00
16.	1	0.5	0.083	0.25	(3.0,1.0)	0.645	0.001	0.1	4	47.519	53.161	10.61
17.	1	1	0.167	0.06	(0.5,2.0)	0.724	0.001	0.1	3.286	15.916	22.057	27.84
18.	1	2	0.03	0.125	(1.5,6.0)	1.015	0.001	0.1	1.962	9.520	11.148	14.60
19.	1	0.5	0.167	0.125	(0.5,6.0)	0.811	0.001	0.1	2.733	37.980	46.572	18.45
20.	1	1	0.03	0.25	(1.5,1.0)	0.766	0.001	0.1	2.991	15.311	21.511	28.82
21.	1	2	0.083	0.06	(3.0,2.0)	0.894	0.001	0.1	2.353	9.648	12.904	25.24
22.	2	0.5	0.167	0.125	(1.5,1.0)	1.229	0.001	0.1	1.545	15.218	17.923	15.10
23.	2	1	0.03	0.25	(3.0,2.0)	0.858	0.001	0.1	2.504	13.319	16.547	19.51
24.	2	2	0.083	0.06	(0.5,6.0)	1.087	0.001	0.1	1.791	6.586	7.715	14.63
25.	0.5	0.5	0.167	0.125	(3.0,2.0)	0.930	0.001	0.1	2.218	90.728	94.882	4.38
26.	0.5	1	0.03	0.25	(0.5,6.0)	1.009	0.001	0.1	1.978	27.047	30.463	11.21
27.	0.5	2	0.083	0.06	(1.5,1.0)	0.953	0.001	0.1	2.141	20.454	24.230	15.59

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Appendix: The calculation of all transition probabilities

Notation:

$$\eta_{Z_{X1}} = P(EWMA_{Z_X} \in I_{Z_{X1}} \mid EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) = 2\Phi(w) - 1$$

$$\eta_{Z_{X2}} = P(EWMA_{Z_X} \in I_{Z_{X2}} \mid EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) = 2\Phi(k) - 2\Phi(w)$$

$$\eta_{Z_{X3}} = P(EWMA_{Z_X} \in I_{Z_{X3}} \mid EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) = 2 - 2\Phi(k)$$

$$\eta_{Z_{e1}} = P(EWMA_{Z_e} \in I_{Z_{e1}} \mid EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda})) = 2\Phi(w) - 1$$

$$\eta_{Z_{e2}} = P(EWMA_{Z_e} \in I_{Z_{e2}} \mid EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda})) = 2\Phi(k) - 2\Phi(w)$$

$$\eta_{Z_{e3}} = P(EWMA_{Z_e} \in I_{Z_{e3}} \mid EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda})) = 2 - 2\Phi(k)$$

$$\beta_{Z_{X1}} = P(EWMA_{Z_X} \in I_{Z_{X1}} \mid EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= P(\left|EWMA_{Z_X}\right| < w\sqrt{\frac{\lambda}{2-\lambda}} \mid EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= P\left(\frac{EWMA_{Z_X} - \delta_1\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}} < \frac{w\sqrt{\frac{\lambda}{2-\lambda}} - \delta_1\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}}\right)$$

$$- P\left(\frac{EWMA_{Z_X} - \delta_1\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}} < \frac{-w\sqrt{\frac{\lambda}{2-\lambda}} - \delta_1\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}}\right)$$

$$= \Phi(w - \delta_1) + \Phi(w + \delta_1) - 1$$

$$\beta_{Z_X 2} = P\left(EWMA_{Z_X} \in I_{Z_{X2}} \middle| EWMA_{Z_X} \sim N\left(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= P\left(\left|EWMA_{Z_X}\right| < k\sqrt{\frac{\lambda}{2-\lambda}} \middle| EWMA_{Z_X} \sim N\left(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$- P\left(\left|EWMA_{Z_X}\right| < w\sqrt{\frac{\lambda}{2-\lambda}} \middle| EWMA_{Z_X} \sim N\left(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= \Phi(k - \delta_1) + \Phi(k + \delta_1) - \Phi(w - \delta_1) - \Phi(w + \delta_1)$$

$$\beta_{Z_x 3} = P\left(EWMA_{Z_X} \in I_{Z_{X3}} \middle| EWMA_{Z_X} \sim N\left(\delta_1\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \frac{\lambda_1}{2-\lambda_1}\right)\right)$$

$$= 1 - \beta_{Z_{X1}} - \beta_{Z_{X2}} = 2 - \Phi(k - \delta_1) - \Phi(k + \delta_1)$$

$$\beta_{Z_{e1}} = P\left(EWMA_{Z_e} \in I_{Z_{e1}} \middle| EWMA_{Z_e} \sim N\left(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= P\left(\left|EWMA_{Z_e}\right| < w\sqrt{\frac{\lambda}{2-\lambda}} \middle| EWMA_{Z_e} \sim N\left(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= P\left(\frac{EWMA_{Ze} - \delta_2\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}} < \frac{w\sqrt{\frac{\lambda}{2-\lambda}} - \delta_2\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}}\right)$$

$$- P\left(\frac{EWMA_{Z_e} - \delta_2\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}} < \frac{-w\sqrt{\frac{\lambda}{2-\lambda}} - \delta_2\sqrt{\frac{\lambda}{2-\lambda}}}{\sqrt{\frac{\lambda}{2-\lambda}}}\right)$$

$$= \Phi(w - \delta_2) + \Phi(w + \delta_2) - 1$$

$$\beta_{Z_e 2} = P\left(EWMA_{Z_e} \in I_{Z_{e2}} \middle| EWMA_{Z_e} \sim N\left(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= P\left(\left|EWMA_{Z_e}\right| < k\sqrt{\frac{\lambda}{2-\lambda}} \middle| EWMA_{Z_e} \sim N\left(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$- P\left(\left|EWMA_{Z_e}\right| < w\sqrt{\frac{\lambda}{2-\lambda}} \middle| EWMA_{Z_e} \sim N\left(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}\right)\right)$$

$$= \Phi(k-\delta_2) + \Phi(k+\delta_2) - \Phi(w-\delta_2) - \Phi(w+\delta_2)$$

$$\beta_{Z_{e3}} = P(EWMA_{Z_e} \in I_{Z_{e3}} \left| EWMA_{Z_e} \sim N(\delta_2 \sqrt{\frac{\lambda_2}{2-\lambda_2}}, \frac{\lambda_2}{2-\lambda_2})\right.)$$
$$= 1 - \beta_{Z_{e1}} - \beta_{Z_{e2}} = 2 - \Phi(k-\delta_2) - \Phi(k+\delta_2)$$

The transition probability can be expressed by the following general form.

$$P_{1,j}(t_3) = P(T_{\mathrm{AC1}} > t_3) \bullet P(T_{\mathrm{AC2}} > t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 = 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 = 0)$$
$$= e^{-\gamma_1 t_3} \bullet e^{-\gamma_2 t_3} \bullet \eta_{z_{xr}} \bullet \eta_{z_{es}} \qquad \text{for } j = 1 \sim 9, r = 1,2,3, s = 1,2,3$$

$$P_{1,j}(t_3) = P(T_{\mathrm{AC1}} > t_3) \bullet P(T_{\mathrm{AC2}} < t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 = 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 \neq 0)$$
$$= e^{-\gamma_1 t_3} \bullet (1 - e^{-\gamma_2 t_3}) \bullet \eta_{z_{xr}} \bullet \beta_{z_{es}} \qquad \text{for } j = 10 \sim 16, r = 1,2,3, s = 1,2,3$$

$$P_{1,j}(t_3) = P(T_{\mathrm{AC1}} < t_3) \bullet P(T_{\mathrm{AC2}} > t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 \neq 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 = 0)$$
$$= (1 - e^{-\gamma_1 t_3}) \bullet e^{-\gamma_2 t_3} \bullet \beta_{z_{xr}} \bullet \eta_{z_{es}} \qquad \text{for } j = 17 \sim 23, r = 1,2,3, s = 1,2,3$$

$$P_{1,j}(t_3) = P(T_{\mathrm{AC1}} < t_3) \bullet P(T_{\mathrm{AC2}} < t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 \neq 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 \neq 0)$$
$$= (1 - e^{-\gamma_1 t_3}) \bullet (1 - e^{-\gamma_2 t_3}) \bullet \beta_{z_{xr}} \bullet \beta_{z_{es}} \qquad \text{for } j = 24 \sim 31, r = 1,2,3, s = 1,2,3$$

$$P_{1,32}(t_3) = 1 - \sum_{j=1}^{31} P_{1,j}(t_3)$$
$$P_{10,j}(t_3) = 0 \qquad \text{for } j = 1 \sim 9, 17 \sim 23$$

$$P_{10,j}(t_3) = P(T_{\mathrm{AC1}} > t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 = 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 \neq 0)$$
$$= e^{-\gamma_1 t_3} \bullet \eta_{z_{xr}} \bullet \beta_{z_{es}} \qquad \text{for } j = 10 \sim 16, r = 1,2,3, s = 1,2,3$$

$$P_{10,j}(t_3) = P(T_{\mathrm{AC1}} < t_3) \bullet P(EWMA_{Z_X} \in I_{Z_{X_r}} | \delta_1 \neq 0) \bullet P(EWMA_{Z_e} \in I_{Z_{e_s}} | \delta_2 \neq 0)$$
$$= (1 - e^{-\gamma_1 t_3}) \bullet \beta_{z_{xr}} \bullet \beta_{z_{es}} \qquad \text{for } j = 24 \sim 31, r = 1,2,3, s = 1,2,3$$

$$P_{10,32}(t_3) = 1 - \sum_{j=1}^{31} P_{10,j}(t_3)$$

$P_{17,j}(t_3)=0 \quad \text{for } j=1\sim16$

$$P_{17,j}(t_3)=P(T_{\mathrm{AC2}}>t_3)\bullet P(EWMA_{Z_X}\in I_{z_{X_r}}\left|\delta_1\neq0)\bullet P(EWMA_{Z_e}\in I_{z_{e_s}}\right|\delta_2=0)$$
$$=e^{-\gamma_2 t_3}\bullet\beta_{z_{xr}}\bullet\eta_{z_{es}} \quad \text{for } j=17\sim23, r=1,2,3, s=1,2,3$$

$$P_{17,j}(t_3)=P(T_{\mathrm{AC2}}<t_3)\bullet P(EWMA_{Z_X}\in I_{z_{X_r}}\left|\delta_1\neq0)\bullet P(EWMA_{Z_e}\in I_{z_{e_s}}\right|\delta_2\neq0)$$
$$=(1-e^{-\gamma_2 t_3})\bullet\beta_{z_{xr}}\bullet\beta_{z_{es}} \quad \text{for } j=24\sim31, r=1,2,3, s=1,2,3$$

$$P_{17,32}(t_3)=1-\sum_{j=1}^{31}P_{17,j}(t_3)$$

$P_{24,j}(t_3)=0 \quad \text{for } j=1\sim23$

$$P_{24,j}(t_3)=P(EWMA_{Z_X}\in I_{z_{X_r}}\left|\delta_1\neq0)*P(EWMA_{Z_e}\in I_{z_{e_s}}\right|\delta_2\neq0)$$
$$=\beta_{z_{xr}}\bullet\beta_{z_{es}} \quad \text{for } j=24\sim31, r=1,2,3, s=1,2,3$$

$$P_{24,32}(t_3)=1-\sum_{j=1}^{31}P_{24,j}(t_3)$$

The transition probabilities for $P_{2,j}(t_2),P_{4,j}(t_2),P_{11,j}(t_2),P_{12,j}(t_2),P_{18,j}(t_2),P_{20,j}(t_2),P_{25,j}(t_2)$ and $P_{27,j}(t_2)$ are calculated by replacing t_2 on t_3 for $P_{1,j}(t_3),P_{10,j}(t_3),P_{17,j}(t_3)$ and $P_{24,j}(t_3), j=1\sim32$.

$P_{2,j}(t_2)=P_{4,j}(t_2)=P_{1,j}(t_2), j=1\sim32$

$P_{11,j}(t_2)=P_{12,j}(t_2)=P_{10,j}(t_2), j=1\sim32$

$P_{18,j}(t_2)=P_{20,j}(t_2)=P_{17,j}(t_2), j=1\sim32$

$P_{25,j}(t_2)=P_{27,j}(t_2)=P_{24,j}(t_2), j=1\sim32$

The transition probabilities for $P_{5,j}(t_1),P_{13,j}(t_1),P_{21,j}(t_1)$ and $P_{28,j}(t_1)$ are calculated by replacing t_1 on t_3 for $P_{1,j}(t_3),P_{10,j}(t_3),P_{17,j}(t_3)$ and $P_{24,j}(t_3), j=1\sim32$

$P_{5,j}(t_1)=P_{1,j}(t_1), j=1\sim32$

$P_{13,j}(t_1)=P_{10,j}(t_1), j=1\sim32$

$P_{21,j}(t_1)=P_{17,j}(t_1), j=1\sim32$

$P_{28,j}(t_1)=P_{24,j}(t_1), j=1\sim32$

$P_{32,j}=0, j\neq32$

$P_{32,32}=1$

$P_{i,j}(t_1^*)=0, \quad i=3,6,23,30,31, j\neq10,11,12,13,32$

$P_{i,j}(t_1^*)=P_{i=10}P_{10,j}(t_3)+P_{i=11}P_{11,j}(t_2)+P_{i=12}P_{12,j}(t_2)+P_{i=13}P_{13,j}(t_1), j=10,11,12,13,32, i=3,6,23,30,31$

Where

$t_1^*=p_{i=10}\bullet t_3+p_{i=11}\bullet t_2+p_{i=12}\bullet t_2+p_{i=13}\bullet t_1$

$$p_{i=10} = P(EWMA_{Z_X} \in I_{Z_{X1}} \middle| EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{Z_{e1}} \middle| EWMA_{Z_e} \sim N(\delta_2 \sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(w)-1}{2\Phi(k)-1} \right) \bullet \left(\frac{\Phi(w-\delta_2)+\Phi(w+\delta_2)-1}{\Phi(k-\delta_2)+\Phi(k+\delta_2)-1} \right)$$

$$p_{i=11} = P(EWMA_{Z_X} \in I_{Z_{X1}} \middle| EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{Z_{e2}} \middle| EWMA_{Z_e} \sim N(\delta_2 \sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(w)-1}{2\Phi(k)-1} \right) \bullet \left(\frac{2\Phi(k-\delta_2)-2\Phi(w-\delta_2)}{\Phi(k-\delta_2)+\Phi(k+\delta_2)-1} \right)$$

$$p_{i=12} = P(EWMA_{Z_X} \in I_{Z_{X2}} \middle| EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{Z_{e1}} \middle| EWMA_{Z_e} \sim N(\delta_2 \sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(k)-2\Phi(w)}{2\Phi(k)-1} \right) \bullet \left(\frac{\Phi(w-\delta_2)+\Phi(w+\delta_2)-1}{\Phi(k-\delta_2)+\Phi(k+\delta_2)-1} \right)$$

$$p_{i=13} = P(EWMA_{Z_X} \in I_{Z_{X2}} \middle| EWMA_{Z_X} \sim N(0, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{Z_{e2}} \middle| EWMA_{Z_e} \sim N(\delta_2 \sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(k)-2\Phi(w)}{2\Phi(k)-1} \right) \bullet \left(\frac{2\Phi(k-\delta_2)-2\Phi(w-\delta_2)}{\Phi(k-\delta_2)+\Phi(k+\delta_2)-1} \right)$$

$$P_{i,j}(t_2^*) = 0, i = 7,8,16,26,29, j \neq 17,18,20,21,32$$

$$P_{i,j}(t_2^*) = P_{i=17} P_{17,j}(t_3) + P_{i=18} P_{18,j}(t_2) + P_{i=20} P_{20,j}(t_2) + P_{i=21} P_{21,j}(t_1), \; j = 17,18,20.21.32, i = 7,8,16,26,29$$

Where

$$t_2{*} = p_{i=17} \bullet t_3 + p_{i=18} \bullet t_2 + p_{i=20} \bullet t_2 + p_{i=21} \bullet t_1$$

$$p_{i=17} = P(EWMA_{Z_X} \in I_{z_{X1}} \middle| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{z_{e1}} \middle| EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{\Phi(w-\delta_1)+\Phi(w+\delta_1)-1}{\Phi(k-\delta_1)+\Phi(k+\delta_1)-1}\right) \bullet \left(\frac{2\Phi(w)-1}{2\Phi(k)-1}\right)$$

$$p_{i=18} = P(EWMA_{Z_X} \in I_{z_{X1}} \middle| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{z_{e}2} \middle| EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{\Phi(w-\delta_1)+\Phi(w+\delta_1)-1}{\Phi(k-\delta_1)+\Phi(k+\delta_1)-1}\right) \bullet \left(\frac{2\Phi(k)-2\Phi(w)}{2\Phi(k)-1}\right)$$

$$p_{i=20} = P(EWMA_{Z_X} \in I_{z_{X2}} \middle| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{z_{e1}} \middle| EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(k-\delta_1)-2\Phi(w-\delta_1)}{\Phi(k-\delta_1)+\Phi(k+\delta_1)-1}\right) \bullet \left(\frac{2\Phi(w)-1}{2\Phi(k)-1}\right)$$

$$p_{i,21} = P(EWMA_{Z_X} \in I_{z_{X}2} \middle| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet$$

$$P(EWMA_{Z_e} \in I_{z_{e2}} \middle| EWMA_{Z_e} \sim N(0, \frac{\lambda}{2-\lambda}))$$

$$= \left(\frac{2\Phi(k-\delta_1)-2\Phi(w-\delta_1)}{\Phi(k-\delta_1)+\Phi(k+\delta_1)-1}\right) \bullet \left(\frac{2\Phi(k)-2\Phi(w)}{2\Phi(k)-1}\right)$$

$$P_{i,j}(t_3^*) = 0, i = 9,14,15,19,22, j \neq 24,25,27,28,32$$

$$P_{i,j}(t_3^*) = P_{i=24}P_{24,j}(t_3) + P_{i=25}P_{25,j}(t_2) + P_{i=27}P_{27,j}(t_2) + P_{i=28}P_{28,j}(t_1),\ j = 24,25,27,28,32, i = 9,14,15,19,22$$

Where

$$t_3{*} = p_{i=24} \bullet t_3 + p_{i=25} \bullet t_2 + p_{i=27} \bullet t_2 + p_{i=28} \bullet t_1$$

$$p_{i=24} = P(EWMA_{Z_X} \in I_{z_{X1}} \left| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet \right.$$

$$P(EWMA_{Z_e} \in I_{z_{e1}} \left| EWMA_{Z_e} \sim N(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \right.$$

$$= \left(\frac{\Phi(w-\delta_1) + \Phi(w+\delta_1) - 1}{\Phi(k-\delta_1) + \Phi(k+\delta_1) - 1} \right) \bullet \left(\frac{\Phi(w-\delta_2) + \Phi(w+\delta_2) - 1}{\Phi(k-\delta_2) + \Phi(k+\delta_2) - 1} \right)$$

$$p_{i=25} = P(EWMA_{Z_X} \in I_{z_{X1}} \left| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet \right.$$

$$P(EWMA_{Z_e} \in I_{z_{e}2} \left| EWMA_{Z_e} \sim N(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \right.$$

$$= \left(\frac{\Phi(w-\delta_1) + \Phi(w+\delta_1) - 1}{\Phi(k-\delta_1) + \Phi(k+\delta_1) - 1} \right) \bullet \left(\frac{2\Phi(k-\delta_2) - 2\Phi(w-\delta_2)}{\Phi(k-\delta_2) + \Phi(k+\delta_2) - 1} \right)$$

$$p_{i=27} = P(EWMA_{Z_X} \in I_{z_{X2}} \left| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet \right.$$

$$P(EWMA_{Z_e} \in I_{z_{e1}} \left| EWMA_{Z_e} \sim N(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \right.$$

$$= \left(\frac{2\Phi(k-\delta_1) - 2\Phi(w-\delta_1)}{\Phi(k-\delta_1) + \Phi(k+\delta_1) - 1} \right) \bullet \left(\frac{\Phi(w-\delta_2) + \Phi(w+\delta_2) - 1}{\Phi(k-\delta_2) + \Phi(k+\delta_2) - 1} \right)$$

$$p_{i=28} = P(EWMA_{Z_X} \in I_{z_{X}2} \left| EWMA_{Z_X} \sim N(\delta_1\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \bullet \right.$$

$$P(EWMA_{Z_e} \in I_{z_{e2}} \left| EWMA_{Z_e} \sim N(\delta_2\sqrt{\frac{\lambda}{2-\lambda}}, \frac{\lambda}{2-\lambda})) \right.$$

$$= \left(\frac{2\Phi(k-\delta_1) - 2\Phi(w-\delta_1)}{\Phi(k-\delta_1) + \Phi(k+\delta_1) - 1} \right) \bullet \left(\frac{2\Phi(k-\delta_2) - 2\Phi(w-\delta_2)}{\Phi(k-\delta_2) + \Phi(k+\delta_2) - 1} \right)$$

$$P_{32,32} = 1$$

CONTROLLING OVER-ADJUSTED PROCESS MEANS AND VARIANCES USING VSI CAUSE SELECTING CONTROL CHARTS

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ABSTRACT

Process adjusted unnecessarily is a common problem in statistical process control. Incorrect adjustment of a process may result in shifts in process mean, process variance, or both, ultimately affecting the quality of products. The article considers the variable process control scheme for two dependent process steps with incorrect adjustment. We construct the variable sampling interval (VSI) $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts in order to effectively monitor the quality variable produced by the first process step with incorrect adjustment and the quality variable produced by the second process step with incorrect adjustment, respectively. The performance of the proposed VSI control charts is measured by the adjusted average time to signal (AATS) derived using a Markov chain approach. An example of process control for automobile braking system shows the application and performance of the proposed joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts in detecting small and median shifts in mean and variance for the two dependent process steps with incorrect adjustment. The performance of the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and the fixed sampling interval (FSI) $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are compared via the numerical analysis results. These demonstrate that the former is much faster in detecting shifts in mean and variance. Whenever quality engineers cannot specify the values of variable sampling intervals, the optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are suggested. Furthermore, the impacts of misusing $Z_{\overline{Y}}$ - $Z_{S_y^2}$ charts to monitoring the process mean and variance in the second step are also investigated.

Key words: Control charts; dependent process steps; optimization technique; Markov chain.

1. INTRODUCTION

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control.

Shewhart (1931) developed the $\overline{X}$ control chart which is easy to implement and has been widely used for industrial processes. However, a common problem in statistical process control is process adjusted unnecessarily (or overadjustment of an in-control process) (see Deming, 1982) due to incorrect use of control chart by the operator or since the only information about the state of the process is available through sampling. A process requires adjustment, when a control chart indicates that it is out of control. However, the process may be adjusted unnecessarily, when a false alarm occurs. Woodall (1986) noted that the effect of overadjustment is a quite significant increase in variability of the quality characteristic. Collani, Saniga and Weigand (1994) proposed an adjustment model for the $\overline{X}$ control chart with a single special cause that considers the effects of process mean with overadjustment from economic viewpoint. Their model determines the optimal design parameters of the $\overline{X}$ control chart which maximize the profitability of the process. Yang and Yang (2004) Yang and Yang (2004) addressed the economic adjustment model for $\overline{X}$ control chart with multiple special causes. Yang and Rahim (2000) addressed an economic adjustment model for the $\overline{\mathrm{X}}$ and S^2 control charts that consider the effects of mean and variance for the process with incorrect adjustment.

However, the above papers, even though Shewhart $\overline{X}$ control charts, always monitor a process by taking samples of equal size at a fixed sampling interval (FSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, in recent years several alternatives have been developed to improve the performance of $\overline{X}$ control charts. One of the useful approaches to improving the detection ability is to use a variable sampling interval (VSI) and/or a variable sample size (VSS) control chart instead of the traditional FSI and/or fixed sample size (FSS). Whenever there is some indication that a process parameter may have changed, the next sampling interval should be shorter and/or the next sample should be larger. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer and/or the next sample should be smaller.

The properties of the $\overline{X}$ chart with VSIs were studied by Reynolds, Amin, Arnold, and Nachlas (1988). Their paper has been extended by several others: Reynolds and Arnold (1989); Amin and Miller (1993); Baxley (1996); Reynolds, and Arnold, and Baik (1996). Tagaras (1998) reviewed the literature on adaptive control charts. Very little work has been down on VSI control charts for simultaneously monitoring process mean and variance. Chengular, Arnold and Reynolds (1989) detected process mean and variance using VSI $\overline{X}$ and R control charts. Reynolds and Stoumbos (2001) discussed the properties of VSI X and MR control charts. These papers show that most work on developing VSI control charts had aimed to solve the problem of monitoring process mean without considering the effects of incorrect adjustment of a process.

However, these articles assume that there is only a single process step whereas many products are currently produced with several dependent process steps. Consequently, it is not appropriate to monitor these process steps by utilizing a control chart for each individual process step. Zhang (1984) proposed the simple cause-selecting control chart to control the specific quality in the current process by adjusting the effect of the in-coming quality variable (X) on out-going quality variable (Y), since the in-coming quality variable on the first process step and the out-going quality variable on the second process step are dependent. The cause-selecting values (e) are Y minus the effect of X, and the cause-selecting control chart is constructed accordingly. Wade and Woodall (1993) reviewed and analyzed the cause-selecting control chart and examine the relationship between the cause-selecting control chart and the Hotelling T^2 control chart. In their opinion the cause-selecting control chart outperforms the Hotelling T^2 control chart, since it is easy to distinguish whether the second step of the process is out-of-control. Linear quality profile calibration and monitoring are discussed by Kim, Mahmoud and Woodall (2004). The relationship of input variable and output variable on dependent process steps is similar to the profile problem. Therefore, it seems reasonable to develop variable control schemes to control dependent process steps. Yang and Yu (2007) construct variable sampling interval EWMA control charts to effectively monitor the means of quality variables on two dependent process steps with unnecessarily adjustment.

However, the properties of the variable sampling interval (VSI) control charts used to control means and variance of quality variables on two dependent process steps with unnecessary adjustment have not yet been investigated. Therefore, studying the performance of the joint VSI control charts on two dependent process steps with unnecessary adjustment is reasonable. In this paper, the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with variable sampling intervals are proposed for the control of mean and variance in two dependent process steps with incorrect adjustment. In the next section, the performance of the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is measured by the adjusted average time to signal (AATS), which is derived using a Markov chain approach. Finally, we illustrate the application of the proposed control charts using the example of automobile braking system with incorrect adjustment on the process steps, and then compare the performance between the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. In case the VSIs cannot be specified by engineers, the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are suggested. Furthermore, the impacts of misusing $Z_{\overline{Y}}$ - $Z_{S_y^2}$ charts are investigated.

2. DESCRIPTION OF THE JOINT VSI $Z_{\overline{X}}$ - Z_{S^2} AND $Z_{\bar{e}}$ - $Z_{S_e^2}$ CHARTS

Consider a process with two dependent process steps controlled by the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. Let X be the measurable in-coming quality variable on the first process step. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X; let Y be the measurable out-going quality characteristic of interest for the second process step, and follow a normal distribution conditional on X. Since the two process steps are dependent, the second process step is affected by the first process step, and following Wade and Woodall (1993), the relationship between X and Y is generally expressed as

$$Y_i|X_i = f(X_i) + \varepsilon_i, i = 1,2,3,...m \qquad (2\text{-}1)$$

where, it is assumed that $\varepsilon_i \sim NID(0,\sigma^2)$. Let Y instead of $Y|X$. If the function $f(X_i)$ is known, the values of the standardized error term $\varepsilon_i^* = \frac{Y_i - f(X_i)}{\sigma}$ are called the cause-selecting values since they are the values of Y_i adjusted for the effects of X_i. In practice, the true function $f(X_i)$ is usually unknown and thus must be estimated using the m observations obtained from the initial m samples of size one, and thus the estimate for $f(X_i)$ will be $\hat{Y}_i$ (Yang (2003)). The residuals $e_i = Y_i - \hat{Y}_i$ are generated by the model used. Hence, e_i~NID(0, σ_e^2). Consequently, the standard residuals $e_i^* = \frac{Y_i - \hat{Y}_i}{\sigma_e}$ are called the cause-selecting values. The X chart is thus constructed to monitor the mean of X_i on the first step, and the e chart is constructed to monitor the mean of e_i on the second step.

However, in our study the chosen sample size is not one and the rational samples of size (n) are taken from the two dependent process steps (see Kim, Mahmoud and Woodall (2003)). Plot the sample data to obtain a sample profile and then establish the reference line of Y and X. To monitor the mean and variance of X on the first step the $\overline{X}$ - S^2 charts should be constructed, and to monitor the mean and variance of e on the second step the $\overline{e}$ - S_e^2 charts should be constructed. The $\overline{X}$ - S^2 charts and $\overline{e}$ - S_e^2 charts are called cause-selecting control charts.

For engineers to use the control charts easily, the sample means and sample variances are standardized as follows.

$$Z_{\overline{X}_i} = \frac{\overline{X}_i - \mu_X}{\sigma_X/\sqrt{n}} \sim N(0,1), \quad Z_{S_i^2} = \frac{(n-1)S_i^2}{\sigma_x^2} \sim \chi^2(\text{n}-1),$$

$$Z_{\overline{e}_i} = \frac{\overline{e}_i}{\sigma_e/\sqrt{n}} \sim N(0,1), \qquad Z_{S_{e_i}^2} = \frac{(n-1)S_{e_i}^2}{\sigma_e^2} \sim \chi^2(\text{n}-1), \tag{2-2}$$

where $\overline{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}$, $S_i^2 = \frac{\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2}{n-1}$, $\overline{e}_i = \frac{\sum_{j=1}^{n} e_{ij}}{n}$ and $S_{e_i}^2 = \frac{\sum_{j=1}^{n}(e_{ij} - \overline{e}_i)^2}{n-1}$

$i = 1,2,3,...,m,\ j = 1,2,3,....,n.$

Assume that once a special cause occurs it affects the X-variable with probability v and the functional relationship (or e-variable) with probability 1-v. That is, the mean of X_{ij} shifts from μ_X to $\mu_X + \delta_1\sigma_X$ ($\delta_1 \neq 0$) and the variance shifts from σ_X to $\delta_2\sigma_X$ ($\delta_2 > 1$) with probability v, and the mean of the e_{ij} shifts from 0 to δ_3 ($\delta_3 \neq 0$) and the variance shifts from σ_e to $\delta_4\sigma_e$ ($\delta_4 > 1$) with probability 1-v. The out-of-control distribution of X_{ij} and/or e_{ij} will be adjusted to an in-control state, as soon as at least one true signal is obtained from the proposed control charts. Let T_{sc} be the time until the occurrence of special cause, and follow an exponential distribution of the form

$$f(t) = \lambda \exp(-\lambda t) \quad t > 0 \tag{2-3}$$

where $1/\lambda$ is the mean time that the process remains in a state of statistical control.

An in-control state analysis for the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\overline{e}}$ - $Z_{S_e^2}$ control charts is performed since the shifts in the mean and variance on step 1 and/or step 2 do not occur when the process is just starting, but rather at some future time. The standardized samples $z_{\overline{X}_i}$ - z_{S^2} and $z_{\overline{e}_i}$ - $z_{S_e^2}$ are plotted on the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\overline{e}}$ - $Z_{S_e^2}$ control charts with warning limits of the form $\pm w_{\overline{X}}$, w_{S^2}, $\pm w_{\overline{e}}$ and $w_{S_e^2}$, and control limits of the form $\pm k_{\overline{X}}$, k_{S^2}, $\pm k_{\bar{e}}$ and $k_{S_e^2}$, respectively, where $0 \le w_{\overline{X}} < k_{\overline{X}}$, $0 \le w_{S^2} < k_{S^2}$, $0 \le w_{\bar{e}} < k_{\bar{e}}$ and $0 \le w_{S_e^2} < k_{S_e^2}$ (see Figure 2.1).

$UCL_{z_{\overline{X}}} = k_{\overline{X}}$	$UCL_{z_{\bar{e}}} = k_{\bar{e}}$
$UWL_{z_{\overline{X}}} = w_{\overline{X}}$	$UWL_{z_{\bar{e}}} = w_{\bar{e}}$
$CL_{z_{\overline{X}}} = 0$	$CL_{z_{\bar{e}}} = 0$
$LWL_{z_{\overline{X}}} = -w_{\overline{X}}$	$LWL_{z_{\bar{e}}} = -w_{\bar{e}}$
$LCL_{z_{\overline{X}}} = -k_{\overline{X}}$	$LCL_{z_{\bar{e}}} = -k_{\bar{e}}$
(1) $Z_{\overline{X}}$ chart	(2) $Z_{\bar{e}}$ chart
$UCL_{z_{S^2}} = k_{S^2}$	$UCL_{z_{S_e^2}} = k_{S_e^2}$
$UWL_{z_{S^2}} = w_{S^2}$	$UWL_{z_{S_e^2}} = w_{S_e^2}$
$LCL_{z_{S^2}} = 0$	$LCL_{z_{S_e^2}} = 0$
(3) Z_{S^2} chart	(4) $Z_{S_e^2}$ chart

Figure 2.1 The control limits of VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts

The search for the special cause and an adjustment in the first process step is undertaken when the sample $z_{\overline{X}_i}$ falls outside the interval $(-k_{\overline{X}}, k_{\overline{X}})$ and/or when the z_{S^2} falls outside the interval $(0, k_{S^2})$, that is when the $Z_{\overline{X}}$ and/or Z_{S^2} charts produce a signal. The search for the special cause and adjustment in the second process step is undertaken when the sample $z_{\bar{e}_i}$ falls outside the interval $(-k_{\bar{e}}, k_{\bar{e}})$ and/or when the sample $z_{S_e^2}$ falls outside the interval $(0, k_{S_e^2})$, that is when the $Z_{\bar{e}}$ and/or $Z_{S_e^2}$ charts produce a signal. For a discontinuous process, the two process steps are stopped for adjustment after a signal is obtained from a control chart. The process adjustment may be incorrect when the signal is false and then the process step is brought to an out-of-control state, but the adjustment may be correct when the signal is true and then the process step is brought back to an in-control state.

The positions of the current samples in the joint $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts construct the sampling interval of the next sample.

We divide the proposed VSI $Z_{\overline{X}}, Z_{S^2}$, $Z_{\bar{e}}$ and $Z_{S_e^2}$ control charts into the following three regions (2-4), respectively.

$$
\begin{aligned}
&I_{Z_{\bar{X}1}} = [-w_{\overline{X}}, w_{\overline{X}}] \ \text{(central region)} && I_{Z_{S_{\bar{x}}^2 1}} = (0, w_{S^2}) \ \text{(central region)} \\
&I_{Z_{\bar{X}2}} = (-k_{\overline{X}}, -w_{\overline{X}}) \cup (w_{\overline{X}}, k_{\overline{X}}) \ \text{(warning region)} && I_{Z_{S_{\bar{x}}^2 2}} = (w_{S^2}, k_{S^2}) \ \text{(warning region)} \quad (2\text{-}4) \\
&I_{Z_{\bar{X}3}} = [-k_{\overline{X}}, k_{\overline{X}}] \ \text{(control region)} && I_{Z_{S_{\bar{x}}^2 3}} = (0, k_{S^2}) \ \text{(control region)}
\end{aligned}
$$

$I_{Z_{\bar{e}1}} = ([-w_{\bar{e}}, w_{\bar{e}}])$ (central region) $I_{Z_{S_e^2 1}} = (0, w_{S_e^2})$ (central region)

$I_{Z_{\bar{e}2}} = (-k_{\bar{e}}, -w_{\bar{e}}) \cup (w_{\bar{e}}, k_{\bar{e}})$ (warning region) $I_{Z_{S_e^2 2}} = (w_{S_e^2}, k_{S_e^2})$ (warning region)

$I_{Z_{\bar{X}3}} = [-k_{\bar{X}}, k_{\bar{e}}]$ (control region) $I_{Z_{S_e^2 3}} = (0, k_{S_e^2})$ (control region)

The first region, within two warning limits, is called the central region. The second region, within warning limit and control limit, is called the warning region. The third region, within control limits, is called the control region.

Five VSIs are adopted: $\infty > t_1 > t_2 > t_3 > t_4 > t_5 > 0$. If the samples, $z_{\bar{X}_i}$, z_{S^2}, $z_{\bar{e}_i}$ and $z_{S_e^2}$, all fall within the central regions, $I_{\bar{X}1}$, $I_{Z_{S_x^2 1}}$, $I_{Z_{\bar{e}1}}$ and $I_{Z_{S_e^2 1}}$, then the next sampling interval should be the longest (t_1). If any three of the samples fall within the central regions but the other falls within the warning region, then the next sampling interval should be long (t_2). If any two of the samples fall within the central regions but the others fall within the warning regions, then the next sampling interval should be in the middle (t_3). If any one of the samples falls within the central region but the others fall within the warning regions, then the next sampling interval should be short (t_4). If all the samples fall within the warning regions, then the next sampling interval should be the shortest (t_5).

The relationship between the next sampling interval $(t_m, m = 1,2,3,4,5)$ and the position of the current samples is expressed as equation (2-5).

$$\begin{cases} t_1 & if \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ t_2 & if \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\ t_3 & if \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\ & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \end{cases}$$

$$
\begin{aligned}
t_m = \quad & or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
& or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\
& or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\
t_4 \quad & if \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\
& or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\
& or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}} \\
& or \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
t_5 \quad & if \quad Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}}
\end{aligned} \qquad (2\text{-}5)
$$

Following Costa (1997), the first sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sampling intervals, including the first one, should have a probability of p_{01} of being the longest, a probability of $p_{02}+p_{03}+p_{04}+p_{05}$ of being long, a probability of $p_{06}+p_{07}+p_{08}+p_{09}+p_{10}+p_{011}$ of being median, a probability of $p_{012}+p_{013}+p_{014}+p_{015}$ of being short, and a probability of p_{016} of being the shortest, where $\sum_{i=1}^{16} p_{0i}=1$, p_{01}, p_{02}, p_{03},… and p_{016} are given by

$$
\begin{aligned}
p_{01} &= P_r(|Z_{\overline{X}}|<w_{\overline{X}} \,\big|\, |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \,\big|\, 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1) \\
&\quad \bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \,\big|\, |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \,\big|\, 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1) \\
&= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$
\begin{aligned}
p_{02} &= P_r(|Z_{\overline{X}}|<w_{\overline{X}} \,\big|\, |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \,\big|\, 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1) \\
&\quad \bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \,\big|\, |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0) \bullet (1-P_r(0<Z_{S_e^2}<w_{S_e^2} \,\big|\, 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1)) \\
&= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$
\begin{aligned}
p_{03} &= P_r(|Z_{\overline{X}}|<w_{\overline{X}} \,\big|\, |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \,\big|\, 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1) \\
&\quad \bullet (1\text{-}P_r(|Z_{\bar{e}}|<w_{\bar{e}} \,\big|\, |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0)) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \,\big|\, 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1) \\
&= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$
\begin{aligned}
p_{04} &= P_r(|Z_{\overline{X}}|<w_{\overline{X}} \,\big|\, |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0) \bullet (1-P_r(0<Z_{S_x^2}<w_{S^2} \,\big|\, 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1)) \\
&\quad \bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \,\big|\, |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \,\big|\, 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1) \\
&= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$p_{05} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1)$$
$$\bullet P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \bullet P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1)$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{06} = P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0) \bullet P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1)$$
$$\bullet (1 - P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{07} = P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0) \bullet (1 - P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2} \delta_2 = 1))$$
$$\bullet P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{08} = P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0) \bullet (1 - P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1))$$
$$\bullet (1 - P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1)$$
$$= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{09} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1)$$
$$\bullet P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{10} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1)$$
$$\bullet (1 - P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1)$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{11} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \,\big|\, |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet (1 - P_r(0 < Z_{S_x^2} < w_{S^2} \,\big|\, 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1))$$
$$\bullet P_r(|Z_{\bar{e}}| < w_{\bar{e}} \,\big|\, |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \bullet P_r(0 < Z_{S_e^2} < w_{S_e^2} \,\big|\, 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1) \tag{2-6}$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{12} = P_r(|Z_{\overline{X}}| < w_{\overline{X}} \big| \ |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0) \bullet (1 - P_r(0 < Z_{S_x^2} < w_{S^2} \big| \ 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1))$$
$$\bullet (1 - P_r(|Z_{\bar{e}}| < w_{\bar{e}} \big| \ |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \big| \ 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(w_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{13} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \big| \ |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet P_r(0 < Z_{S_x^2} < w_{S^2} \big| \ 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1)$$
$$\bullet \ (1 - P_r(|Z_{\bar{e}}| < w_{\bar{e}} \big| \ |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \big| \ 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{14} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \big| \ |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet (1 - P_r(0 < Z_{S_x^2} < w_{S^2} \big| \ 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1))$$
$$\bullet P_r(|Z_{\bar{e}}| < w_{\bar{e}} \big| \ |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \bullet (1 - P_r(0 < Z_{S_e^2} < w_{S_e^2} \big| \ 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1))$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{15} = (1 - P_r(|Z_{\overline{X}}| < w_{\overline{X}} \big| \ |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0)) \bullet (1 \text{-} P_r(0 < Z_{S_x^2} < w_{S^2} \big| \ 0 < Z_{S_x^2} < k_{S_x^2}, \delta_2 = 1))$$
$$\bullet (1 \text{-} P_r(|Z_{\bar{e}}| < w_{\bar{e}} \big| \ |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0)) \bullet P_r(0 < Z_{S_e^2} < w_{S_e^2} \big| \ 0 < Z_{S_e^2} < k_{S_e^2}, \delta_4 = 1)$$
$$= \frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})] \bullet [F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}$$

$$p_{016} = 1 - \sum_{i=01}^{15} p_{0i}$$

To facilitate the computation of the performance measures, $w_{\overline{X}}, w_{S^2}$ $k_{\overline{X}}$, k_{S^2}, $w_{\bar{e}}$, $w_{S_e^2}$, $k_{\bar{e}}$ and $k_{S_e^2}$ will be specified with the constraint that the probability of a sample falling in the warning limits is same for both the $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts, when the process is in control. Thus,

$$P_r(|Z_{\overline{X}}| < w_{\overline{X}} \big| \ |Z_{\overline{X}}| < k_{\overline{X}}, \delta_1 = 0) \cdot P_r(|Z_{S^2}| < w_{S^2} \big| \ |Z_{S^2}| < k_{S^2}, \delta_2 = 1)$$
$$= P_r(|Z_{\bar{e}}| < w_{\bar{e}} \big| \ |Z_{\bar{e}}| < k_{\bar{e}}, \delta_3 = 0) \cdot P_r(|Z_{S_e^2}| < w_{S_e^2} \big| \ |Z_{S_e^2}| < k_{S_e^2}, \delta_4 = 1) \quad (2\text{-}7)$$

implying, $w_{\overline{X}} = w_{\bar{e}} = w_1$, $k_{\overline{X}} = k_{\bar{e}} = k_1$, $w_{S^2} = w_{S_e^2} = w_2$, and $k_{S^2} = k_{S_e^2} = k_2$. Hence, $p_{04} = p_{02}$, $p_{05} = p_{03}$, $p_{08} = p_{09} = p_{011} = p_{06}$, $p_{014} = p_{012}$ and $p_{015} = p_{013}$.

If both $w_{\overline{X}} = w_{\bar{e}} =0$, $w_{S^2} = w_{S_e^2} =0$ and $t_1 = t_2 = t_3 = t_4 = t_5 = t_0$, then the joint VSI $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts reduce to the joint $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts with fixed sampling interval (FSI) t_0 .

3. COMPARISON OF CONTROL CHARTS

Sampling schemes should be compared under equal conditions; that is, VSI and FSI schemes should demand the same average sampling interval under the in-control period. That is,

$$E\left[t_m \mid Z_{\bar{X}} \mid < k_1, \mid Z_{S^2} \mid < k_2, \mid Z_{\bar{e}} \mid < k_1 \mid Z_{S_e^2} \mid < k_2, \mid \delta_1 = 0, \delta_2 = 0\right] = t_0 \tag{3-1}$$

Based on the equation (3-1), equation (3-2) can be formulated as follows.

$$t_1\ p_{01} + t_2\,(p_{02} + p_{03} + p_{04} + p_{05}) + t_3\,(p_{06} + p_{07} + p_{08} + p_{09} + p_{10} + p_{011}) + t_4\ p_{04} + t_5\,(1 - p_{01} - p_{02} - p_{03} - p_{04}) = t_0 \tag{3-2}$$

Simplifying,

$$\begin{aligned}
&(\Phi(w_1))^2 [\,(4t_1\text{-}16t_2 + 24t_3\text{-}16t_4 + 4t_5)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_3\text{-}8t_4 + 4t_5)\bullet (F_{\chi^2}(k_2))^2\,] \\
&+ \Phi(w_1)\,[(\text{-}4t_1 + 12t_2\text{-}12t_3 + 4t_4)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (\text{-}8t_2 + 16t_3\text{-}8t_4)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (16t_3\text{-}32t_4 + 16t_5)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (\text{-}4t_3 + 4t_4)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2\,] \\
&+ [(t_1\text{-}2t_2 + t_3)\bullet (F_{\chi^2}(w_2))^2 + (\text{-}4t_2 + 8t_3\text{-}4t_4)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (2t_2\text{-}2t_3)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (\text{-}8t_3 + 8t_4)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (4t_3\text{-}8t_4 + 4t_5)\bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(w_2))^2 + (t_3\text{-}t_0)\bullet (F_{\chi^2}(k_2))^2 \\
&+ (\text{-}4t_4 + 4t_0)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet (\Phi(k_1))^2 \bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (4t_5\text{-}4t_0) * (\Phi(k_1))^2 * (F_{\chi^2}(k_2))^2\,] = 0
\end{aligned} \tag{3-3}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative function, and $F_{x^2}(\cdot)$ denotes the X^2 cumulative function.

The warning limit is derived as follows.

$$w_1 = \Phi^{\text{-}1}\left(\frac{\text{-}b \pm \sqrt{b^2\text{-}4ac}}{2a}\right) \tag{3-4}$$

where

$$a = (4t_1\text{-}16t_2 + 24t_3\text{-}16t_4 + 4t_5)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_3\text{-}8t_4 + 4t_5)\bullet (F_{\chi^2}(k_2))^2$$

$$\begin{aligned}
b = {} & (\text{-}4t_1 + 12t_2\text{-}12t_3 + 4t_4)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 + (\text{-}8t_2 + 16t_3\text{-}8t_4)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
& + (16t_3\text{-}32t_4 + 16t_5)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (\text{-}4t_3 + 4t_4)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2
\end{aligned}$$

$$\begin{aligned}
c = {} & (t_1\text{-}2t_2 + t_3)\bullet (F_{\chi^2}(w_2))^2 + (\text{-}4t_2 + 8t_3\text{-}4t_4)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 + (2t_2\text{-}2t_3)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
& + (\text{-}8t_3 + 8t_4)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_3\text{-}8t_4 + 4t_5)\bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(w_2))^2 + (t_3\text{-}t_0)\bullet (F_{\chi^2}(k_2))^2 \\
& + (\text{-}4t_4 + 4t_0)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet (\Phi(k_1))^2 \bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_5\text{-}4t_0)\bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(k_2))^2
\end{aligned}$$

However, to obtain w_1 and let $0 < w_1 < k_1$, the constraints $\infty > t_1 > t_2 > t_0 > t_3 > t_4 > t_5 > 0$, and $0 < w_2 < k_2$ are required. Thus, the warning limit w_1 can be obtained by using equation (3-4) and choosing a combination of specified five

VSIs, $(t_1, t_2, t_3, t_4, t_5)$, w_2 and the FSI, t_0.

In this paper, the VSI scheme is compared with the FSI scheme and the sampling scheme was considered to be better than another when it allowed the joint $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts to detect changes in the means and variances on step 1 and step 2 faster.

4. PERFORMANCE MEASUREMENT

The speed with which a control chart detects process shifts measures the chart's statistical efficiency. For a VSI, the detection speed is measured by the average time from either mean or variance or both shifting until either $Z_{\overline{X}}$ - Z_{S^2} or $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts or both signal, which is known as the adjusted average time to signal (AATS). That is, the AATS is the mean time that the process remains out of control.

Since $T_{SC} \sim \exp(-\lambda t)$, $t > 0$, the occurrence time until the special cause occurs. Hence,

$$AATS = ATC - \frac{1}{\lambda} \tag{4-1}$$

The average time of the cycle (ATC) is the average time from the start of process until a true signal is obtained from one of the proposed charts (see Costa (1997)). The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 51 states is assigned based on whether the process step is in or out of control and the position of samples (see Table 4.1 for the 51 states of the process). The status of the process when the $(i+1)^{th}$ sample is taken and the position of the i^{th} samples on the joint $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts define the transition states of the Markov chain. The joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts produce a signal when at least one of the samples falls outside the control limits. If the current state is any one of the States 1~48, then there is no signal. If the current state is State 49, it indicates that at least one false signal came from the first process step and that the process is adjusted unnecessarily. State 49 instantly becomes any one of the States 17~32 with a probability of $P_{49=j}$, $\mathrm{j} = 17 \sim 32$. Any one of the States 17~32 thus transits to any other state after a sampling time interval. State 50 is similar to State 49. If the current state is any one of the States 1~50, then it may transit to any other state, hence States 1~50 are transient states. The absorbing state (State 51) is reached when a true signal occurs.

Insert Table 4.1

Let $\mathbf{P}$ be the transition probability matrix, where $\mathbf{P}$ is a square matrix of order 51. Let $P_{i,j}(t_m)$ be the transition probability from prior state i to the current state j with sampling interval t_m, where t_m is determined by the prior state i, $i=1,2,...,51, j=1,2,...,51, m=1,2,3,4,5.$ The transition probability, for example, from state 1 to state 4 with sampling interval t_1 and fixed sample size n is calculated as

$$P_{1,4}(t_1)=P(T_{sc}>t_1)\bullet P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}_1}}, Z_{S^2_{xi}}\in I_{Z_{S^2_{\bar{x}2}}}, Z_{\bar{e}_i}\in I_{Z_{\bar{e}1}}, Z_{S^2_{ei}}\in I_{Z_{S^2_{\bar{e}}1}} \mid \delta_1=\delta_3=0\,,\delta_2=\delta_4=1)$$
$$=\left(e^{-\lambda t_1}\right)(2\Phi(w_1)-1)^2(F_{x^2}(k_2)-F_{x^2}(w_2))(F_{x_2}(w_2))$$

The calculation of all transition probabilities is shown in Appendix.

From the elementary properties of Markov chains (see, e.g., Cinlar (1975)), the ATC is derived as:

$$ATC=\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{t}+\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}M_f+\mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(A\,)T_r$$

(4-2)

where $\mathbf{b}'=(p_{01},p_{02},p_{03},p_{02},p_{03},p_{06},p_{07},p_{06},p_{06},p_{10},p_{06},p_{012},p_{013},p_{012},p_{013},p_{016},0,......0)$ is the vector of starting probabilities for State 1~ State 50, where the first sampling interval has probability p_{01} (see equation (2-6) for calculation) of being the longest (or State 1 with probability p_{01}), the probability $p_{02}+p_{03}+p_{02}+p_{03}$ of being long (or State 2~ State 5 with probability $p_{02}, p_{03}, p_{02}, p_{03}$, respectively) and the probability $p_{06}+p_{07}+p_{06}+p_{06}+p_{10}+p_{06}$ of being median (or State 6~State 11 with probability $p_{06}, p_{07}, p_{06}, p_{06}, p_{10}, p_{06}$, respectively), the probability $p_{012}+p_{013}+p_{012}+p_{013}$ of being short (or State 12 ~ State 15 with probability $p_{12}, p_{13}, p_{12}, p_{13}$, respectively) and the probability p_{016} of being the shortest (or State 16 with probability p_{016}); $\mathbf{I}$ is the identity matrix of order 51; $\mathbf{Q}$ is the transition probability matrix where elements represent the transition probability, $P_{i,j}(t_m)$, from transient state i, i=1,…,50, to transient state j, j=1,…,50, m=1,2,3,4,5; $M'_f=(0,0,\ldots,0,T_f,T_f)$ is the vector of in-correct adjustment time for State1 ~ State50; $\mathbf{t}'=(t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t^*_{49},t^*_{50})$ is the vector of the VSIs for state1~state 50, where t^*_{49} is the average time of sampling interval for State 49, and t^*_{50} is the average time of sampling interval for State 50, the calculation of t^*_{49} and t^*_{50} is shown in Appendix; A is the vector of transition probability,

$P_{i,51}(t_m)$, form transient state i, i=1,…,50, to absorbing state 51, m=1,2,3,4,5; T_r is the time to adjust any process step correctly.

5. AN EXAMPLE

An example of process control for automobile braking system is presented, and the data of the process are measurements of roll weight and bake weight. Let variables X=roll weight and Y= bake weight be measured from the end of the second process step. The bake weight produced in the second step is influenced by the roll weight produced in the first step. Thirty-five samples of size two (X, Y) are taken from Wade and Woodall (1993). A machine is used in the process steps. The machine could only fail in the first process step and affects the X-variable with probability 0.75 (v=0.75), or only fail in the second process step and affects the functional relationship (or e-variable) with probability 0.25. The failure rate of the machine is 0.1 time per hour (or $\lambda = 0.1$). Presently, the joint FSI $Z_{\overline{X}}$ - Z_{S^2} or $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are used to monitor the shifts in means and variances on the two process steps every hour. Information about the state of the process steps is available only through sampling. When the control charts indicate that at least one of the process steps is out of control, it requires adjustment. Sometimes, the process steps may be adjusted unnecessarily when at least one false signal occurs. The average incorrect adjustment time of the process step is 1.0 hours ($T_f = 1.0$), and The average correct adjustment time of the process step is 2.0 hours (or $T_r = 2.0$). The FSI $Z_{\overline{X}}$ and $Z_{\bar{e}}$ charts have control limits placed at ± 3 with a false signal rate of 0.0027 and the Z_{S^2} and $Z_{S_e^2}$ charts have control limits placed at 9 with false signal rate 0.0027, respectively. Thus, approximately 10.8 false alarms are expected per 1,000 samples and have an in-control average run length (ARL) of 92.59 hours if $T_f = 0$ and $T_r = 0$. The calculated AATS of FSI charts is 2.1 hours using equation (4-2). The slowness with which the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts detect shifts in the out-of-control process steps has led the quality manager to propose building the $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with specified VSIs, $(t_5, t_4, t_3, t_2, t_1)$=(0.001,0.002,0.01,0.02,1.1).

The first 25 samples are used to establish the reference line of Y and X and then construct the $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. After delete the out-of-control sample no. 3, the fitted regression model is

$$\hat{Y}_{ij} = a_0 + a_1 X_{ij} = 82.82 + 0.56229 X_{ij}.$$

Thus, the residuals (e_{ij}) are obtained by $Y_{ij} - \hat{Y}_{ij}$. The estimated in-control distributions of $\overline{X}$ and $\overline{e}$ are $\overline{X} \sim N\left(210.125, \frac{1.3525^2}{2}\right)$ and $\overline{e} \sim N\left(0, \frac{0.7873^2}{2}\right)$.

The estimated out-of-control distributions of $\overline{X}$ and $\overline{e}$ are $\overline{X} \sim N\left(211.2, \frac{2.9746^2}{2}\right)$ (or $\delta_1 = 1.1241$ and $\delta_2 = 2.1993$) and $\overline{e} \sim N\left(0.7868, \frac{2.1911^2}{2}\right)$ (or $\delta_3 = 1.4133$ and $\delta_4 = 2.7831$).

The construction and application of the proposed VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is illustrated. The following are the guidelines for using the proposed charts:

Step 1: Let the factors of control limits $k_1 = 3$ and $k_2 = 9.0$, in order to maintain the average false signal rate at around 10.8 per 1000 samples. The reciprocal of 10.8 false signals is also the ARL, but for the in-control case of $\delta_1 = \delta_3 = 0$ and $\delta_2 = \delta_4 = 1$.

Step 2: The average incorrect adjustment time of the process steps is 1.0 hours ($T_f = 1.0$). The average correct adjustment time of the process steps is 2.0 hours (or $T_r = 2.0$).

Step 3: For out-of-control step1, the estimated shift scales are $\delta_1 = 1.1241$ and $\delta_2 = 2.1993$; for out-of-control step2, the estimated shift scales are $\delta_3 = 1.4133$ and $\delta_4 = 2.7831$.

Step 4: Since $0 < t_5 < t_4 < t_3 < t_2 < t_1 < \infty$ is required, and for performance of process control we adopt the combination (t_5, t_4, t_3, t_2, t_1)=(0.001,0.002,0.01,0.02,1.1).

Step 5: Letting t_5 =0.001, t_4 =0.002, t_3 =0.01, t_2 =0.02, t_1 = 1.1, t_0 =1, $k_1 = 3$ and $k_2 = 9.0$ in the equation (3-4) leads to w_1 =1.5 and w_2 =5.5.

Step 6: From equations (4-1) and (4-2), the AATS of the VSI charts is 1.17h. Compared to FSI charts, the VSI charts save detection time 44.29%.

The structures of the proposed VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are in Figure 4.1.

$\text{UCL}_{z_{\overline{X}}} = k_1 = 3$ $\quad$ $\text{U CL}_{z_{\bar{e}}} = k_1 = 3$

$UWL_{z_{\overline{X}}} = w_1 = 1.26$ $\quad$ $\text{UWL}_{z_{\bar{e}}} = w_1 = 1.26$

$\text{CL}_{z_{\overline{X}}} = 0$ $\quad$ $\text{CL}_{z_{\bar{e}}} = 0$

$\text{LWL}_{z_{\overline{X}}} = -w_1 = \text{-}1.26$ $\quad$ $\text{LWL}_{z_{\bar{e}}} = -w_1 = -1.26$

$\text{LCL}_{z_{\overline{X}}} = -k_{\overline{X}} = \text{-}3$ $\quad$ $\text{LCL}_{z_{\bar{e}}} = -k_{\bar{e}} = -3$

(1) $Z_{\overline{X}}$ chart (2) $Z_{\bar{e}}$ chart

$\mathrm{UCL}_{z_{s^2}} = k_2 = 9$ $\mathrm{UCL}_{z_{s_e^2}} = k_2 = 9$

$UWL_{z_{s^2}} = w_2 = 4.5$ $\mathrm{UWL}_{z_{s_e^2}} = w_2 = 4.5$

$\mathrm{LCL}_{z_{s^2}} = 0$ $\mathrm{LCL}_{z_{s_e^2}} = 0$

(3) Z_{S^2} chart (4) $Z_{S_e^2}$ chart

Figure 5.1 the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control limits

With the design parameters determined, the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts can be used for controlling the process of automobile braking system with incorrect adjustment. According to the VSI scheme, the first sampling interval is determined by random and the other sampling intervals are determined by the position of samples. An example using the VSI charts is introduced. When the process starts, a random procedure decides the first sampling interval t_1=1.1 hours with sample of size two. The first sample with means and variances is $\left(Z_{\bar{x}_1} = 0.461, Z_{s^2} = 0.2765, Z_{\bar{e}_1} = -2.1259, Z_{s_e^2} = 0.2550\right)$. Since three of the plotted points fall within the central regions but the other falls within the warning regions, the second sample will be observed adopting a sample of size two after t_2=0.02h. The second sample is ($z_{\overline{X}}$=-2.1691, z_{S^2} =0.00, $z_{\bar{e}}$=-0.4990, $z_{S_e^2}$ =0.8066). Since three of the plotted points fall within the central regions but the other falls within the warning regions, the third sample (sample no. 4) will be observed adopting a sample of size two after t_2 =0.02h. The third sample is ($z_{\overline{X}}$ =0.9860, z_{S^2} =0.00, $z_{\bar{e}}$ =0.0635, $z_{S_e^2}$ =0.8066). Since all plotted points fall within central regions, the fourth sample will be observed after t_1=1.1h. The process continues until at least one signal is obtained and the process is stopped and adjusted.

The constructed VSI charts are used to monitor the samples from number 26-35 (see Figures 5-1).

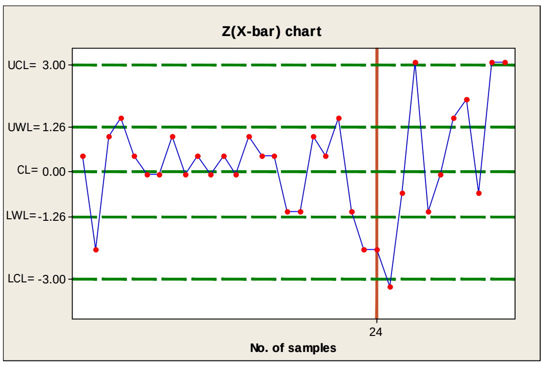

Fig. 5-1-a

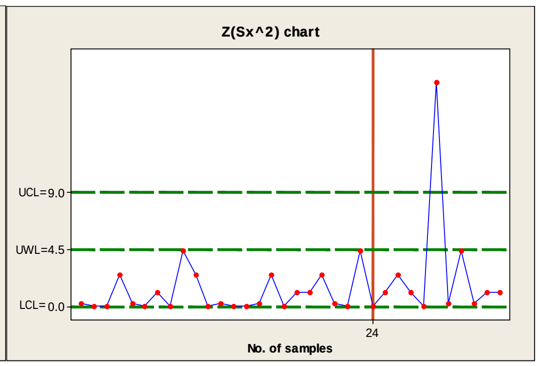

Fig. 5-1-b

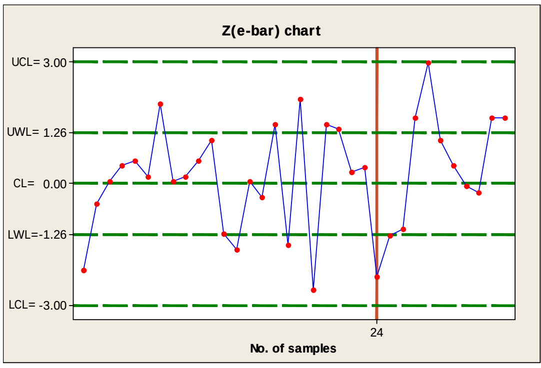

Fig. 5-1-a

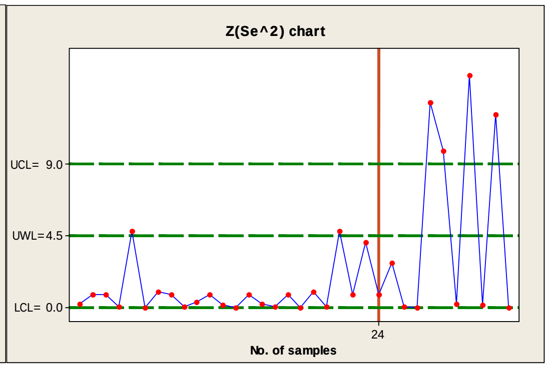

Fig. 5-1-b

We find outliers, sample no. 26, 28, 34 and 35, occur on $Z_{\bar{x}}$ chart; outlier, sample no. 30, occurs on $Z_{S_x^2}$ chart; no outlier occurs on $Z_{\bar{e}}$ chart; outliers, sample no. 29, 30, 32, and 34, occur on $Z_{S_e^2}$ chart.

To compare the performance between the VSI and FSI control charts in the example, we consider nine combinations of v, λ, T_f and T_r using orthogonal array $L_9(3^2)$. Table 5.1 shows the AATSs of VSI and FSI charts. The detection time of the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts has been reduced from 13.64% to 80%. The VSI scheme improves the sensitivity of the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts. Hence, the proposed VSI control charts outperform FSI control charts.

Insert Table 5.1

Furthermore, sometimes engineers cannot specify the VSIs. The optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are thus suggested. The optimal VSI of the proposed charts are determined using optimization technique, Quasi-Newton approach in Fortran IMSL BCONF subroutine, to minimum AATS under the constraints $0 < w_1 < 3$, $0 < w_2 < 9$, $0<t_5<t_4<t_3<t_2<1<t_1<2$ and nine combinations of v, λ, T_f and T_r as described in Table 5.1. The optimum VSIs and minimal AATS are illustrated in Table 5.2. We found that the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts all work better than the $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts with specified VSIs. Compare to the FSI control charts, the optimal VSI charts may save the detection time from 17.53% to 90.91%. Consequently, the performance of the optimal VSI charts is better than the specified VSI charts, and the specified VSI charts outperform the FSI charts.

From Table 5.2, we find that the optimal VSIs are very robust in the 9 combinations of parameters. The robust optimal variable sampling intervals are $t_5^* = 0.002$, $t_4^* = 0.002$, $t_3^* = 0.019$, $t_2^* = 0.020$ and $t_1^* = 1.000$. Technically, we may let $t_5^* = t_4^* = 0.002$ and $t_3^* = t_2^* = 0.020$. Consequently, the five VSIs may be reduced to three VSIs, $t_5^* = t_4^* = 0.002$, $t_3^* = t_2^* = 0.020$ and $t_1^* = 1.000$, for simplifying the proposed optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and giving better performance. An application of using the robust $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with three VSIs is introduced. If all plotted points fall within central regions then the longest sampling interval, $t_1^* = 1.000$, is adopted. If any two or three plotted points fall within central regions and the others fall within warning regions then the medium sampling interval, $t_3^* = t_2^* = 0.020$, is adopted. If no or one plotted point falls within the central region and the others fall within warning regions, then the next sample should be taken instantly ($t_5^* = t_4^* = 0.002$).

Insert Table 5.2

To examine the effects of detecting small and median shifts ($0.5 \le \delta_1 \le 1.5, 1.5 \le \delta_2 \le 2.0$, $0.5 \le \delta_3 \le 1.5$ and $1.5 \le \delta_4 \le 2.0$) in process means and variances for the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. Table 5.3 provides the AATS of the optimum VSI and FSI schemes, which are obtained under the constraints 0 < w_1 < 3, 0 < w_2 < 9, 0<t_5<t_4<t_3<t_2<1<t_1<2 and sixteen combinations of v, λ, T_f, T_r, δ_1, δ_2, δ_3 and δ_4 based on orthogonal array $L_{16}(2^{15})$ table. The two levels of these parameters are v=0.1,0.75, $\lambda = 0.5, 1.0$, $\delta_1 = 0.5, 1.5$, $\delta_2 = 1.5, 2.0$, $\delta_3 = 0.5,\ 1.5$, $\delta_4 = 1.5,\ 2.0$, $t_0 = 1.0$, $t_5 = 0.001,\ 0.0035$, $t_4 = 0.004, 0.006$, $t_3 = 0.0065,\ 0.001$, $t_2 = 0.16,\ 0.18$, $t_1 = 1.5,\ 2.0$, $(T_f, T_r) = (0.5,1.0), (1.5,3.0)$, and $n = 5$.

When comparing the AATS between the FSI and the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts it can be seen that the performance of the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is better for detecting small and median shifts in process means and variances. The optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts save detection time from 17.40% to 51.23% (in the cases examined) compared to the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts.

Insert Table 5.3

From Table 5.3, we find that the optimal t_5^*, t_4^* and t_3^* are very small. Technically, we may adopt $t_5^* = t_4^* = t_3^* = 0.01$, $t_2^* = 0.13$ and $t_1^* = 1.35$. Consequently, the five VSIs may be reduced to three VSIs for simplifying the proposed optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and giving better performance.

From the results of data analyses in Table 5.2 and Table 5.3, we conclude that the five VSIs may reduce to three VSIs in reality.

6. MISUSING $\overline{Y}$ AND S_y^2 CONTROL CHARTS

In many real situations, engineers may misuse $\overline{Y}$ and S_y^2 control charts to monitor mean and variance in the seconds step. Figure 6.1 shows the monitoring results of using $\overline{X}$ - S_x^2 and $\overline{Y}$ - S_y^2 control charts. On the second step, there are four outliers, sample no.26, 28, 34 and 35, occur on the $\overline{Y}$ chart, and one outlier, sample no.34, occurs on S_y^2 chart. Compare to $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts, it indicates that misusing $\overline{Y}$ and S_y^2 control charts will lead to unnecessarily adjust the mean and underadjust the variance on the second step.

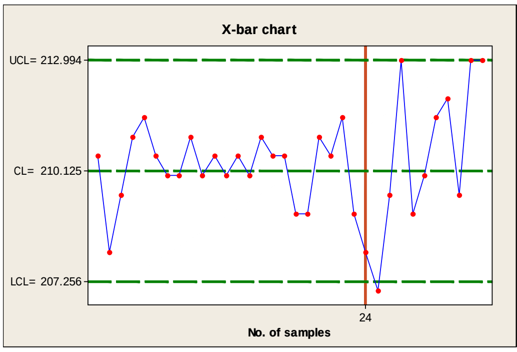

Fig. 6-1-a

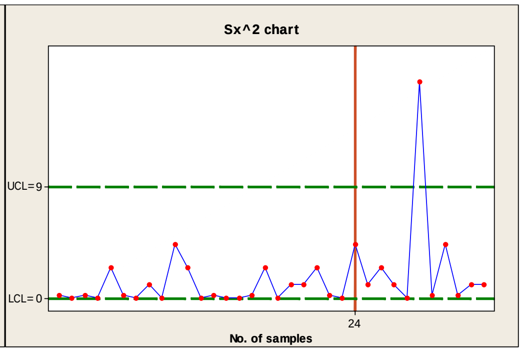

Fig. 6-1-b

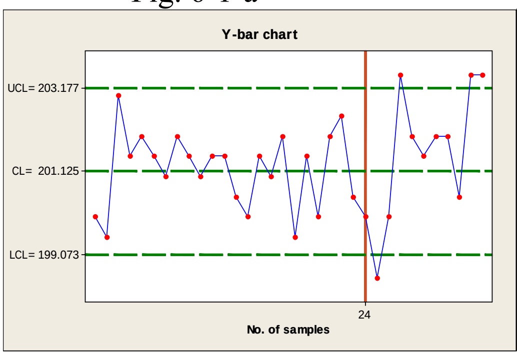

Fig. 6-1-c

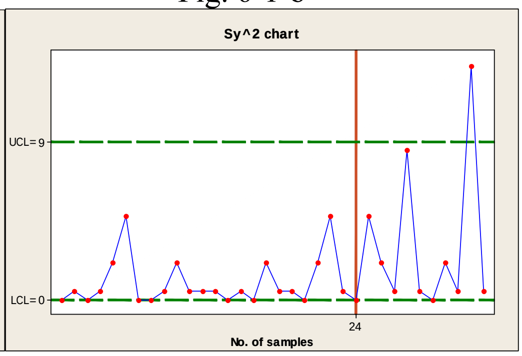

Fig. 6-1-d

7. CONCLUSIONS

The proposed VSI scheme, which controls the means and variances in two dependent process steps with incorrect adjustment, substantially improves the performance of the FSI scheme by increasing the speed with which shifts in the means and variances of process steps are detected. We have found that the VSI $Z_{\overline{X}} - Z_{S^2}$ and $Z_{\bar{e}} - Z_{S_e^2}$ control charts always work better (in the cases examined) than the FSI $Z_{\overline{X}} - Z_{S^2}$ and $Z_{\bar{e}} - Z_{S_e^2}$ control charts. Furthermore, the performance of the optimum VSI $Z_{\overline{X}} - Z_{S^2}$ and $Z_{\bar{e}} - Z_{S_e^2}$ control charts outperforms the $Z_{\overline{X}} - Z_{S^2}$ and $Z_{\bar{e}} - Z_{S_e^2}$ control charts with specified VSIs, and is thus recommended when quality engineers cannot specify the VSIs. The effects of misusing control charts are also investigated.

This paper considered two dependent process steps which had incorrect adjustment. However, a study of the variable sample size (VSS), variable sample size and sampling interval (VSSI) or variable parameters (VP) $Z_{\overline{X}} - Z_{S^2}$ and $Z_{\bar{e}} - Z_{S_e^2}$ control charts under multiple dependent process steps with incorrect adjustment is an interesting topic for future research. Other important extensions of the proposed model can also be developed. It is straight forward to extend the proposed model to study VP control charts for attribute data, EWMA-charts, CUSUM-charts or multivariate charts.

Table 4.1 Definition of 51 process states

state	SC occurs?	which step?	the position	the position	the position	the position	which step over-adjustment?

			of $Z_{\bar{x}_i}$	of $Z_{S^2_{xi}}$	of $Z_{\bar{e}_i}$	of $Z_{S^2_{ei}}$	
1	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
2	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
3	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
4	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
5	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
6	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
7	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
8	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
9	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
10	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
11	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
12	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
13	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
14	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
15	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
16	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$\mathrm{I}_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
17	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
18	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
19	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
20	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
21	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
22	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
23	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
24	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
25	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No
26	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e1}}$	No
27	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e1}}$	No
28	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
29	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S^2_e2}}$	No
30	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S^2_x2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S^2_e2}}$	No

31	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
32	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
33	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
34	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
35	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
36	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
37	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
38	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
39	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
40	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
41	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
42	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
43	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
44	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
45	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
46	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
47	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
48	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
49	false signal comes from $Z_{\bar{X}}$ and/or Z_{S^2} charts , but no signal comes from the $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts						first step overadjustment
50	no signal comes from the $Z_{\bar{X}}$ - Z_{S^2} charts , but false signal comes from the $Z_{\bar{e}}$ and/or $Z_{S_e^2}$ charts						second step overadjustment
51	true signal comes from $Z_{\bar{X}}$ - Z_{S^2} charts and/or $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts						Correct adjustment

Table 5.1. The AATS of VSI and FSI charts under various combinations of parameters

L_9 table	**combination of parameters**			AATS		*save time (%)
No.	v	λ	(T_f, T_r)	VSI	FSI	
1	0.25	0.25	(0.5,1.0)	1.45	1.91	24.08

L

2	0.25	0.50	(1.0,2.0)	2.66	3.08	13.64
3	0.25	0.10	(0.5,1.5)	0.64	1.52	57.89
4	0.50	0.25	(1.0,2.0)	2.43	2.93	17.06
5	0.50	0.50	(0.5,1.5)	2.18	2.62	16.79
6	0.50	0.10	(0.5,1.0)	0.22	1.10	80.00
7	0.75	0.25	(0.5,1.5)	1.95	2.49	21.69
8	0.75	0.50	(0.5,1.0)	1.72	2.21	22.17
9	0.75	0.10	(1.0,2.0)	1.17	2.10	44.29

*save time(%)=[(FSI AATS-VSI AATS)/(FSI AATS)]%

Table 5.2 AATS of the optimal VSI and FSI charts under various combinations of parameters

L_9 table	**combination of other parameters**			optimum solution		AATS		saved time (%)
No.	v	λ	(T_f, T_r)	$(t_5^*, t_4^*, t_3^*, t_2^*, t_1^*)$	(w_1^*, w_2^*)	VSI*	FSI	
1	0.25	0.25	(0.5,1.0)	(0.002,0.002,0.019,0.020,1.001)	(1.15,4.50)	1.30	1.91	31.94
2	0.25	0.50	(1.0,2.0)	(0.001,0.002,0.019,0.020,1.001)	(1.23,3.50)	2.54	3.08	17.53
3	0.25	0.10	(0.5,1.5)	(0.002,0.002,0.019,0.020,1.001)	(1.09,6.79)	0.25	1.52	83.55
4	0.50	0.25	(1.0,2.0)	(0.002,0.002,0.019,0.050,1.001)	(0.73,7.00)	1.89	2.93	35.49
5	0.50	0.50	(0.5,1.5)	(0.001,0.002,0.019,0.020,1.001)	(1.20,3.80)	2.06	2.62	21.37
6	0.50	0.10	(0.5,1.0)	(0.002,0.002,0.019,0.020,1.001)	(1.09,6.75)	0.10	1.10	90.91
7	0.75	0.25	(0.5,1.5)	(0.002,0.002,0.019,0.020,1.001)	(1.15,4.60)	1.79	2.49	28.11
8	0.75	0.50	(0.5,1.0)	(0.001,0.002,0.019,0.050,1.001)	(0.73,6.70)	1.39	2.21	37.10
9	0.75	0.10	(1.0,2.0)	(0.002,0.002,0.019,0.020,1.001)	(1.10,6.63)	0.78	2.10	62.86

*constraints 0 < w1 < 3 , 2 < w2 < 9, 0<t5<t4<t3<t2<1<t1<2. *save time(%)=[(FSI AATS-VSI AATS)/(FSI AATS)]%

Table 5.3 AATS of the optimum VSI and FSI charts under various combinations of parameters

combination of various parameters								VSI								FSI	save time
No.	v	δ_1	δ_2	δ_3	δ_4	λ	(T_f, T_r)	w_1^*	w_2^*	t_5^*	t_4^*	t_3^*	t_2^*	t_1^*	AATS	AATS	percentage
1	0.10	0.5	1.5	0.5	1.5	0.5	(0.5, 1.0)	0.5190	5.651	0.002	0.002	0.010	0.142	1.491	1.29	2.62	50.76
2	0.10	0.5	2.0	1.5	2.0	1.0	(1.5, 3.0)	0.5564	5.326	0.002	0.010	0.010	0.153	1.286	3.30	4.18	21.05
3	0.10	1.5	1.5	0.5	1.5	0.5	(1.5, 3.0)	0.5299	5.801	0.002	0.002	0.010	0.135	1.493	3.21	4.58	29.91
4	0.10	1.5	2.0	1.5	2.0	1.0	(0.5, 1.0)	0.6505	5.370	0.002	0.010	0.010	0.136	1.287	1.44	2.17	33.64
5	0.10	1.5	1.5	0.5	2.0	1.0	(0.5, 1.0)	0.6542	5.414	0.002	0.010	0.010	0.133	1.308	1.47	2.30	36.09
6	0.10	1.5	2.0	1.5	1.5	0.5	(1.5, 3.0)	0.5038	6.471	0.002	0.002	0.010	0.124	1.294	3.08	4.42	30.32
7	0.10	0.5	1.5	0.5	2.0	1.0	(1.5, 3.0)	0.6516	5.384	0.002	0.010	0.010	0.135	1.289	3.40	4.31	21.11
8	0.10	0.5	2.0	1.5	1.5	0.5	(0.5, 1.0)	0.5063	6.301	0.002	0.002	0.010	0.128	1.293	1.19	2.44	51.23
9	0.75	1.5	1.5	1.5	1.5	1.0	(0.5, 1.0)	0.5392	6.118	0.002	0.002	0.010	0.126	1.341	1.46	2.62	44.27
10	0.75	1.5	2.0	0.5	2.0	0.5	(1.5, 3.0)	0.5380	5.871	0.002	0.002	0.010	0.134	1.350	3.10	4.02	22.89
11	0.75	0.5	1.5	1.5	1.5	1.0	(1.5, 3.0)	0.7330	6.381	0.001	0.010	0.010	0.094	1.276	3.58	4.78	25.10
12	0.75	0.5	2.0	0.5	2.0	0.5	(0.5, 1.0)	0.5006	5.881	0.002	0.002	0.010	0.140	1.319	1.15	2.09	44.98
13	0.75	0.5	1.5	1.5	2.0	0.5	(0.5, 1.0)	0.5292	5.901	0.002	0.002	0.010	0.135	1.322	1.22	2.43	49.79
14	0.75	0.5	2.0	0.5	1.5	1.0	(1.5, 3.0)	0.7354	5.231	0.010	0.010	0.010	0.124	1.458	3.56	4.31	17.40
15	0.75	1.5	1.5	1.5	2.0	0.5	(1.5, 3.0)	0.5418	6.201	0.002	0.002	0.010	0.124	1.324	3.13	4.31	27.38
16	0.75	1.5	2.0	0.5	1.5	1.0	(0.5, 1.0)	0.5554	5.701	0.001	0.010	0.010	0.139	1.218	1.37	2.25	39.11

*constraints 0 < w1 < 3 , 0 < w2 <16.25, 0<t5<t4<t3<t2<1<t1<2. *save time(%)=[(FSI AATS-VSI AATS)/(FSI AATS)]%

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Appendix: The calculation of all transition probabilities

Notation:

$$\beta_{Z_{\bar{X}1}}=P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \mid \bar{X}_i \sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))= P(-w_1 < Z_{\bar{x}_i} < w_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))$$

$$=P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2} < \frac{w_1-\delta_1}{\delta_2})-P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2} < \frac{-w_1-\delta_1}{\delta_2})= \Phi\left(\frac{w_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{w_1+\delta_1}{\delta_2}\right)-1$$

$$\beta_{Z_{\bar{X}2}}=P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \mid \bar{X}_i \sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))= P(-k_1 < Z_{\bar{x}_i} < -w_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))+P(w_1 < Z_{\bar{x}_i} < k_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))$$

$$=P(\left|Z_{\bar{x}_i}\right| < k_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))-P(\left|Z_{\bar{x}_i}\right| < w_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))=\Phi\left(\frac{k_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{k_1+\delta_1}{\delta_2}\right)-\Phi\left(\frac{w_1-\delta_1}{\delta_2}\right)-\Phi\left(\frac{w_1+\delta_1}{\delta_2}\right)$$

$$\beta_{Z_{\bar{X}3}}=P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}3}} \mid \bar{X}_i \sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))= P(-k_1 < Z_{\bar{x}_i} < k_1 \mid Z_{\bar{x}_i} \sim N(\delta_1,\delta_2^2))$$

$$=P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2} < \frac{k_1-\delta_1}{\delta_2})-P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2} < \frac{-k_1-\delta_1}{\delta_2})= \Phi\left(\frac{k_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{k_1+\delta_1}{\delta_2}\right)-1$$

$$\beta_{Z_{S_x^2 1}}=P(Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \mid \bar{X}_i \sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))= P(0 < Z_{S_{xi}^2} < w_2 \mid Z_{S_{xi}^2} \sim \delta_2^2\chi^2(n-1))= F_{\chi^2}\left(\frac{w_2}{\delta_2^2}\right)$$

$$\beta_{Z_{S_x^2 2}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 2}}\ \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(w_2<Z_{S_{xi}^2}<k_2\ \mid\ Z_{S_{xi}^2}\sim\delta_2^2\chi^2(n\text{-}1))=F_{\chi^2}\left(\frac{k_2}{\delta_2^2}\right)\text{-}F_{\chi^2}\left(\frac{w_2}{\delta_2^2}\right)$$

$$\beta_{Z_{S_x^2 3}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}}\ \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(0<Z_{S_{xi}^2}<k_2\ \mid\ Z_{S_{xi}^2}\sim\delta_2^2\chi^2(n\text{-}1))=F_{\chi^2}\left(\frac{k_2}{\delta_2^2}\right)$$

$$\beta_{Z_{\bar{e}1}}=P(Z_{\bar{e}_i}\in \mathrm{I}_{Z_{\bar{e}1}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(\text{-}w_1<Z_{\bar{e}_i}<w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\frac{Z_{\bar{e}_i}\text{-}\delta_3}{\delta_4}<\frac{w_1\text{-}\delta_3}{\delta_4})\text{-}P(\frac{Z_{\bar{e}_i}\text{-}\delta_3}{\delta_4}<\frac{\text{-}w_1\text{-}\delta_3}{\delta_4})=\Phi\left(\frac{w_1\text{-}\delta_3}{\delta_4}\right)+\Phi\left(\frac{w_1+\delta_3}{\delta_4}\right)\text{-}1$$

$$\beta_{Z_{\bar{e}2}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}2}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(\text{-}k_1<Z_{\bar{e}_i}<\text{-}w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))+P(w_1<Z_{\bar{e}_i}<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\left|Z_{\bar{e}_i}\right|<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))\text{-}P(\left|Z_{\bar{e}_i}\right|<w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))=\Phi\left(\frac{k_1\text{-}\delta_3}{\delta_4}\right)+\Phi\left(\frac{k_1+\delta_3}{\delta_4}\right)\text{-}\Phi\left(\frac{w_1\text{-}\delta_3}{\delta_4}\right)\text{-}\Phi\left(\frac{w_1+\delta_3}{\delta_4}\right)$$

$$\beta_{Z_{\bar{e}3}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(\text{-}k_1<Z_{\bar{e}_i}<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\frac{Z_{\bar{e}_i}\text{-}\delta_3}{\delta_4}<\frac{k_1\text{-}\delta_3}{\delta_4})\text{-}P(\frac{Z_{\bar{e}_i}\text{-}\delta_3}{\delta_4}<\frac{\text{-}k_1\text{-}\delta_3}{\delta_4})=\Phi\left(\frac{k_1\text{-}\delta_3}{\delta_4}\right)+\Phi\left(\frac{k_1+\delta_3}{\delta_4}\right)\text{-}1$$

$$\beta_{Z_{S_e^2 1}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 1}}\ \mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(0<Z_{S_{ei}^2}<w_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n\text{-}1))=F_{\chi^2}\left(\frac{w_2}{\delta_4^2}\right)$$

$$\beta_{Z_{S_e^2 2}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 2}}\ \mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(w_2<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n\text{-}1))=F_{\chi^2}\left(\frac{k_2}{\delta_4^2}\right)\text{-}F_{\chi^2}\left(\frac{w_2}{\delta_4^2}\right)$$

$$\beta_{Z_{S_e^2 3}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 3}}\ \mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(0<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n\text{-}1))=F_{\chi^2}\left(\frac{k_2}{\delta_4^2}\right)$$

$$\gamma_{Z_{\bar{X}1}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}1}}\mid Z_{\bar{x}_i}\sim N(0,1))=P(\text{-}w_1<Z_{\bar{x}_i}<w_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(w_1)\text{-}1$$

$$\gamma_{Z_{\bar{X}2}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}2}}\ \mid Z_{\bar{x}_i}\sim N(0,1))=P(\text{-}k_1<Z_{\bar{x}_i}<\text{-}w_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))$$

$$+P(w_1<Z_{\bar{x}_i}<k_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(k_1)\text{-}2\Phi(w_1)$$

$$\gamma_{Z_{\bar{X}3}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}}\mid Z_{\bar{x}_i}\sim N(0,1))=P(\text{-}k_1<Z_{\bar{x}_i}<k_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(k_1)\text{-}1$$

$$\gamma_{Z_{S_x^2 1}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 1}}\mid Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=P(0<Z_{S_{xi}^2}<w_2\mid Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_x^2 2}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 2}}\ \mid\ Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=P(w_2<Z_{S_{xi}^2}<k_2\ \mid\ Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_x^2 3}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}}\ \mid\ Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=P(0<Z_{S_{xi}^2}<k_2\ \mid\ Z_{S_{xi}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(k_2)$$

$$\gamma_{Z_{\bar{e}1}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}1}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(\text{-}w_1<Z_{\bar{e}_i}<w_1\mid\ Z_{\bar{e}_i}\sim N(0,1))=2\Phi(w_1)\text{-}1$$

$$\gamma_{Z_{\bar{e}2}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}2}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(\text{-}k_1<Z_{\bar{e}_i}<\text{-}w_1\ \mid Z_{\bar{e}_i}\sim N(0,1))$$

$$+P(w_1<Z_{\bar{e}_i}<k_1\mid Z_{\bar{e}_i}\sim N(0,1))=2\Phi(k_1)\text{-}2\Phi(w_1)$$

$$\gamma_{Z_{\bar{e}3}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(\text{-}k_1<Z_{\bar{e}_i}<k_1\mid\ Z_{\bar{e}_i}\sim N(0,1))=2\Phi(k_1)\text{-}1$$

$$\gamma_{Z_{S_e^2 1}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 1}}\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=P(0<Z_{S_{ei}^2}<w_2\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_e^2 2}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 2}}\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=P(w_2<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_e^2 3}}=P(Z_{S_{ei}^2}\in I_{Z_{S_e^2 3}}\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=P(0<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\chi^2(n\text{-}1))=F_{\chi^2}(k_2)$$

The transition probability can be expressed by the following general form.

$$P_{i,j}(t_m) = P(Tsc > t_m)*P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}v_1}}, Z_{S^2_{xi}} \in I_{Z_{S^2_x v_2}}, Z_{\bar{e}_i} \in I_{Z_{\bar{e}v_3}}, Z_{S^2_{ei}} \in I_{Z_{S^2_e v_4}} | \bar{X}_i \sim N\left(\mu, \frac{\sigma_x^2}{n}\right), \bar{e}_i \sim N\left(0, \frac{\sigma_e^2}{n}\right))$$

$$= \left(e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S^2_x v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S^2_e v_4}}$$

$$\begin{aligned}
\text{where } & i = 1 && ;m = 1;j = 1,2,\dots,16;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 2,3,4,5 && ;m = 2;j = 1,2,\dots,16;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 6,7,\dots,11 && ;m = 3;j = 1,2,\dots,16;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 12,13,14,15 && ;m = 4;j = 1,2,\dots,16;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 16 && ;m = 5;j = 1,2,\dots,16;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2
\end{aligned}$$

$$P_{i,j}(t_m) = P(Tsc < t_m)*v*P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}v_1}}, Z_{S^2_{xi}} \in I_{Z_{S^2_x v_2}}, Z_{\bar{e}_i} \in I_{Z_{\bar{e}v_3}}, Z_{S^2_{ei}} \in I_{Z_{S^2_e v_4}} |$$

$$\bar{X}_i \sim N\left(\mu + \delta_1 \frac{\sigma_x}{\sqrt{n}}, \delta_2^2 \frac{\sigma_x^2}{n}\right), \bar{e}_i \sim N\left(0, \frac{\sigma_e^2}{n}\right))$$

$$= v*\left(1 - e^{-\lambda t_m}\right)*\beta_{Z_{\bar{X}v_1}}*\beta_{Z_{S^2_x v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S^2_e v_4}}$$

$$\begin{aligned}
\text{where } & i = 1 && ;m = 1;j = 17,18,\dots,32;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 2,3,4,5 && ;m = 2;j = 17,18,\dots,32;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 6,7,\dots,11 && ;m = 3;j = 17,18,\dots,32;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 12,13,14,15 && ;m = 4;j = 17,18,\dots,32;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 16 && ;m = 5;j = 17,18,\dots,32;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2
\end{aligned}$$

$$P_{i,j}(t_m) = P(Tsc < t_m)*(1-v)*P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}v_1}}, Z_{S^2_{xi}} \in I_{Z_{S^2_x v_2}}, Z_{\bar{e}_i} \in I_{Z_{\bar{e}v_3}}, Z_{S^2_{ei}} \in I_{Z_{S^2_e v_4}} |$$

$$\bar{X}_i \sim N\left(\mu, \frac{\sigma_x^2}{n}\right), \bar{e}_i \sim N\left(\delta_3 \frac{\sigma_e}{\sqrt{n}}, \delta_4^2 \frac{\sigma_e^2}{n}\right))$$

$$= (1-v)*\left(1 - e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S^2_x v_2}}*\beta_{Z_{\bar{e}v_3}}*\beta_{Z_{S^2_e v_4}}$$

$$\begin{aligned}
\text{where } & i = 1 && ;m = 1;j = 33,34,\dots,48;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 2,3,4,5 && ;m = 2;j = 33,34,\dots,48;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 6,7,\dots,11 && ;m = 3;j = 33,34,\dots,48;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 12,13,14,15 && ;m = 4;j = 33,34,\dots,48;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2 \\
& i = 16 && ;m = 5;j = 33,34,\dots,48;v_1 = 1,2;v_2 = 1,2;v_3 = 1,2;v_4 = 1,2
\end{aligned}$$

$$P_{i,49}(t_m) = P(Tsc > t_m)*P[(Z_{\bar{x}_i} \in I_{Z_{\bar{X}4}} \cup Z_{S^2_{xi}} \in I_{Z_{S^2_x 4}}) \cap (Z_{\bar{e}_i} \in I_{Z_{\bar{e}3}} \cap Z_{S^2_{ei}} \in I_{Z_{S^2_e 3}}) | \bar{X}_i \sim N\left(\mu, \frac{\sigma_x^2}{n}\right), \bar{e}_i \sim N\left(0, \frac{\sigma_e^2}{n}\right))$$

$$= \left(e^{-\lambda t_m}\right)*\left[1 - \left(\gamma_{Z_{\bar{X}3}}*\gamma_{Z_{S^2_{x3}}}\right)\right]*\gamma_{Z_{\bar{e}3}}*\gamma_{Z_{S^2_e 3}}$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1\\
& i=2,3,4,5 && ;m=2\\
& i=6,7,\ldots,11 && ;m=3\\
& i=12,13,14,15 && ;m=4\\
& i=16 && ;m=5
\end{aligned}
$$

$$
P_{i,50}(t_m)=P(Tsc>t_m)*P[(Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}}\cap Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}})\cap(Z_{\bar{e}_i}\in I_{Z_{\bar{e}4}}\cup Z_{S_{ei}^2}\in I_{Z_{S_e^2 4}})\ |\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))
$$

$$
=\left(e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}3}}*\gamma_{Z_{S_{x3}^2}}*\left(1-\left(\gamma_{Z_{\bar{e}_3}}*\gamma_{Z_{S_e^2 3}}\right)\right)
$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1\\
& i=2,3,4,5 && ;m=2\\
& i=6,7,\ldots,11 && ;m=3\\
& i=12,13,14,15 && ;m=4\\
& i=16 && ;m=5
\end{aligned}
$$

$$
P_{i,51}(t_m)=1-\sum_{j=1}^{50}P_{i,j}(t_m)\ ,\textit{where}\ \ i=1,2,\ldots,16;m=1,2,3,4,5
$$

$$
P_{i,j}(t_m)=0 \qquad\qquad j=1,2,3,\ldots,16,33,34,\ldots,50,\ m=1,2,3.
$$

$$
P_{i,j}(t_m)=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S_{xi}^2}\in I_{Z_{S_x^2 v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S_{ei}^2}\in I_{Z_{S_e^2 v_4}}|\bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))
$$

$$
=\beta_{Z_{\bar{X}v_1}}*\beta_{Z_{S_x^2 v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S_e^2 v_4}}
$$

$$
\begin{aligned}
\textit{where}\ \ & i=17 && ;m=1;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=18,19,20,21 && ;m=2;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=22,23,\ldots,27 && ;m=3;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=28,29,30,31 && ;m=4;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=32 && ;m=5;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2
\end{aligned}
$$

$$
P_{i,51}(t_m)=1-\sum_{j=1}^{50}P_{i,j}(t_m)\ ,\textit{where}\ \ i=17,18,\ldots,32;m=1,2,3,4,5
$$

$$
P_{i,j}(t_m)=0 \qquad\qquad j=1,2,3,\ldots,32,49,50,\ m=1,2,3.
$$

$$
P_{i,j}(t_m)=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S_{xi}^2}\in I_{Z_{S_x^2 v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S_{ei}^2}\in I_{Z_{S_e^2 v_4}}|\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))
$$

$$
=\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S_x^2 v_2}}*\beta_{Z_{\bar{e}v_3}}*\beta_{Z_{S_e^2 v_4}}
$$

$$
\begin{aligned}
\textit{where}\ \ & i=33 && ;m=1;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=34,35,36,37 && ;m=2;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=38,39,\ldots,43 && ;m=3;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=44,45,46,47 && ;m=4;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=48 && ;m=5;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2
\end{aligned}
$$

$$
P_{i,51}(t_m)=1-\sum_{j=1}^{50}P_{i,j}(t_m)\ ,\textit{where}\ \ i=33,34,\ldots,48;m=1,2,3,4,5
$$

$$P_{49,j}\left(t_{49}^{*}\right)=0,\ \ j=1,2,3,\ldots,16,33,34,\ldots,51.$$

$$P_{49,j}\left(t_{49}^{*}\right)=P_{49=17}P_{17,j}\left(t_{1}\right)+\sum_{k=18}^{21}P_{49=k}P_{k,j}\left(t_{2}\right)+\sum_{k=22}^{27}P_{49=k}P_{k,j}\left(t_{3}\right)+\sum_{k=28}^{31}P_{49=k}P_{k,j}\left(t_{4}\right)+P_{49=32}P_{32,j}\left(t_{5}\right),\ \ j=17,18,\ldots,32$$

where

$$P_{49=j}=P\left(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S_{xi}^2}\in I_{Z_{S_x^2 v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S_{ei}^2}\in I_{Z_{S_e^2 v_4}}\mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right),\bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right),\ Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}},Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}},\ Z_{S_{ei}^2}\in I_{Z_{S_e^2 3}}\right)$$

$$=\frac{\beta_{Z_{\bar{X}v_1}}\bullet\beta_{Z_{S_x^2 v_2}}\bullet\gamma_{Z_{\bar{e}v_3}}\bullet\gamma_{Z_{S_e^2 v_4}}}{\beta_{Z_{\bar{X}3}}\bullet\beta_{Z_{S_x^2 3}}\bullet\gamma_{Z_{\bar{e}3}}\bullet\gamma_{Z_{S_e^2 3}}},\ \ v1=1,2,\ v2=1,2,\ v3=1,2,\ v4=1,2,\ j=17,18,19,\ldots,32\ \text{ and }\ \sum_{j=17}^{32}P_{49=j}=1$$

and $t_{49}^{*}=t_1\bullet P_{49=17}+t_2\bullet\sum_{i=18}^{21}P_{49=i}+t_3\bullet\sum_{i=22}^{27}P_{49=i}+t_4\bullet\sum_{i=28}^{31}P_{49=i}+t_5\bullet P_{49=32}$

$$P_{50,j}\left(t_{50}^{*}\right)=0,\ \ j=1,2,3,\ldots,32,49,50,51.$$

$$P_{50,j}\left(t_{50}^{*}\right)=P_{50=33}P_{33,j}\left(t_{1}\right)+\sum_{k=34}^{37}P_{50=k}P_{k,j}\left(t_{2}\right)+\sum_{k=38}^{43}P_{50=k}P_{k,j}\left(t_{3}\right)+\sum_{k=44}^{47}P_{50=k}P_{k,j}\left(t_{4}\right)+P_{50=48}P_{48,j}\left(t_{5}\right),\ \ j=33,34,\ldots,48$$

where

$$P_{50=j}=P\left(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},\ Z_{S_{xi}^2}\in I_{Z_{S_x^2 v_2}},\ Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},\ Z_{S_{ei}^2}\in I_{Z_{S_e^2 v_4}}\mid \bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right),\ Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}},Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}},\ Z_{S_{ei}^2}\in I_{Z_{S_e^2 3}}\right)$$

$$=\frac{\gamma_{Z_{\bar{X}v_1}}\bullet\gamma_{Z_{S_x^2 v_2}}\bullet\beta_{Z_{\bar{e}v_3}}\bullet\beta_{Z_{S_e^2 v_4}}}{\gamma_{Z_{\bar{X}3}}\bullet\gamma_{Z_{S_x^2 3}}\bullet\beta_{Z_{\bar{e}3}}\bullet\beta_{Z_{S_e^2 3}}},v1=1,2,\ v2=1,2,\ v3=1,2,\ v4=1,2,\ j=33,34,\ldots,48\ \text{and}\ \sum_{j=33}^{48}P_{49=j}=1$$

and $t_{50}^{*}=t_1\bullet P_{50=33}+t_2\bullet\sum_{i=34}^{37}P_{50=i}+t_3\bullet\sum_{i=38}^{43}P_{50=i}+t_4\bullet\sum_{i=44}^{47}P_{50=i}+t_5\bullet P_{50=48}$

$$P_{51,51}=1$$

ON-LINE MONITORING USING VSI CAUSE SELECTING CONTROL CHARTS

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ABSTRACT

The article considers the variable process control scheme for two dependent process steps. We construct the variable sampling interval (VSI) $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts in order to effectively monitor the quality variable produced by the first process step and the quality variable produced by the second process step, respectively. The performance of the proposed VSI control charts is measured by the adjusted average time to signal (AATS) derived using a Markov chain approach. An example of the process control for automobile braking system shows the application and performance of the proposed joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts in detecting shifts in mean and variance for the two dependent process steps. Furthermore, the performance of the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and the fixed sampling interval (FSI) $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are compared via the numerical analysis results. These demonstrate that the former is much faster in detecting shifts in mean and variance. Whenever quality engineers cannot specify the values of variable sampling intervals, the optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are recommended. Furthermore, the impacts of misusing $Z_{\overline{Y}}$ - $Z_{S_y^2}$ charts to monitoring the process mean and variance in the second step are also investigated.

Key words: Control charts; dependent process steps; optimization technique; Markov chain.

1. INTRODUCTION

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control. Shewhart (1931) developed the $\overline{X}$ control chart which is easy to implement and has been widely used for industrial processes. However, even though Shewhart $\overline{X}$ control charts, always monitor a process by taking samples of equal size at a fixed sampling interval (FSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, in recent years several alternatives have been developed to improve the performance of $\overline{X}$ control charts. One of the useful approaches to improving the detection ability is to use a variable sampling interval (VSI) and/or a variable sample size (VSS) control chart instead of the traditional FSI and/or fixed sample size (FSS). Whenever there is some indication that a process parameter may have changed, the next sampling interval should be shorter and/or the next sample should be larger. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer and/or the next sample should be smaller.

The properties of the $\overline{X}$ chart with VSIs were studied by Reynolds, et al. (1988). Their paper has been extended by several others: Reynolds and Arnold (1989); Amin and Miller (1993); Baxley (1996); Reynolds, and Arnold, and Baik (1996). Tagaras (1998) reviewed the literature on adaptive control charts. Very little work has been down on VSI control charts for simultaneously monitoring process mean and variance. Chengular, et al. (1989) detected process mean and variance using VSI $\overline{X}$ and R control charts. Reynolds and Stoumbos (2001) discussed the properties of VSI X and MR control charts. These papers show that most work on developing VSI control charts had aimed to solve the problem of monitoring process mean.

However, these articles assume that there is only a single process step whereas many products are currently produced with several dependent process steps. Consequently, it is not appropriate to monitor these process steps by utilizing a control chart for each individual process step. Zhang (1984) proposed the simple cause-selecting control chart to control the specific quality in the current process by adjusting the effect of the in-coming quality variable (X) on out-going quality variable (Y), since the in-coming quality variable on the first process step and the out-going quality variable on the second process step are dependent. The cause-selecting values (e) are Y minus the effect of X, and the cause-selecting control chart is constructed accordingly. Wade and Woodall (1993) reviewed and analyzed the cause-selecting control chart and examine the relationship between the cause-selecting control chart and the Hotelling T^2 control chart. In their opinion the cause-selecting control chart outperforms the Hotelling T^2 control chart, since it is easy to distinguish whether the second step of the process is out-of-control. Therefore, it seems reasonable to develop variable control schemes to control dependent process steps. However, the properties of the variable sampling interval (VSI) control charts used to control mean and variance on two dependent process steps have not yet been addressed. Therefore, studying the performance of the joint VSI control charts on two dependent process steps is reasonable. In this paper, the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with variable sampling intervals are proposed for the control of mean and variance in two dependent process steps. In the next section, the performance of the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is measured by the adjusted average time to signal (AATS), which is derived using a Markov chain approach. Finally, we illustrate the application of the proposed control charts using the example of process control for automobile braking system on the process steps, and then compare the performance between the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts and FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts**.** Whenever quality engineers cannot specify the values of variable sampling intervals, the optimal VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are suggested. Furthermore, the impacts of misusing $Z_{\overline{Y}}$ - $Z_{S_y^2}$ charts to monitoring the process mean and variance in the second step are also investigated.

2. DESCRIPTION OF THE JOINT VSI $Z_{\overline{X}}$ - Z_{S^2} AND $Z_{\bar{e}}$ - $Z_{S_e^2}$ CHARTS

Consider a process with two dependent process steps controlled by the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. Let X be the measurable in-coming quality variable on the first process step. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X; let Y be the measurable out-going quality characteristic of interest for the second process step, and follow a normal distribution conditional on X. Since the two process steps are dependent, and the second process step is affected by the first process step, then following Wade and Woodall (1993), the relationship between X and Y is generally expressed as

$$Y_i | X_i = f(X_i) + \varepsilon_i, i = 1,2,3,...m \qquad (2\text{-}1)$$

where, it is assumed that $\varepsilon_i \sim NID(0, \sigma^2)$. Let Y instead of $Y \mid X$. If the function $f(X_i)$ is known, the values of the standardized error term $\varepsilon_i^* = \dfrac{Y_i - f(X_i)}{\sigma}$ are called the cause-selecting values since they are the values of Y_i adjusted for the effects of X_i. In practice, the true function $f(X_i)$ is usually unknown and thus must be estimated using the m observations obtained from the initial m samples of size one, and thus the estimate for

$f(X_i)$ will be $\hat{Y}_i$ (Yang (2003)). The residuals, $e_i = Y_i - \hat{Y}_i$, are generated by the model used. Hence, e_i ~NID$(0, \sigma_e^2)$. Consequently, the standard residuals $e_i^* = \dfrac{Y_i - \hat{Y}_i}{\sigma_e}$ are called the cause-selecting values. The X chart is thus constructed to monitor the mean of X_i on the first step, and the e chart is constructed to monitor the mean of e_i on the second step.

However, in our study the chosen sample size is not one and the rational samples of size (n) are taken from the two dependent process steps. Plotting the sample data to obtain a sample profile and then establish the reference line of Y and X (see Kim, Mahmoud and Woodall (2003)). To monitor the mean and variance of X on the first step the $\overline{X}$-S^2 charts should be constructed, and to monitor the mean and variance of e on the second step the $\bar{e}$-S_e^2 charts should be constructed. The $\overline{X}$-S^2 charts and $\bar{e}$-S_e^2 charts are called cause-selecting control charts.

For engineers to use the control charts easily, the sample means and sample variances are standardized as follows.

$$Z_{\overline{X}_i} = \frac{\overline{X}_i - \mu_X}{\sigma_X/\sqrt{n}} \sim N(0,1), \quad Z_{S_i^2} = \frac{(n\text{-}1)S_i^2}{\sigma_x^2} \sim \chi^2(\mathrm{n}-1),$$

$$Z_{\bar{e}_i} = \frac{\bar{e}_i}{\sigma_e/\sqrt{n}} \sim N(0,1), \quad Z_{S_{e_i}^2} = \frac{(n\text{-}1)S_{e_i}^2}{\sigma_e^2} \sim \chi^2(\mathrm{n}-1), \tag{2-2}$$

where $\overline{X}_i = \dfrac{\sum_{j=1}^{n} X_{ij}}{n}$, $S_i^2 = \dfrac{\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2}{n\text{-}1}$, $\bar{e}_i = \dfrac{\sum_{j=1}^{n} e_{ij}}{n}$ and $S_{e_i}^2 = \dfrac{\sum_{j=1}^{n}(e_{ij} - \bar{e}_i)^2}{n\text{-}1}$

$i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots.,n.$

Assume that once a special cause occurs it affects the X-variable with probability v and the functional relationship (or e-variable) with probability 1-v. That is, the mean of X_{ij} shifts from μ_X to $\mu_X + \delta_1\sigma_X$ ($\delta_1 \neq 0$) and the variance shifts from σ_X to $\delta_2\sigma_X$ ($\delta_2 > 1$) with probability v, and the mean of the e_{ij} shifts from 0 to δ_3 ($\delta_3 \neq 0$) and the variance shifts from σ_e to $\delta_4\sigma_e$ ($\delta_4 > 1$) with probability 1-v. The out-of-control distribution of X_{ij} and/or e_{ij} will be adjusted to in-control state, once at least one true signal is obtained from the proposed control charts. Let T_{sc} be the time until the occurrence of special cause, and follow an exponential distribution of the form

$$f(t) = \lambda \exp(-\lambda t) \quad t > 0 \tag{2-3}$$

where $1/\lambda$ is the mean time that the process remains in a state of statistical control.

An in-control state analysis for the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is performed since the shifts in the process mean and variance on step 1 and/or step 2 do not occur when the process is just starting, but occur at some time in the future. The standardized samples $z_{\overline{X}_i}$ - z_{S^2} and $z_{\bar{e}_i}$ - $z_{S_e^2}$ are plotted on the joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with warning limits of the form $\pm w_{\overline{X}}$, w_{S^2}, $\pm w_{\bar{e}}$ and $w_{S_e^2}$, and control limits of the form $\pm k_{\overline{X}}$, k_{S^2}, $\pm k_{\bar{e}}$ and $k_{S_e^2}$, respectively, where $0 \le w_{\overline{X}} < k_{\overline{X}}$, $0 \le w_{S^2} < k_{S^2}$, $0 \le w_{\bar{e}} < k_{\bar{e}}$ and $0 \le w_{S_e^2} < k_{S_e^2}$ (see figure 2.1).

$UCL_{z_{\overline{X}}} = k_{\overline{X}}$	$UCL_{z_{\bar{e}}} = k_{\bar{e}}$
$UWL_{z_{\overline{X}}} = w_{\overline{X}}$	$UWL_{z_{\bar{e}}} = w_{\bar{e}}$
$CL_{z_{\overline{X}}} = 0$	$CL_{z_{\bar{e}}} = 0$
$LWL_{z_{\overline{X}}} = -w_{\overline{X}}$	$LWL_{z_{\bar{e}}} = -w_{\bar{e}}$
$LCL_{z_{\overline{X}}} = -k_{\overline{X}}$	$LCL_{z_{\bar{e}}} = -k_{\bar{e}}$
(1) $Z_{\overline{X}}$ chart	(2) $Z_{\bar{e}}$ chart
$UCL_{z_{S^2}} = k_{S^2}$	$UCL_{z_{S_e^2}} = k_{S_e^2}$
$UWL_{z_{S^2}} = w_{S^2}$	$UWL_{z_{S_e^2}} = w_{S_e^2}$
$LCL_{z_{S^2}} = 0$	$LCL_{z_{S_e^2}} = 0$
(3) Z_{S^2} chart	(4) $Z_{S_e^2}$ chart

Figure 2.1 The control limits of VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts

The search for the special cause and adjustment in the first process step is undertaken when the sample $z_{\overline{X}_i}$ falls outside the interval $(-k_{\overline{X}}, k_{\overline{X}})$ and/or when the z_{S^2} falls outside the interval $(0, k_{S^2})$, that is when the $Z_{\overline{X}}$ and/or Z_{S^2} charts produce a signal. The search for the special cause and adjustment in the second process step is undertaken when the sample $z_{\bar{e}_i}$ falls outside the interval $(-k_{\bar{e}}, k_{\bar{e}})$ and/or when the sample $z_{S_e^2}$ falls outside the interval $(0, k_{S_e^2})$, that is when the $Z_{\bar{e}}$ and/or $Z_{S_e^2}$ charts produce a signal. For a discontinuous process, the process steps are stopped to adjustment and then brought back to an in-control state after a true signal is obtained from a control chart. The process is not adjusted but continues when a false alarm is obtained from a control chart.

The positions of the current samples in the joint $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts construct the sampling interval of the next sample.

We divide the proposed VSI $Z_{\overline{X}}$, Z_{S^2}, $Z_{\bar{e}}$ and $Z_{S_e^2}$ control charts into the following three regions (2-4), respectively.

$$
\begin{aligned}
&I_{Z_{\overline{X}1}} = \left[-w_{\overline{X}}, w_{\overline{X}}\right] && \text{(central region)} && I_{Z_{S_x^2 1}} = \left(0, w_{S^2}\right) && \text{(central region)} \\
&I_{Z_{\overline{X}2}} = \left(-k_{\overline{X}}, -w_{\overline{X}}\right) \cup \left(w_{\overline{X}}, k_{\overline{X}}\right) && \text{(warning region)} && I_{Z_{S_x^2 2}} = \left(w_{S^2}, k_{S^2}\right) && \text{(warning region)} \quad (2\text{-}4) \\
&I_{Z_{\overline{X}3}} = \left[-k_{\overline{X}}, k_{\overline{X}}\right] && \text{(control region)} && I_{Z_{S_x^2 3}} = \left(0, k_{S^2}\right) && \text{(control region)} \\
&I_{Z_{\bar{e}1}} = \left(\left[-w_{\bar{e}}, w_{\bar{e}}\right]\right) && \text{(central region)} && I_{Z_{S_e^2 1}} = \left(0, w_{S_e^2}\right) && \text{(central region)} \\
&I_{Z_{\bar{e}2}} = \left(-k_{\bar{e}}, -w_{\bar{e}}\right) \cup \left(w_{\bar{e}}, k_{\bar{e}}\right) && \text{(warning region)} && I_{Z_{S_e^2 2}} = \left(w_{S_e^2}, k_{S_e^2}\right) && \text{(warning region)} \\
&I_{Z_{\overline{X}3}} = \left[-k_{\overline{X}}, k_{\bar{e}}\right] && \text{(control region)} && I_{Z_{S_e^2 3}} = \left(0, k_{S_e^2}\right) && \text{(control region)}
\end{aligned}
$$

The first region, within two warning limits, is called the central region. The second region, within warning limit and control limit, is called the warning region. The third region, within control limits, is called the control region.

Since there are four charts, five VSIs are adopted, $\infty > t_1 > t_2 > t_3 > t_4 > t_5 > 0$. If the samples, $z_{\overline{X}_i}$, z_{S^2}, $z_{\bar{e}_i}$ and $z_{S_e^2}$, all fall within the central regions, $I_{\overline{X}1}$, $I_{Z_{S_x^2 1}}$, $I_{Z_{\bar{e}1}}$ and $I_{Z_{S_e^2 1}}$, then the next sampling interval should be the longest (t_1). If any three of the samples fall within the central regions but the other falls within the warning region, then the next sampling interval should be long (t_2). If any two of the samples fall within the central regions but the others fall within the warning regions, then the next sampling interval should be in the middle (t_3). If any one of the samples falls within the central region but the others fall within the warning regions, then the next sampling interval should be short (t_4). If all the samples fall within the warning regions, then the next sampling interval should be the shortest (t_5).

The relationship between the next sampling interval $(t_m, m = 1,2,3,4,5)$ and the position of the current samples is expressed as equation (2-5).

$$
\begin{cases}
t_1 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\overline{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
t_2 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\overline{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
 & \text{or } Z_{\bar{x}_i} \in I_{Z_{\overline{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
 & \text{or } Z_{\bar{x}_i} \in I_{Z_{\overline{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 1}} \\
 & \text{or } Z_{\bar{x}_i} \in I_{Z_{\overline{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_x^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_e^2 2}}
\end{cases}
$$

$$
t_m = \begin{cases}
t_3 & if\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 1}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 1}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 1}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
t_4 & if\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 1}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}} \\
 & or\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 1}} \\
t_5 & if\ Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S_{xi}^2} \in I_{Z_{S_{\bar{x}}^2 2}} \cap Z_{\bar{e}_i} \in I_{Z_{\bar{e}2}} \cap Z_{S_{e_i}^2} \in I_{Z_{S_{\bar{e}}^2 2}}
\end{cases} \qquad (2\text{-}5)
$$

Following Costa (1997), the first sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sampling intervals, including the first one, should have a probability of p_{01} of being the longest, a probability of $p_{02}+p_{03}+p_{04}+p_{05}$ of being long, a probability of $p_{06}+p_{07}+p_{08}+p_{09}+p_{10}+p_{011}$ of being median, a probability of $p_{012}+p_{013}+p_{014}+p_{015}$ of being short, and a probability of p_{016} of being the shortest, where $\sum_{i=1}^{16} p_{0i}=1$, p_{01}, p_{02}, p_{03},… and p_{016} are given by

$$
\begin{aligned}
p_{01} &= P_r(|Z_{\bar{X}}|< w_{\bar{X}} \big| \ |Z_{\bar{X}}|< k_{\bar{X}}, \delta_1=0) \bullet P_r(0< Z_{S_x^2} < w_{S^2} \big| \ 0< Z_{S_x^2} < k_{S_x^2}, \delta_2=1) \\
&\quad \bullet P_r(|Z_{\bar{e}}|< w_{\bar{e}} \big| \ |Z_{\bar{e}}|< k_{\bar{e}}, \delta_3=0) \bullet P_r(0< Z_{S_e^2} < w_{S_e^2} \big| \ 0< Z_{S_e^2} < k_{S_e^2}, \delta_4=1) \\
&= \frac{([2\Phi(w_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$
\begin{aligned}
p_{02} &= P_r(|Z_{\bar{X}}|< w_{\bar{X}} \big| \ |Z_{\bar{X}}|< k_{\bar{X}}, \delta_1=0) \bullet P_r(0< Z_{S_x^2} < w_{S^2} \big| \ 0< Z_{S_x^2} < k_{S_x^2}, \delta_2=1) \\
&\quad \bullet P_r(|Z_{\bar{e}}|< w_{\bar{e}} \big| \ |Z_{\bar{e}}|< k_{\bar{e}}, \delta_3=0) \bullet (1-P_r(0< Z_{S_e^2} < w_{S_e^2} \big| \ 0< Z_{S_e^2} < k_{S_e^2}, \delta_4=1)) \\
&= \frac{([2\Phi(w_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(w_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$
\begin{aligned}
p_{03} &= P_r(|Z_{\bar{X}}|< w_{\bar{X}} \big| \ |Z_{\bar{X}}|< k_{\bar{X}}, \delta_1=0) \bullet P_r(0< Z_{S_x^2} < w_{S^2} \big| \ 0< Z_{S_x^2} < k_{S_x^2}, \delta_2=1) \\
&\quad \bullet (1\text{-}P_r(|Z_{\bar{e}}|< w_{\bar{e}} \big| \ |Z_{\bar{e}}|< k_{\bar{e}}, \delta_3=0)) \bullet P_r(0< Z_{S_e^2} < w_{S_e^2} \big| \ 0< Z_{S_e^2} < k_{S_e^2}, \delta_4=1) \\
&= \frac{([2\Phi(w_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(w_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})] \bullet [F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1] \bullet [F_{\chi^2}(k_{S^2})] \bullet [2\Phi(k_{\bar{e}})\text{-}1] \bullet [F_{\chi^2}(k_{S_e^2})])}
\end{aligned}
$$

$$p_{04} = P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0) \bullet (1-P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1))$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1)$$
$$= \frac{([2\Phi(w_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{05} = (1-P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0)) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1)$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1)$$
$$= \frac{([2\Phi(k_{\bar{X}})\text{-}2\Phi(w_{\bar{X}})]\bullet[F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{06} = P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1)$$
$$\bullet (1-P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0)) \bullet (1-P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1))$$
$$= \frac{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{07} = P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0) \bullet (1-P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}\delta_2=1))$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0) \bullet (1-P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1))$$
$$= \frac{([2\Phi(w_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{08} = P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0) \bullet (1-P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1))$$
$$\bullet (1-P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0)) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1)$$
$$= \frac{([2\Phi(w_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{09} = (1-P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0)) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1)$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0) \bullet (1-P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1))$$
$$= \frac{([2\Phi(k_{\bar{X}})\text{-}2\Phi(w_{\bar{X}})]\bullet[F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{010} = (1\text{-}P_r(|Z_{\bar{X}}|<w_{\bar{X}} \mid |Z_{\bar{X}}|<k_{\bar{X}}, \delta_1=0)) \bullet P_r(0<Z_{S_x^2}<w_{S^2} \mid 0<Z_{S_x^2}<k_{S_x^2}, \delta_2=1)$$
$$\bullet (1\text{-}P_r(|Z_{\bar{e}}|<w_{\bar{e}} \mid |Z_{\bar{e}}|<k_{\bar{e}}, \delta_3=0)) \bullet P_r(0<Z_{S_e^2}<w_{S_e^2} \mid 0<Z_{S_e^2}<k_{S_e^2}, \delta_4=1)$$
$$= \frac{([2\Phi(k_{\bar{X}})\text{-}2\Phi(w_{\bar{X}})]\bullet[F_{\chi^2}(w_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\bar{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{011}=(1-P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0))\bullet(1\text{-}P_r(0<Z_{S_x^2}<w_{S^2}\big|\ 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1))$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0)\bullet P_r(0<Z_{S_e^2}<w_{S_e^2}\big|\ 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1)$$
$$=\frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

(2-6)

$$p_{012}=P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0)\bullet(1-P_r(0<Z_{S_x^2}<w_{S^2}\big|\ 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1))$$
$$\bullet(1-P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0))\bullet(1-P_r(0<Z_{S_e^2}<w_{S_e^2}\big|\ 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1))$$
$$=\frac{([2\Phi(w_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{013}=(1-P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0))\bullet P_r(0<Z_{S_x^2}<w_{S^2}\big|\ 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1)$$
$$\bullet\ (1-P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0))\bullet(1-P_r(0<Z_{S_e^2}<w_{S_e^2}\big|\ 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1))$$
$$=\frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(w_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{014}=(1-P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0))\bullet(1-P_r(0<Z_{S_x^2}<w_{S^2}\big|\ 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1))$$
$$\bullet P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0)\bullet(1-P_r(0<Z_{S_e^2}<w_{S_e^2}\big|\ 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1))$$
$$=\frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})]\bullet[F_{\chi^2}(w_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(k_{S_e^2})\text{-}F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{015}=(1-P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0))\bullet(1\text{-}P_r(0<Z_{S_x^2}<w_{S^2}\big|\ 0<Z_{S_x^2}<k_{S_x^2},\delta_2=1))$$
$$\bullet(1\text{-}P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0))\bullet P_r(0<Z_{S_e^2}<w_{S_e^2}\big|\ 0<Z_{S_e^2}<k_{S_e^2},\delta_4=1)$$
$$=\frac{([2\Phi(k_{\overline{X}})\text{-}2\Phi(w_{\overline{X}})]\bullet[F_{\chi^2}(k_{S^2})\text{-}F_{\chi^2}(w_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}2\Phi(w_{\bar{e}})]\bullet[F_{\chi^2}(w_{S_e^2})])}{([2\Phi(k_{\overline{X}})\text{-}1]\bullet[F_{\chi^2}(k_{S^2})]\bullet[2\Phi(k_{\bar{e}})\text{-}1]\bullet[F_{\chi^2}(k_{S_e^2})])}$$

$$p_{016}=1-\sum_{i=01}^{15}p_{0i}$$

To facilitate the computation of the performance measures, $w_{\overline{X}}, w_{S^2}$ $k_{\overline{X}}$, k_{S^2}, $w_{\bar{e}}$, $w_{S_e^2}$, $k_{\bar{e}}$ and $k_{S_e^2}$ will be specified with the constraint that the probability of a sample falling in the warning limits is same for both the $Z_{\overline{X}}\text{-}Z_{S^2}$ charts and $Z_{\bar{e}}\text{-}Z_{S_e^2}$ charts, when the process is in control. Thus,

$$\begin{aligned}&P_r(|Z_{\overline{X}}|<w_{\overline{X}}\big|\ |Z_{\overline{X}}|<k_{\overline{X}},\delta_1=0)\cdot P_r(|Z_{S^2}|<w_{S^2}\big|\ |Z_{S^2}|<k_{S^2},\delta_2=1)\\&=P_r(|Z_{\bar{e}}|<w_{\bar{e}}\big|\ |Z_{\bar{e}}|<k_{\bar{e}},\delta_3=0)\cdot P_r(|Z_{S_e^2}|<w_{S_e^2}\big|\ |Z_{S_e^2}|<k_{S_e^2},\delta_4=1)\end{aligned}\qquad(2\text{-}7)$$

implying, $w_{\overline{X}}=w_{\bar{e}}=w_1$, $k_{\overline{X}}=k_{\bar{e}}=k_1$, $w_{S^2}=w_{S_e^2}=w_2$, and $k_{S^2}=k_{S_e^2}=k_2$. Hence, $p_{04}=p_{02}, p_{05}=p_{03}, p_{08}=p_{09}=p_{011}=p_{06}, p_{014}=p_{012}$ and $p_{015}=p_{013}$.

If both $w_{\overline{X}} = w_{\bar{e}} = 0$, $w_{S^2} = w_{S_e^2} = 0$ and $t_1 = t_2 = t_3 = t_4 = t_5 = t_0$, then the joint VSI $Z_{\overline{X}}$-Z_{S^2} charts and $Z_{\bar{e}}$-$Z_{S_e^2}$ charts reduce to the joint $Z_{\overline{X}}$-Z_{S^2} charts and $Z_{\bar{e}}$-$Z_{S_e^2}$ charts with fixed sampling interval (FSI) t_0.

3. COMPARISON OF CONTROL CHARTS

Sampling schemes should be compared under equal conditions; that is, VSI and FSI schemes should demand the same average sampling interval under the in-control period. That is,

$$E\left[t_m \middle| |Z_{\overline{X}}| < k_1, |Z_{S^2}| < k_2, |Z_{\bar{e}}| < k_1\ |Z_{S_e^2}| < k_2, \delta_1 = 0, \delta_2 = 0\right] = t_0$$

(3-1)

Based on the equation (3-1), equation (3-2) can be formulated as follows.

$$t_1\ p_{01} + t_2 \left(\sum_{j=2}^{5} p_{0j}\right) + t_3 \left(\sum_{j=6}^{11} p_{0j}\right) + t_4 \sum_{j=7}^{15} p_{0j} + t_5\ p_{016} = t_0$$

(3-2)

Simplifying,

$$\begin{aligned}
&(\Phi(w_1))^2 [(4t_1\text{-}16t_2 + 24t_3\text{-}16t_4 + 4t_5)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_3\text{-}8t_4 + 4t_5)\bullet (F_{\chi^2}(k_2))^2] \\
&+ \Phi(w_1) [(\text{-}4t_1 + 12t_2\text{-}12t_3 + 4t_4)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (\text{-}8t_2 + 16t_3\text{-}8t_4)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (16t_3\text{-}32t_4 + 16t_5)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (\text{-}4t_3 + 4t_4)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2] \\
&+ [(t_1\text{-}2t_2 + t_3)\bullet (F_{\chi^2}(w_2))^2 + (\text{-}4t_2 + 8t_3\text{-}4t_4)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 \\
&+ (2t_2\text{-}2t_3)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (\text{-}8t_3 + 8t_4)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (4t_3\text{-}8t_4 + 4t_5)\bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(w_2))^2 + (t_3\text{-}t_0)\bullet (F_{\chi^2}(k_2))^2 \\
&+ (\text{-}4t_4 + 4t_0)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet (\Phi(k_1))^2 \bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (4t_5\text{-}4t_0) * (\Phi(k_1))^2 * (F_{\chi^2}(k_2))^2] = 0
\end{aligned}$$

(3-3)

where $\Phi(\cdot)$ denotes the standard normal cumulative function, and $F_{x^2}(\cdot)$ denotes the X^2 cumulative function.

The warning limit is derived as follows.

$$w_1 = \Phi^{-1}\left(\frac{-b \pm \sqrt{b^2\text{-}4ac}}{2a}\right) \qquad (3\text{-}4)$$

where

$$a = (4t_1\text{-}16t_2 + 24t_3\text{-}16t_4 + 4t_5)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (4t_3\text{-}8t_4 + 4t_5)\bullet (F_{\chi^2}(k_2))^2$$

$$\begin{aligned}
b = &(\text{-}4t_1 + 12t_2\text{-}12t_3 + 4t_4)\bullet (F_{\chi^2}(w_2))^2 + (8t_2\text{-}24t_3 + 24t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(w_2))^2 + (\text{-}8t_2 + 16t_3\text{-}8t_4)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) \\
&+ (16t_3\text{-}32t_4 + 16t_5)\bullet \Phi(k_1)\bullet F_{\chi^2}(w_2)\bullet F_{\chi^2}(k_2) + (\text{-}4t_3 + 4t_4)\bullet (F_{\chi^2}(k_2))^2 + (8t_4\text{-}8t_5)\bullet \Phi(k_1)\bullet (F_{\chi^2}(k_2))^2
\end{aligned}$$

$$c = (t_1 - 2t_2 + t_3) \bullet (F_{\chi^2}(w_2))^2 + (-4t_2 + 8t_3 - 4t_4) \bullet \Phi(k_1) \bullet (F_{\chi^2}(w_2))^2 + (2t_2 - 2t_3) \bullet F_{\chi^2}(w_2) \bullet F_{\chi^2}(k_2)$$
$$+ (-8t_3 + 8t_4) \bullet \Phi(k_1) \bullet F_{\chi^2}(w_2) \bullet F_{\chi^2}(k_2) + (4t_3 - 8t_4 + 4t_5) \bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(w_2))^2 + (t_3 - t_0) \bullet (F_{\chi^2}(k_2))^2$$
$$+ (-4t_4 + 4t_0) \bullet \Phi(k_1) \bullet (F_{\chi^2}(k_2))^2 + (8t_4 - 8t_5) \bullet (\Phi(k_1))^2 \bullet F_{\chi^2}(w_2) \bullet F_{\chi^2}(k_2) + (4t_5 - 4t_0) \bullet (\Phi(k_1))^2 \bullet (F_{\chi^2}(k_2))^2$$

However, to obtain w_1 and let $0 < w_1 < k_1$, the constraints $\infty > t_1 > t_2 > t_0 > t_3 > t_4 > t_5 > 0$, and $0 < w_2 < k_2$ are required. Thus, the warning limit w_1 can be obtained by using equation (3-4) and choosing a combination of the five VSIs, $(t_1, t_2, t_3, t_4, t_5)$, w_2 and the FSI, t_0.

In this paper, the VSI scheme is compared with the FSI scheme and sampling scheme was considered to be better than another when it allowed the joint $Z_{\overline{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts to detect changes in the means and variances on step 1 and step 2 faster.

4. PERFORMANCE MEASUREMENT

The speed with which a control chart detects process shifts measures the chart's statistical efficiency. For a VSI, the detection speed is measured by the average time from either mean or variance or both shifting until either $Z_{\overline{X}}$ - Z_{S^2} or $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts or both signal, which is known as the adjusted average time to signal (AATS). That is, the AATS is the mean time that the process remains out of control.

Since $T_{SC} \sim \exp(-\lambda t), \quad t > 0$, the occurrence time until the special cause occurs. Hence,

$$AATS = ATC - \frac{1}{\lambda}$$

(4-1)

The average time of the cycle (ATC) is the average time from the start of process until a true signal obtained from one of the proposed charts (see Costa (1997)). The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 51 states is assigned based on whether the process step is in or out of control and the position of samples (see Table 4.1 for the 51 states of the process). The status of the process when the $(i+1)^{th}$ sample is taken, and the position of the i^{th} samples on the joint $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts define the transition states of the Markov chain. The joint VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts produce a signal when at least one of the samples falls outside the control limits. If the current state is any one of the States 1~48, then there is no signal. If the current state is

State 49, it indicates at least one false signal comes from the first process step then the process is not adjusted and continues but State 49 instantly becomes any one of the States 1~16 with probability $P_{49=j}$, $j = 1 \sim 16$, $\sum_{j=1}^{16} P_{49=j}$. Any one of the States 1~16 thus transits to any state of states 1~51 after a sampling time interval. State 50 is similar to State 49. If the current state is any one of the States 1~50, then it may transit to any other state, hence States 1~50 are transient states. The absorbing state (State 51) is reached when a true signal occurs.

Insert Table 4.1

Let $\mathbf{P}$ be the transition probability matrix, where $\mathbf{P}$ is a square matrix of order 51. Let $P_{i,j}(t_m)$ be the transition probability from prior state i to the current state j with sampling interval t_m , where t_m is determined by the prior state i, $i = 1,2,...,51, j = 1,2,...,51, m = 1,2,3,4,5.$ The transition probability, for example, from state 1 to state 4 with sampling interval t_1 and fixed sample size n is calculated as

$$\begin{aligned} P_{1,4}(t_1) &= P(T_{sc} > t_1) \bullet P(Z_{\bar{x}_i} \in I_{Z_{\bar{X}_1}}, Z_{S^2_{xi}} \in I_{Z_{S^2_{x2}}}, Z_{\bar{e}_i} \in I_{Z_{\bar{e}1}}, Z_{S^2_{ei}} \in I_{Z_{S^2_{\bar{e}1}}} \mid \delta_1 = \delta_3 = 0, \delta_2 = \delta_4 = 1) \\ &= \left(e^{-\lambda t_1}\right)(2\Phi(w_1) - 1)^2 (F_{x^2}(k_2) - F_{x^2}(w_2))(F_{x_2}(w_2)) \end{aligned}$$

The calculation of all transition probabilities is shown in Appendix.

From the elementary properties of Markov chains (see, e.g., Cinlar (1975)), the ATC is derived as

$$ATC = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{t} + \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1} M_f + \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}(A\ )T_r$$

(4-2)

where $\mathbf{b}' = (p_{01}, p_{02}, p_{03}, p_{02}, p_{03}, p_{06}, p_{07}, p_{06}, p_{06}, p_{10}, p_{06}, p_{012}, p_{013}, p_{012}, p_{013}, p_{016}, 0, ...0)$ is the vector of starting probabilities for State 1~ State 50, where the first sampling interval has probability p_{01} (see equation (2-6) for calculation) of being the longest (or State 1 with probability p_{01}) , the probability $p_{02} + p_{03} + p_{02} + p_{03}$ of being long (or State 2~ State 5 with probability $p_{02}, p_{03}, p_{02}, p_{03}$, respectively) and the probability $p_{06} + p_{07} + p_{06} + p_{06} + p_{10} + p_{06}$ of being median (or State 6~State 11 with probability $p_{06}, p_{07}, p_{06}, p_{06}, p_{10}, p_{06}$, respectively), the probability $p_{012} + p_{013} + p_{012} + p_{013}$ of being short (or State 12 ~ State 15 with probability $p_{12}, p_{13}, p_{12}, p_{13}$, respectively) and the probability p_{016} of being the shortest (or State 16 with probability p_{016}); $\mathbf{I}$ is the identity matrix of order 51; $\mathbf{Q}$ is the transition probability matrix where elements represent the transition probability, $P_{i,j}(t_m)$, from transient state i, i=1,…,50, to transient state j, j=1,…,50, m=1,2,3,4,5; $M'_f = (0,0,\ldots,0, T_f, T_f)$ is the vector of in-correct adjustment time for State1 ~ State50; $\mathbf{t}' = (t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t_1,t_2,t_2,t_2,t_2,t_3,t_3,t_3,t_3,t_3,t_3,t_4,t_4,t_4,t_4,t_5,t^*_{49},t^*_{50})$ is the vector of the VSIs for state1~state 50, where t^*_{49} is the average time of sampling interval for State 49, and t^*_{50} is the average time of sampling interval for State 50, the calculation of t^*_{49} and t^*_{50} is shown in Appendix; A is the vector of transition probability, $P_{i,51}(t_m)$, form transient state i, i=1,…,50, to absorbing state 51, m=1,2,3,4,5; T_r is the time to adjust any process step correctly.

5. AN EXAMPLE

An example of process control for automobile braking system is presented, and the data of the process are measurements of roll weight and bake weight. Let variables X=roll weight and Y= bake weight be measured from the end of the second process step. The bake weight produced in the second step is influenced by the roll weight produced in the first step. Thirty-five samples of size two (X, Y) are taken from Wade and Woodall (1993). A machine is used in the process steps. The machine could only fail in the first process step and affects the X-variable with probability 0.75 (v=0.75), or only fail in the second process step and affects the functional relationship (or e-variable) with probability 0.25. The failure rate of the machine is 0.1 time per hour (or $\lambda = 0.1$). Presently, the joint FSI $Z_{\overline{X}}$ - Z_{S^2} or $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are used to monitor the shifts in means and variances on the two process steps every hour. When a true signal occurs from the step1/step2, it indicates that it requires adjustment. The average time of searching a false signal is 1.0 hours ($T_f = 1.0$), and The average time of correct adjustment on the process step is 2.0 hours (or $T_r = 2.0$). The FSI $Z_{\overline{X}}$ and $Z_{\bar{e}}$ charts have control limits placed at ± 3 with a false signal rate of 0.0027 and the Z_{S^2} and $Z_{S_e^2}$ charts have control limits placed at 9 with false signal rate 0.0027, respectively. Thus, approximately 10.8 false alarms are expected per 1,000 samples and have an in-control average run length (ARL) of 92.59 hours if $T_f = 0$ and $T_r = 0$. The calculated AATS of FSI charts is 3.52 hours using equation (4-2). The slowness with which the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts detect shifts in the out-of-control process steps has led the quality manager to propose building the $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with specified VSIs, $(t_5, t_4, t_3, t_2, t_1)$=(0.001,0.002,0.01,0.02,1.1).

The first 25 samples are used to establish the reference line of Y and X and then construct the $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts. After delete the out-of-control sample number 3, the fitted regression model is

$$\hat{Y}_{ij} = a_0 + a_1 X_{ij} = 82.82 + 0.56229 X_{ij} .$$

Thus, the residuals (e_{ij}) are obtained by $Y_{ij} - \hat{Y}_{ij}$. The estimated in-control distributions of $\overline{X}$ and $\bar{e}$ are $\overline{X} \sim \mathrm{N}\left(210.0625, \frac{1.405252^2}{2}\right)$ and $\bar{e} \sim \mathrm{N}\left(0, \frac{0.858^2}{2}\right)$.

Since the out-of-control sample number 3 is occurs on the step 2 not the step 1, hence it is used to estimate the out-of-control distribution of $\bar{e}$. That is, $\bar{e} \sim N\left(0.7868, \frac{2.1911^2}{2}\right)$ (or $\delta_3 = 1.4133$ and $\delta_4 = 2.7831$). The distribution of $\overline{X}$ is unchanged. That is, $\overline{X} \sim \mathrm{N}\left(210.0625, \frac{1.405252^2}{2}\right)$ (or $\delta_1 = 0.0$ and $\delta_2 = 0.0$).

The construction and application of the proposed VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts is illustrated. The following are the guidelines for using the proposed charts:

Step 1: Let the factors of control limits $k_1 = 3$ and $k_2 = 9.0$, in order to maintain the average false signal rate at around 10.8 per 1000 samples. The reciprocal of 10.8 false signals is also the ARL, but for the in-control case of $\delta_1 = \delta_3 = 0$ and $\delta_2 = \delta_4 = 1$.

Step 2: The average time of searching a false alarm is 1.0 hours ($T_f = 1.0$). The average time of correct adjustment on the process steps is 2.0 hours (or $T_r = 2.0$).

Step 3: For out-of-control step1, the estimated shift scales are $\delta_1 = 0.0$ and $\delta_2 = 0.0$; for out-of-control step2, the estimated shift scales are $\delta_3 = 1.4133$ and $\delta_4 = 2.7831$.

Step 4: Since $0 < t_5 < t_4 < t_3 < t_2 < t_1 < \infty$ is required, and for performance of process control we adopt the combination (t_5, t_4, t_3, t_2, t_1)=(0.001,0.002,0.01,0.02,1.1).

Step 5: Letting t_5 =0.001, t_4 =0.002, t_3 =0.01, t_2 =0.02, t_1 = 1.1, t_0 =1, $k_1 = 3$ and $k_2 = 9.0$ in the equation (3-4) leads to w_1=1.188 and w_2=7.001.

Step 6: From equations (4-1) and (4-2), the AATS of the VSI charts is 3.04h. Compared to FSI charts, the VSI charts save detection time 13.64%.

The structures of the proposed VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are in Figure

4.1.

$$\begin{aligned} UCL_{z_{\overline{X}}} &= k_1 = 3 \\ UWL_{z_{\overline{X}}} &= w_1 = 1.188 \\ CL_{z_{\overline{X}}} &= 0 \\ LWL_{z_{\overline{X}}} &= -w_1 = \text{-}1.188 \\ LCL_{z_{\overline{X}}} &= -k_{\overline{X}} = \text{-}3 \end{aligned}$$

(1) $Z_{\overline{X}}$ chart

$$\begin{aligned} UCL_{z_{\bar{e}}} &= k_1 = 3 \\ UWL_{z_{\bar{e}}} &= w_1 = 1.188 \\ CL_{z_{\bar{e}}} &= 0 \\ LWL_{z_{\bar{e}}} &= -w_1 = -1.188 \\ LCL_{z_{\bar{e}}} &= -k_{\bar{e}} = -3 \end{aligned}$$

(2) $Z_{\bar{e}}$ chart

$$\begin{aligned} UCL_{z_{s^2}} &= k_2 = 9 \\ UWL_{z_{s^2}} &= w_2 = 7.001 \\ LCL_{z_{s^2}} &= 0 \end{aligned}$$

(3) Z_{S^2} chart

$$\begin{aligned} UCL_{z_{s_e^2}} &= k_2 = 9 \\ UWL_{z_{s_e^2}} &= w_2 = 7.001 \\ LCL_{z_{s_e^2}} &= 0 \end{aligned}$$

(4) $Z_{S_e^2}$ chart

Figure 5.1 the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control limits

With the design parameters determined, the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts can be used for controlling the automobile braking system. According to the VSI scheme, the first sampling interval is determined by random and the other sampling intervals are determined by the position of samples. An example using the VSI charts is introduced. To monitor the process, a random procedure decides the first sampling interval t_1=1.1 hours with sample of size two. The first sample with means and variances is $\left(Z_{\bar{x}_1} = 0.004, Z_{s^2} = 0.261, Z_{\bar{e}_1} = -1.807, Z_{s_e^2} = 0.184\right)$. Since one of the plotted points falls in the warning region, but the other falls within the central regions, the second sample will be observed adopting a sample of size two after long sampling time interval t_2=0.02h. The second sample is ($z_{\overline{X}}$=-2.076, z_{S^2} =0.00, $z_{\bar{e}}$=-0.4240, $z_{S_e^2}$ =0.583). Since one of the plotted points falls in the warning region, but the other falls within the central regions, the third sample will be observed adopting a sample of size two after long sampling time interval t_2=0.02h. The third sample is ($z_{\overline{X}}$=0.9430, z_{S^2} =0.00, $z_{\bar{e}}$=0.0540, $z_{S_e^2}$ =0.583). Since all plotted points fall within central regions, the fourth sample will be observed after t_1=1.1h. The process continues until at least one true signal is obtained and the process is stopped and adjusted.

The constructed VSI charts are used to monitor the samples from number 26-35 (see Figures 5-1).

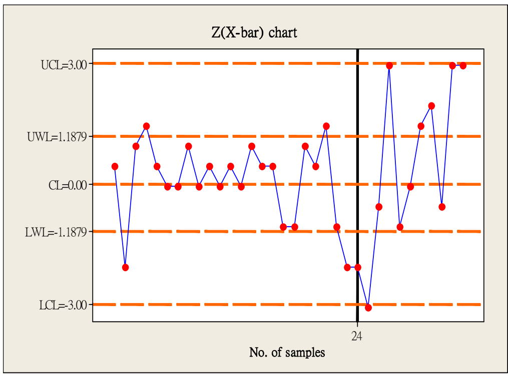

Fig. 5-1-a

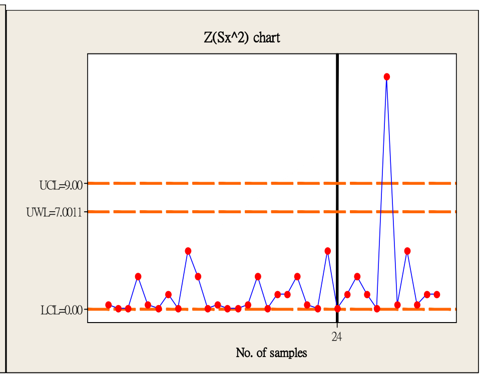

Fig. 5-1-b

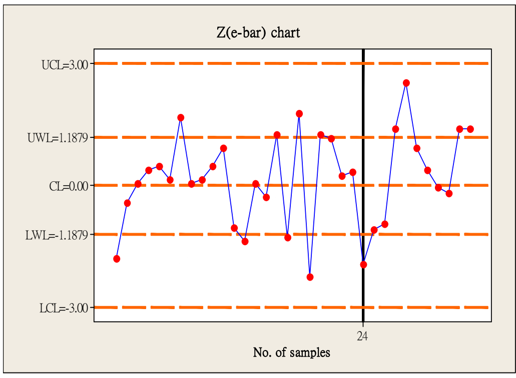

Fig. 5-1-c

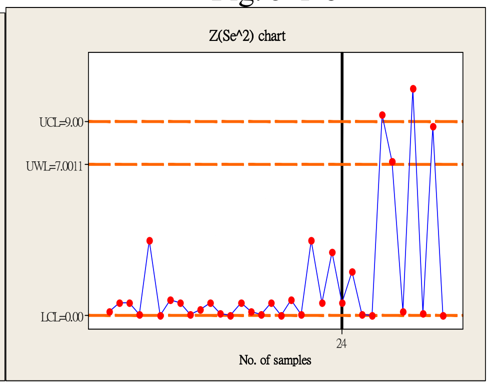

Fig. 5-1-d

We find outlier, sample no. 26, occurs on $Z_{\bar{x}}$ chart; outlier, sample no. 30, occurs on $Z_{S_x^2}$ chart; no outlier occurs on $Z_{\bar{e}}$ chart; outliers, samples no. 29 and 32, occur on $Z_{S_e^2}$ chart.

To compare the performance between the specified VSI and FSI control charts, we consider sixteen combinations of v, λ, $t_1 \sim t_5$, $\delta_1 \sim \delta_4$, T_f and T_r using orthogonal array $L_{16}(2^{15})$. The levels of the parameters are b=0.1,0.5, $\lambda = 0.05, 0.1$, $\delta_1 = 0.5, 1.5$, $\delta_2 = 1.5, 2.0$, $\delta_3 = 0.5, 1.5$, $\delta_4 = 1.5, 2.0$, $t_0 = 1.0$, $t_5 = 0.001, 0.0035$, $t_4 = 0.004, 0.006$, $t_3 = 0.0065, 0.009$, $t_2 = 0.12, 0.15$, $t_1 = 1.0, 1.5$ and n=5.

Table 5.1 shows the AATSs of specified VSI and FSI charts. The detection time of the specified VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts has been reduced from 28.10% to 95.14%. The specified VSI scheme improves the sensitivity of the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts. Hence, the proposed VSI control charts outperform FSI control charts.

Insert Table 5.1

Furthermore, sometimes engineers cannot specify the VSIs. The optimal VSI

$Z_{\bar{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts are thus suggested. The optimal VSI of the proposed charts are determined using optimization technique, Quasi-Newton approach in Fortran IMSL BCONF subroutine, to minimum AATS under the constraints 0 < w_1 < 3, 0 < w_2 < 9, $0<t_5<t_4<t_3<t_2<1<t_1<2$, and parameters as described in Table 5.1. The optimum VSIs and minimal AATS under various combinations of parameters are illustrated in Table 5.2. We found that the optimum VSI $Z_{\bar{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts all work better than the $Z_{\bar{X}}$ - Z_{S^2} charts and $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts with specified VSIs. Compare to the FSI control charts, the optimal VSI charts may save the detection time from 34.64% to 98.61%. Consequently, the performance of the optimal VSI charts is better than the specified VSI charts, and the specified VSI charts outperform the FSI charts.

Insert Table 5.2

6. MISUSING $\bar{Y}$ AND S_y^2 CONTROL CHARTS

In many real situations, engineers may misuse $\bar{Y}$ and S_y^2 control charts to monitor mean and variance in the seconds step. Figure 6.1 shows the monitoring results of using $\bar{X}$ - S_x^2 and $\bar{Y}$ - S_y^2 control charts. On the second step, there are four outliers, sample no.26, 28, 34 and 35, occur on the $\bar{Y}$ chart, and one outlier, sample no.34, occurs on S_y^2 chart. Compare to $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts, it indicates that misusing $\bar{Y}$ and S_y^2 control charts will lead to unnecessarily adjust the mean and underadjust/overadjust the variance on the second step. Incorrect adjustment of a process will increase in variability of the quality of products and cost (Woodall (1986)).

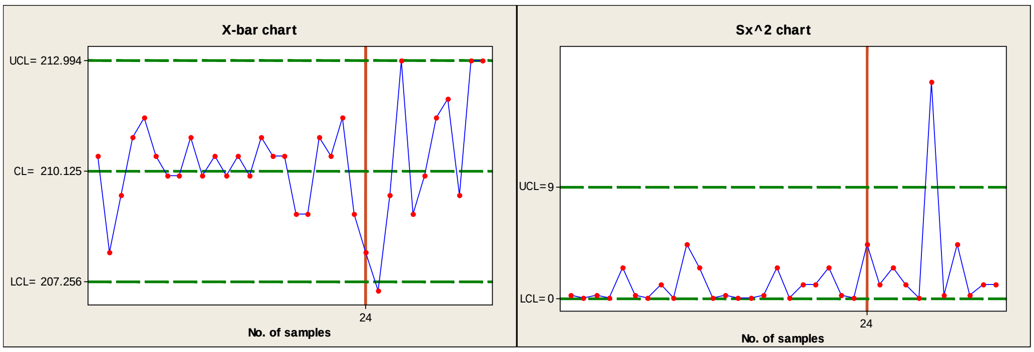

Fig. 6-1-a Fig. 6-1-b

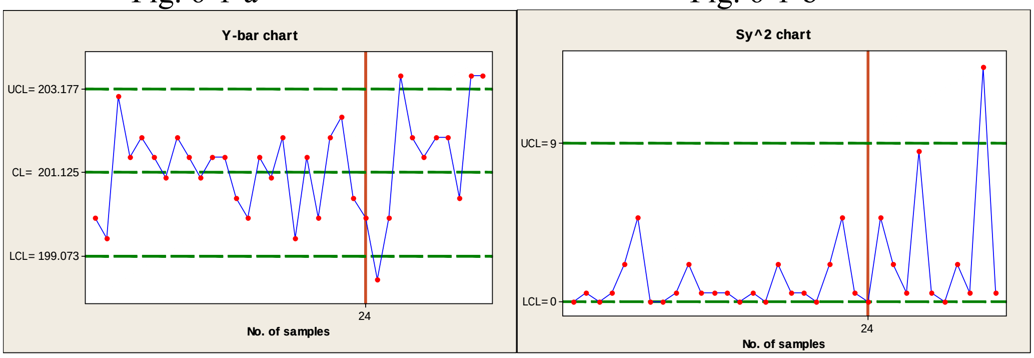

Fig. 6-1-c Fig. 6-1-d

7. CONCLUSIONS

The proposed VSI scheme controlling two dependent process steps substantially improves the performance of the FSI scheme by increasing the speed with which small and median shifts in the means and variances of process steps are detected. We have found that the VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts always work better (in the cases examined) than the FSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts for small and median $\delta_1, \delta_2, \delta_3$ and δ_4 values. Furthermore, the performance of the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts outperforms the $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts with specified VSIs, and is thus recommended when quality engineers cannot specify the VSIs.

This paper considered to monitoring mean and variance on two dependent process steps with variable sampling intervals. However, a study of the variable sample size (VSS), variable sample size and sampling interval (VSSI) or variable parameters (VP) $Z_{\overline{X}}$ - Z_{S^2} and $Z_{\bar{e}}$ - $Z_{S_e^2}$ control charts under two or multiple dependent process steps is an interesting topic for future research. Other important extensions of the proposed model can also be developed. It is straight forward to extend the proposed model to study VP control charts or other control charts, such as attribute charts, EWMA-charts, CUSUM-charts or multivariate charts.

Table 4.1 Definition of 51 process states

state	SC occurs?	which step?	the position of $Z_{\bar{x}_i}$	the position of $Z_{S_{xi}^2}$	the position of $Z_{\bar{e}_i}$	the position of $Z_{S_{ei}^2}$	process adjustment?
1	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
2	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
3	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
4	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
5	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
6	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
7	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
8	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
9	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
10	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No

11	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
12	No	-	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
13	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
14	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
15	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
16	No	-	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$\mathrm{I}_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
17	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
18	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
19	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
20	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
21	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
22	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
23	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
24	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
25	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
26	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
27	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
28	Yes	first	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
29	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
30	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
31	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
32	Yes	first	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
33	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
34	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
35	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
36	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
37	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
38	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
39	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
40	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
41	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No

42	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
43	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 1}}$	No
44	Yes	second	$I_{Z_{\bar{X}1}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
45	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 1}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
46	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}1}}$	$I_{Z_{S_e^2 2}}$	No
47	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 1}}$	No
48	Yes	second	$I_{Z_{\bar{X}2}}$	$I_{Z_{S_x^2 2}}$	$I_{Z_{\bar{e}2}}$	$I_{Z_{S_e^2 2}}$	No
49	false signal comes from $Z_{\overline{X}}$ and/or Z_{S^2} charts , but no signal comes from the $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts						No
50	no signal comes from the $Z_{\overline{X}}$ - Z_{S^2} charts , but false signal comes from the $Z_{\bar{e}}$ and/or $Z_{S_e^2}$ charts						No
51	true signal comes from $Z_{\overline{X}}$ - Z_{S^2} charts and/or $Z_{\bar{e}}$ - $Z_{S_e^2}$ charts						Correct adjustment

Table 5.1 AATS comparisons for the specified VSI and FSI charts under various combinations of parameters

combination of various parameters													VSI				FSI		save time percentage
No.	v	t_5	t_4	t_3	t_2	t_1	δ_1	δ_2	δ_3	δ_4	λ	(T_f, T_r)	w_1^*	w_2^*	ATC	AATS	ATC	AATS	
1	0.1	0.001	0.004	0.0065	0.12	1	0.5	1.5	0.5	1.5	0.05	(0.1, 0.5)	0.4113	8.1520	20.97	0.97	21.86	1.86	47.85
2	0.1	0.001	0.004	0.0065	0.12	1	0.5	2	1.5	2	0.1	(0.05, 0.1)	0.2506	13.6145	10.42	0.42	11.09	1.09	61.47
3	0.1	0.001	0.004	0.009	0.15	1.5	1.5	1.5	0.5	1.5	0.05	(0.05, 0.1)	0.0135	16.2499	20.07	0.07	21.44	1.44	95.14
4	0.1	0.001	0.004	0.009	0.15	1.5	1.5	2	1.5	2	0.1	(0.1, 0.5)	0.1132	9.2368	10.94	0.94	11.49	1.49	36.91
5	0.1	0.0035	0.006	0.0065	0.12	1.5	1.5	1.5	0.5	2	0.1	(0.1, 0.5)	0.2361	13.2497	11.06	1.06	11.57	1.57	32.48
6	0.1	0.0035	0.006	0.0065	0.12	1.5	1.5	2	1.5	1.5	0.05	(0.05, 0.1)	0.2235	16.2163	20.56	0.56	21.33	1.33	57.89
7	0.1	0.0035	0.006	0.009	0.15	1	0.5	1.5	0.5	2	0.1	(0.05, 0.1)	0.1167	9.2362	10.32	0.32	11.17	1.17	72.65
8	0.1	0.0035	0.006	0.009	0.15	1	0.5	2	1.5	1.5	0.05	(0.1, 0.5)	0.1938	7.9820	20.89	0.89	21.74	1.74	48.85
9	0.5	0.001	0.006	0.0065	0.15	1	1.5	1.5	1.5	1.5	0.1	(0.1, 0.5)	0.2153	8.0078	10.87	0.87	11.77	1.77	50.85
10	0.5	0.001	0.006	0.0065	0.15	1	1.5	2	0.5	2	0.05	(0.05, 0.1)	0.1820	8.4704	20.30	0.30	21.09	1.09	72.48
11	0.5	0.001	0.006	0.009	0.12	1.5	0.5	1.5	1.5	1.5	0.1	(0.05, 0.1)	0.2010	16.2499	10.57	0.57	11.42	1.42	59.86
12	0.5	0.001	0.006	0.009	0.12	1.5	0.5	2	0.5	2	0.05	(0.1, 0.5)	0.3170	8.7220	21.10	1.10	21.53	1.53	28.10
13	0.5	0.0035	0.004	0.0065	0.15	1.5	0.5	1.5	1.5	2	0.05	(0.1, 0.5)	0.1985	8.0880	21.03	1.03	21.66	1.66	37.95
14	0.5	0.0035	0.004	0.0065	0.15	1.5	0.5	2	0.5	1.5	0.1	(0.05, 0.1)	0.0363	15.9615	10.36	0.36	11.30	1.30	72.31
15	0.5	0.0035	0.004	0.009	0.12	1	1.5	1.5	1.5	2	0.05	(0.05, 0.1)	0.2139	16.2499	20.35	0.35	21.21	2.21	84.16
16	0.5	0.0035	0.004	0.009	0.12	1	1.5	2	0.5	1.5	0.1	(0.1, 0.5)	0.2139	16.2499	10.88	0.88	11.67	1.67	47.31

*Save time percentage=[(AATS of FSI charts-AATS of VSI charts)/ AATS of FSI charts]100%

Table 5.2 AATS comparisons for the optimum VSI and FSI charts under various combinations of parameters

combination of various parameters								VSI									FSI		save time
No.	v	δ_1	δ_2	δ_3	δ_4	λ	(T_f, T_r)	w_1^*	w_2^*	t_5^*	t_4^*	t_3^*	t_2^*	t_1^*	ATC	AATS	ATC	AATS	percentage
1	0.1	0.5	1.5	0.5	1.5	0.05	(0.1, 0.5)	0.1936	8.4578	0.0001	0.010	0.010	0.143	1.000	20.90	0.90	21.86	1.86	51.61
2	0.1	0.5	2.0	1.5	2.0	0.10	(0.05, 0.1)	0.0290	10.0024	0.0001	0.010	0.010	0.158	1.491	10.31	0.31	11.09	1.09	71.56
3	0.1	1.5	1.5	0.5	1.5	0.05	(0.05, 0.1)	0.0003	9.9999	0.0009	0.002	0.010	0.164	1.500	20.02	0.02	21.44	1.44	98.61
4	0.1	1.5	2.0	1.5	2.0	0.10	(0.1, 0.5)	0.1050	10.0091	0.0097	0.010	0.010	0.145	1.455	10.92	0.92	11.49	1.49	38.26
5	0.1	1.5	1.5	0.5	2.0	0.10	(0.1, 0.5)	0.1045	9.9583	0.0001	0.010	0.010	0.145	1.518	10.94	0.94	11.57	1.57	40.13
6	0.1	1.5	2.0	1.5	1.5	0.05	(0.05, 0.1)	0.0180	9.9999	0.0009	0.002	0.010	0.160	1.496	20.09	0.09	21.33	1.33	93.23
7	0.1	0.5	1.5	0.5	2.0	0.10	(0.05, 0.1)	0.0337	10.0405	0.0001	0.010	0.010	0.157	1.459	10.31	0.31	11.17	1.17	73.50
8	0.1	0.5	2.0	1.5	1.5	0.05	(0.1, 0.5)	0.1780	10.1910	0.0001	0.007	0.010	0.133	1.000	20.89	0.89	21.74	1.74	48.85
9	0.5	1.5	1.5	1.5	1.5	0.10	(0.1, 0.5)	0.1498	10.2486	0.0001	0.010	0.010	0.137	1.000	10.86	0.86	11.77	1.77	51.41
10	0.5	1.5	2.0	0.5	2.0	0.05	(0.05, 0.1)	0.0142	9.9998	0.0008	0.002	0.010	0.161	1.496	20.07	0.07	21.09	1.09	93.58
11	0.5	0.5	1.5	1.5	1.5	0.10	(0.05, 0.1)	0.0396	9.9988	0.0054	0.008	0.010	0.156	1.489	10.32	0.32	11.42	1.42	77.46
12	0.5	0.5	2.0	0.5	2.0	0.05	(0.1, 0.5)	0.1617	10.0067	0.0085	0.009	0.010	0.136	1.497	21.00	1.00	21.53	1.53	34.64
13	0.5	0.5	1.5	1.5	2.0	0.05	(0.1, 0.5)	0.1228	9.9714	0.0001	0.010	0.010	0.142	1.461	20.98	0.98	21.66	1.66	40.96
14	0.5	0.5	2.0	0.5	1.5	0.10	(0.05, 0.1)	0.0320	9.9982	0.0001	0.010	0.010	0.158	1.483	10.32	0.32	11.30	1.30	75.38
15	0.5	1.5	1.5	1.5	2.0	0.05	(0.05, 0.1)	0.0158	10.0005	0.0015	0.003	0.010	0.161	1.498	20.08	0.08	21.21	2.21	96.38
16	0.5	1.5	2.0	0.5	1.5	0.10	(0.1, 0.5)	0.1325	10.1686	0.0001	0.010	0.010	0.140	1.098	10.87	0.87	11.67	1.67	47.90

*Save time percentage=[(optimum AATS of FSI charts-optimum AATS of VSI charts)/ AATS of FSI charts]100%

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Appendix: The calculation of all transition probabilities

Notation:

$$\beta_{Z_{\bar{X}1}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}1}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(-w_1<Z_{\bar{x}_i}<w_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))$$

$$=P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2}<\frac{w_1-\delta_1}{\delta_2})-P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2}<\frac{-w_1-\delta_1}{\delta_2})=\Phi\left(\frac{w_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{w_1+\delta_1}{\delta_2}\right)-1$$

$$\beta_{Z_{\bar{X}2}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}2}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(-k_1<Z_{\bar{x}_i}<-w_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))+P(w_1<Z_{\bar{x}_i}<k_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))$$

$$=P(\left|Z_{\bar{x}_i}\right|<k_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))-P(\left|Z_{\bar{x}_i}\right|<w_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))=\Phi\left(\frac{k_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{k_1+\delta_1}{\delta_2}\right)-\Phi\left(\frac{w_1-\delta_1}{\delta_2}\right)-\Phi\left(\frac{w_1+\delta_1}{\delta_2}\right)$$

$$\beta_{Z_{\bar{X}3}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(-k_1<Z_{\bar{x}_i}<k_1 \mid Z_{\bar{x}_i}\sim N(\delta_1,\delta_2^2))$$

$$=P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2}<\frac{k_1-\delta_1}{\delta_2})-P(\frac{Z_{\bar{X}_i}-\delta_1}{\delta_2}<\frac{-k_1-\delta_1}{\delta_2})=\Phi\left(\frac{k_1-\delta_1}{\delta_2}\right)+\Phi\left(\frac{k_1+\delta_1}{\delta_2}\right)-1$$

$$\beta_{Z_{S_x^2 1}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 1}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(0<Z_{S_{xi}^2}<w_2 \mid Z_{S_{xi}^2}\sim\delta_2^2\chi^2(n-1))=F_{\chi^2}\left(\frac{w_2}{\delta_2^2}\right)$$

$$\beta_{Z_{S_x^2 2}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 2}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(w_2<Z_{S_{xi}^2}<k_2 \mid Z_{S_{xi}^2}\sim\delta_2^2\chi^2(n-1))=F_{\chi^2}\left(\frac{k_2}{\delta_2^2}\right)-F_{\chi^2}\left(\frac{w_2}{\delta_2^2}\right)$$

$$\beta_{Z_{S_x^2 3}}=P(Z_{S_{xi}^2}\in I_{Z_{S_x^2 3}} \mid \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right))=P(0<Z_{S_{xi}^2}<k_2 \mid Z_{S_{xi}^2}\sim\delta_2^2\chi^2(n-1))=F_{\chi^2}\left(\frac{k_2}{\delta_2^2}\right)$$

$$\beta_{Z_{\bar{e}1}}=P(Z_{\bar{e}_i}\in \mathrm{I}_{Z_{\bar{e}1}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(-w_1<Z_{\bar{e}_i}<w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\frac{Z_{\bar{e}_i}-\delta_3}{\delta_4}<\frac{w_1-\delta_3}{\delta_4})-P(\frac{Z_{\bar{e}_i}-\delta_3}{\delta_4}<\frac{-w_1-\delta_3}{\delta_4})=\Phi\left(\frac{w_1-\delta_3}{\delta_4}\right)+\Phi\left(\frac{w_1+\delta_3}{\delta_4}\right)-1$$

$$\beta_{Z_{\bar{e}2}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}2}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(-k_1<Z_{\bar{e}_i}<-w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))+P(w_1<Z_{\bar{e}_i}<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\left|Z_{\bar{e}_i}\right|<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))-P(\left|Z_{\bar{e}_i}\right|<w_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))=\Phi\left(\frac{k_1-\delta_3}{\delta_4}\right)+\Phi\left(\frac{k_1+\delta_3}{\delta_4}\right)-\Phi\left(\frac{w_1-\delta_3}{\delta_4}\right)-\Phi\left(\frac{w_1+\delta_3}{\delta_4}\right)$$

$$\beta_{Z_{\bar{e}3}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}}\mid\ \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(-k_1<Z_{\bar{e}_i}<k_1\ \mid\ Z_{\bar{e}_i}\sim N(\delta_3,\delta_4^2))$$

$$=P(\frac{Z_{\bar{e}_i}-\delta_3}{\delta_4}<\frac{k_1-\delta_3}{\delta_4})-P(\frac{Z_{\bar{e}_i}-\delta_3}{\delta_4}<\frac{-k_1-\delta_3}{\delta_4})=\Phi\left(\frac{k_1-\delta_3}{\delta_4}\right)+\Phi\left(\frac{k_1+\delta_3}{\delta_4}\right)-1$$

$$\beta_{Z_{S_{\bar{e}}^2 1}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 1}}\mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(0<Z_{S_{ei}^2}<w_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n-1))=F_{\chi^2}\left(\frac{w_2}{\delta_4^2}\right)$$

$$\beta_{Z_{S_{\bar{e}}^2 2}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 2}}\mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(w_2<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n-1))=F_{\chi^2}\left(\frac{k_2}{\delta_4^2}\right)-F_{\chi^2}\left(\frac{w_2}{\delta_4^2}\right)$$

$$\beta_{Z_{S_{\bar{e}}^2 3}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 3}}\mid \bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))=P(0<Z_{S_{ei}^2}<k_2\ \mid\ Z_{S_{ei}^2}\sim\delta_4^2\chi^2(n-1))=F_{\chi^2}\left(\frac{k_2}{\delta_4^2}\right)$$

$$\gamma_{Z_{\bar{X}1}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}1}}\mid Z_{\bar{x}_i}\sim N(0,1))=P(-w_1<Z_{\bar{x}_i}<w_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(w_1)-1$$

$$\gamma_{Z_{\bar{X}2}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}2}}\mid Z_{\bar{x}_i}\sim N(0,1))=P(-k_1<Z_{\bar{x}_i}<-w_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))$$

$$+P(w_1<Z_{\bar{x}_i}<k_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(k_1)-2\Phi(w_1)$$

$$\gamma_{Z_{\bar{X}3}}=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}}\mid Z_{\bar{x}_i}\sim N(0,1))=P(-k_1<Z_{\bar{x}_i}<k_1\ \mid\ Z_{\bar{x}_i}\sim N(0,1))=2\Phi(k_1)-1$$

$$\gamma_{Z_{S_{\bar{x}}^2 1}}=P(Z_{S_{xi}^2}\in I_{Z_{S_{\bar{x}}^2 1}}\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=P(0<Z_{S_{xi}^2}<w_2\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_{\bar{x}}^2 2}}=P(Z_{S_{xi}^2}\in I_{Z_{S_{\bar{x}}^2 2}}\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=P(w_2<Z_{S_{xi}^2}<k_2\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=F_{\chi^2}(k_2)-F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_{\bar{x}}^2 3}}=P(Z_{S_{xi}^2}\in I_{Z_{S_{\bar{x}}^2 3}}\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=P(0<Z_{S_{xi}^2}<k_2\mid Z_{S_{xi}^2}\sim\chi^2(n-1))=F_{\chi^2}(k_2)$$

$$\gamma_{Z_{\bar{e}1}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}1}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(-w_1<Z_{\bar{e}_i}<w_1\mid\ Z_{\bar{e}_i}\sim N(0,1))=2\Phi(w_1)-1$$

$$\gamma_{Z_{\bar{e}2}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}2}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(-k_1<Z_{\bar{e}_i}<-w_1\mid Z_{\bar{e}_i}\sim N(0,1))$$

$$+P(w_1<Z_{\bar{e}_i}<k_1\mid Z_{\bar{e}_i}\sim N(0,1))=2\Phi(k_1)-2\Phi(w_1)$$

$$\gamma_{Z_{\bar{e}3}}=P(Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}}\mid Z_{\bar{e}_i}\sim N(0,1))=P(-k_1<Z_{\bar{e}_i}<k_1\mid\ Z_{\bar{e}_i}\sim N(0,1))=2\Phi(k_1)-1$$

$$\gamma_{Z_{S_{\bar{e}}^2 1}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 1}}\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=P(0<Z_{S_{ei}^2}<w_2\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_{\bar{e}}^2 2}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 2}}\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=P(w_2<Z_{S_{ei}^2}<k_2\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=F_{\chi^2}(k_2)-F_{\chi^2}(w_2)$$

$$\gamma_{Z_{S_{\bar{e}}^2 3}}=P(Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 3}}\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=P(0<Z_{S_{ei}^2}<k_2\mid Z_{S_{ei}^2}\sim\chi^2(n-1))=F_{\chi^2}(k_2)$$

The transition probability can be expressed by the following general form.

$$P_{i,j}(t_m)=P(Tsc>t_m)*P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S_{xi}^2}\in I_{Z_{S_{\bar{x}}^2 v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S_{ei}^2}\in I_{Z_{S_{\bar{e}}^2 v_4}}\mid$$

$$\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))$$

$$=\left(e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S_{\bar{x}}^2 v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S_{\bar{e}}^2 v_4}}$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1;j=1,2,\ldots,16;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=2,3,4,5 && ;m=2;j=1,2,\ldots,16;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=6,7,\ldots,11 && ;m=3;j=1,2,\ldots,16;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=12,13,14,15 && ;m=4;j=1,2,\ldots,16;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=16 && ;m=5;j=1,2,\ldots,16;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2
\end{aligned}
$$

$$
\begin{aligned}
P_{i,j}(t_m) &= P(Tsc<t_m)*v*P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S^2_{xi}}\in I_{Z_{S^2_x v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S^2_{ei}}\in I_{Z_{S^2_e v_4}}\ |\\
&\qquad \bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))\\
&= v*\left(1-e^{-\lambda t_m}\right)*\beta_{Z_{\bar{X}v_1}}*\beta_{Z_{S^2_x v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S^2_e v_4}}
\end{aligned}
$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=2,3,4,5 && ;m=2;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=6,7,\ldots,11 && ;m=3;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=12,13,14,15 && ;m=4;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=16 && ;m=5;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2
\end{aligned}
$$

$$
\begin{aligned}
P_{i,j}(t_m) &= P(Tsc<t_m)*(1-v)*P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S^2_{xi}}\in I_{Z_{S^2_x v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S^2_{ei}}\in I_{Z_{S^2_e v_4}}\ |\\
&\qquad \bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))\\
&= (1-v)*\left(1-e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S^2_x v_2}}*\beta_{Z_{\bar{e}v_3}}*\beta_{Z_{S^2_e v_4}}
\end{aligned}
$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=2,3,4,5 && ;m=2;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=6,7,\ldots,11 && ;m=3;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=12,13,14,15 && ;m=4;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2\\
& i=16 && ;m=5;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2
\end{aligned}
$$

$$
\begin{aligned}
P_{i,49}(t_m) &= P(Tsc>t_m)*P[(Z_{\bar{x}_i}\in I_{Z_{\bar{X}4}}\cup Z_{S^2_{xi}}\in I_{Z_{S^2_x 4}})\cap(Z_{\bar{e}_i}\in I_{Z_{\bar{e}3}}\cap Z_{S^2_{ei}}\in I_{Z_{S^2_e 3}})\ |\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))\\
&\quad + P(Tsc>t_m)*P[(Z_{\bar{x}_i}\in I_{Z_{\bar{X}3}}\cap Z_{S^2_{xi}}\in I_{Z_{S^2_x 3}})\cap(Z_{\bar{e}_i}\in I_{Z_{\bar{e}4}}\cup Z_{S^2_{ei}}\in I_{Z_{S^2_e 4}})\ |\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))\\
&= \left(e^{-\lambda t_m}\right)*\left[1-\left(\gamma_{Z_{\bar{X}3}}*\gamma_{Z_{S^2_{x3}}}\right)\right]*\gamma_{Z_{\bar{e}_3}}*\gamma_{Z_{S^2_e 3}}+\left(e^{-\lambda t_m}\right)*\gamma_{Z_{\bar{X}3}}*\gamma_{Z_{S^2_{x3}}}*\left(1-\left(\gamma_{Z_{\bar{e}_3}}*\gamma_{Z_{S^2_e 3}}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
\textit{where}\ \ & i=1 && ;m=1\\
& i=2,3,4,5 && ;m=2\\
& i=6,7,\ldots,11 && ;m=3\\
& i=12,13,14,15 && ;m=4\\
& i=16 && ;m=5
\end{aligned}
$$

$$P_{i,50}(t_m)=P(Tsc>t_m)*P[(Z_{\bar{x}_i}\in I_{Z_{\bar{X}4}}\cup Z_{S^2_{xi}}\in I_{Z_{S^2_x 4}})\cap(Z_{\bar{e}_i}\in I_{Z_{\bar{e}4}}\cup Z_{S^2_{ei}}\in I_{Z_{S^2_e 4}})\ |\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))$$

$$=\left(e^{-\lambda t_m}\right)*\left(1-\left(\gamma_{Z_{\bar{X}3}}*\gamma_{Z_{S^2_{x3}}}\right)\right)*\left(1-\left(\gamma_{Z_{\bar{e}3}}*\gamma_{Z_{S^2_e 3}}\right)\right)$$

where $i=1\quad;m=1$

$i=2,3,4,5\quad;m=2$

$i=6,7,\ldots,11\quad;m=3$

$i=12,13,14,15\ ;m=4$

$i=16\quad;m=5$

$$P_{i,51}(t_{\mathrm{m}})=1-\sum_{j=1}^{50}P_{i,j}(t_{\mathrm{m}})\ ,where\ \ i=1,2,\ldots,16;\mathrm{m}=1,2,3,4,5\,.$$

$$P_{i,j}(t_m)=0,\ i=17\sim 32,\ j=1\sim 16,\ 33\sim 50\,.$$

$$P_{i,j}(t_m)=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S^2_{xi}}\in I_{Z_{S^2_x v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S^2_{ei}}\in I_{Z_{S^2_e v_4}}\,|\bar{X}_i\sim N\left(\mu+\delta_1\frac{\sigma_x}{\sqrt{n}},\delta_2^2\frac{\sigma_x^2}{n}\right),\ \bar{e}_i\sim N\left(0,\frac{\sigma_e^2}{n}\right))$$

$$=\beta_{Z_{\bar{X}v_1}}*\beta_{Z_{S^2_x v_2}}*\gamma_{Z_{\bar{e}v_3}}*\gamma_{Z_{S^2_e v_4}}$$

where $i=17\quad;m=1;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=18,19,20,21\ ;m=2;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=22,23,\ldots,27\ ;m=3;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=28,29,30,31;m=4;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=32\quad;m=5;j=17,18,\ldots,32;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$$P_{i,51}(t_m)=1-\sum_{j=1}^{50}P_{i,j}(t_m)\ ,where\ \ i=17,18,\ldots,32;m=1,2,3,4,5$$

$$P_{i,j}(t_m)=0,\ i=33\sim 48,\ j=1\sim 32,\ 49\sim 50$$

$$P_{i,j}(t_m)=P(Z_{\bar{x}_i}\in I_{Z_{\bar{X}v_1}},Z_{S^2_{xi}}\in I_{Z_{S^2_x v_2}},Z_{\bar{e}_i}\in I_{Z_{\bar{e}v_3}},Z_{S^2_{ei}}\in I_{Z_{S^2_e v_4}}\,|\bar{X}_i\sim N\left(\mu,\frac{\sigma_x^2}{n}\right),\bar{e}_i\sim N\left(\delta_3\frac{\sigma_e}{\sqrt{n}},\delta_4^2\frac{\sigma_e^2}{n}\right))$$

$$=\gamma_{Z_{\bar{X}v_1}}*\gamma_{Z_{S^2_x v_2}}*\beta_{Z_{\bar{e}v_3}}*\beta_{Z_{S^2_e v_4}}$$

where $i=33\quad;m=1;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=34,35,36,37\ ;m=2;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=38,39,\ldots,43\ ;m=3;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=44,45,46,47;m=4;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$i=48\quad;m=5;j=33,34,\ldots,48;v_1=1,2;v_2=1,2;v_3=1,2;v_4=1,2$

$$P_{i,51}(t_m)=1-\sum_{j=1}^{50}P_{i,j}(t_m)\ ,where\ \ i=33,34,\ldots,48;m=1,2,3,4,5$$

$$P_{49,j}(t^*_{49})=P_{49=1}P_{1,j}(t_1)+\sum_{\mathrm{k}=2}^{5}P_{49=\mathrm{k}}P_{\mathrm{k},j}(t_2)+\sum_{\mathrm{k}=6}^{11}P_{49=\mathrm{k}}P_{\mathrm{k},j}(t_3)+\sum_{\mathrm{k}=12}^{15}P_{49=\mathrm{k}}P_{\mathrm{k},j}(t_4)+P_{49=16}P_{16,j}(t_5),\ j=1,2,\ldots,51$$

where $P_{49=\mathrm{k}}=p_{0k}$, $k=1,2,3,\ldots,16$, and

$$t_{49}^{*}=t_1\bullet(P_{49=1})+t_2\bullet\sum_{i=2}^{5}P_{49=i}+t_3\bullet\sum_{i=6}^{11}P_{49=i}+t_4\bullet\sum_{i=12}^{15}P_{49=i}+t_5\bullet(P_{49=16})=t_0$$

$$P_{50,j}\left(t_{49}^{*}\right)=P_{49,j}\left(t_{50}^{*}\right)=P_{49,j}\left(t_0\right),\ j=1,2,3,\ldots,51$$
$$P_{51,51}=1$$

出席國際學術會議心得報告

參加 ISBIS 2007 會議報告

楊素芬

此次第六屆商業與工業統計會議於葡萄牙亞宿島(Acores, Portugal)舉行，目的在讓工業統計和商業統計的學者，研究者或實務者能聚在一起，分享新想法和技術。發表的文章廣泛包含統計在各領域的應用，如工業上品質和可靠度上的應用，生物科技的應用，健康，管理，財務，商業上的應用，和資料擷取等。這是本人第四次參加 ISBIS，第一次到葡萄牙，心中既期待又興奮。

會議共 3 天(8 月 18 至 20 日)。第一天一早是 Opening session，由 ISBIS 理事長和會議負責人歡迎與會者及說明會議籌備經過。接著由美國學者發表 text document data mining 的研究情形。接下來每段時間都有六場平行的演講;爲實驗設計、統計製程管制、財務統計問題、商業統計、多變量統計和統計理論等，而我都選擇與研究領域相關的統計製程管制和品質改善的場次聽講。8 月 18 日 11:00~12:30am 統計製程管制場次首由葡萄牙代表 Fernanda Otilia Sousa Figueiredo 報告如何建立 Robust control chart 以追蹤非常態的製程管制問題，並以模擬數據分析和比較其績效。接著由葡萄牙 Galp Energia 公司報告其公司之品質管理過程(distributed control system and automatic control process)，績效非常好。我問報告者數據是否符合常態分配他說系統會自動調整數據。這倒是令我不敢相信其正確性。最後由葡萄牙博士生 Elisabete Carolino 報告非常態分配(exponential and Weibull distributions)之允收抽樣方法和以模擬數據分析比較其績效。這場讓我了解葡萄牙學界和業界在品質管理領域的研究和實用情形。

8 月 19 日早上第一場由日本學者介紹日本最近的 TQM 活動。Dr. Amasaka 提出新 TQM 原理，包含 TMS (total marketing system), TDS (total development system), TPS (total production system), TIS (total intelligence management system) and TJS (total job quality management system) 。這是新解。 接著, Dr Tsubaki 說明統計方法在技術發展上包括 Taguchi method and QFD 的重要,而 QI story 則用到簡單的統計方法。此外行銷研究也越重視統計方法的應用。最後由提出二維品質創新的 Dr Kano 演講 six sigma 的改善步驟 DMADV 來自何處? Dr Kano 認爲 GE company 並未說明 DMADV 出處，而他認爲 DMADV 其實就是 QI story。這點也是在校上品管課時我對學生說明的見解。

8 月 19 日第二場主題是製程管制與改善。主持人是巴西的 Dr. Epprecht。第一個演講者 Dr. Denby 說明如何對工作流程做分析以改善過程的品質與速度。非常實用，這是目前火紅的六標準差所強調的。

接著 Dr. Epprecht 報告多注頭製程的 SPC 在理論和實務的差異。他除了回顧相關文獻外，也說明實務應用上的困難和問題。多年前，我曾對多注頭製程做相關研究，有二篇文章刊登於期刊上，後來沒再繼續。看來多注頭製程管制是還有一些研究題目可做。

最後 Dr. Czitrom 報告工業實驗圖解法，說明如何對工程師以圖形解釋資料分析結果，

才能使工程師容易接受以統計方法做品質改善。這在實務上非常有效。

下午，刁錦寰大師受邀演講統計學在商學院領域的發展與應用。刁大師是時間數列的大師，又服務於芝加哥大學商學院，他說明統計在商學各領域的重要，尤其是財務金融方面。

最後一場的 SPC 由美國健康統計中心的 Dr. Choi 主持。首先由南韓 Dr. Park 介紹如何對非對稱分配的數據建立百分比管制圖以追蹤製程，效果比用 Shewhat Chart 好。

接著 Dr. Lahirl 提出實證貝氏品質量測計畫。最後 Dr. Choi 報告多變量管制圖可以貝氏方法降低其維度爲 2，以提昇多變量管制圖之績效，作者並以半導體製程爲例。

晚上大會準備 Gala dinner 歡迎與會的人。地點在學校附近的大餐廳。晚餐品嚐葡式料理和甜點並有民俗表演，令賓主盡歡。

本人的論文發表在 8 月 20 日早上第一場。第一位是義大利的 Dr. Fichera 報告 S^2-Cusum 管制圖的建立方法。Ficheraa 和 Castagliolu 有幾篇文章是有關 S^2-EWMA 管制圖的探討。這次，他們將此方法應用於 Cusum 圖的建立。接著由我介紹不正確製程調整下 VSI 管制圖的建立。有聽者問到如何調整製程，也有聽者認爲實務上變異數調整並不容易。我接受他們的看法，因爲本文是完全在理論上做推導的，實務上可能碰到的問題都假設可以解決的。事實上，製程調整我認爲是可以由另一種 Engineering Process Control 的技術來完成的，而這並不是 SPC 的功能。接著，由來自巴西的學者報告，多變量製程管制的優缺點，和多變量管制圖在包裝產品封口的應用，非常實用且績效不錯。

大會最後一天下午安排半日 Tour，遊到亞宿島中西部參觀有名的火山溫泉，地熱和製茶廠品茶。聰明的亞宿島人並以地熱煮熟食物，這些豐盛的食物變成我們的午宴，非常美味。大會用心的安排與服務，使得參與者盡興而歸。

ISBIS 3 天的研討會到此結束。此研討會強調統計方法在工業和商業等理論和實務問題的解決、創新和應用，所以偏重應用績效和理論發展，這可以很容易令人了解統計的重要並進一步促進產學合作和多多進用統計人才，讓統計人發揮最大的功效。類似這種研討會應可以多多舉辦。

國內統計界的研討會多偏重學術，甚少辦較應用的研討會，但應多鼓勵以使一部份的統計人能走出外面與實務結合，並使參與學生體會統計方法的應用與績效且能廣泛被業界所招覽而發揮學以致用之功效。

攜回論文集摘要本和研討會議程表一份。

此次台灣來參加研討會的只有 2 人；清大曾勝滄教授和我。亞宿島人普遍不知道亞洲的台灣，但因是觀光島所以英文都很好，所以溝通上問題少。出遊參加旅行團是較好之選擇。會議後，我和曾教授夫婦三人自行到亞宿島市西北邊看火山湖，看溫泉，一路上繡球花奇大鮮艷而黃薑花盛開且整齊排列真是漂亮。據說是此火山島土地肥沃造成。這些景物真是奇美無比。這次所到之處我們都告訴葡萄牙人，我們來自台灣。參加研討會及出遊不僅可促進學術交流，可進行國民外交且增廣見聞，建議國科會能多鼓勵和補助學者到較無邦交的國家參加國際研討會，特別是多鼓勵以團對方式參加國際研討會以更提升台灣的知名度和促進國民外交。

發表之論文

VARIABLE CONTROL SCHEME FOR A PROCESS WITH INCORRECT ADJUSTMENT

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ABSTRACT

The article considers the variable process control scheme for a process with incorrect adjustment. Incorrect adjustment of a process may result in shifts in process mean and variance, ultimately affecting the quality of products. We construct variable sampling interval (VSI) $Z_{\overline{X}}$ and Z_{S^2} control charts to control the quality variable produced by the process with incorrect adjustment. The performance of the proposed VSI control charts is measured by the adjusted average time to signal (AATS) derived using a Markov chain approach. An example of producing the metallic film thickness of computer connectors shows the application and performance of the proposed VSI $Z_{\overline{X}}$ and Z_{S^2} control charts in detecting small and median shifts in mean and variance for the process with incorrect adjustment. Furthermore, the performance of the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts and FSI $Z_{\overline{X}}$ and Z_{S^2} control charts are compared by some numerical analysis results. These demonstrate that the former is much faster in detecting small and median shifts in mean and variance.

Key words: Control charts; special cause; incorrect adjustment; Markov chain.

1. INTRODUCTION

Control charts are important tools in statistical quality control. They are used to effectively monitor and determine whether a process is in-control or out-of-control. A common problem in statistical process control is process adjusted unnecessarily (or overadjustment of a process) (see Deming, 1982) due to incorrect use of control charts by the operator or since the only information about the state of the process is available through sampling. A process requires adjustment, when a control chart indicates that it is out of control. However, the process may be adjusted unnecessarily, when a false alarm occurs. Woodall (1986) noted that the effect of incorrect adjustment is a quite significant increase in variability of the quality characteristic. To effectively control the significant increase in variability of a process with incorrect adjustment is important. Collani and Saniga (1994) proposed an economic adjustment model for the $\overline{X}$ control chart with a single special cause that considers the effects of a process with incorrect adjustment. Their model determines the optimal design parameters of the $\overline{X}$ control chart which maximize the profitability of the process with incorrect adjustment. Yang and Yang (2004) derived economic adjustment model for the $\overline{X}$ control chart with multiple special causes.

However, the above papers, even Shewhart $\overline{X}$ control charts, always monitor a process by taking equal samples of size at a fixed sampling interval (FSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, several alternatives have been developed to improve the performance of $\overline{X}$ control charts in recent years. One of the useful approaches to improve the detecting ability is to use a VSI control chart instead of the traditional FSI. Whenever there is some indication that a process parameter may have changed, the next sampling interval should be shorter. On the other hand, if there is no indication of a parameter change, then the next sampling interval should be longer.

The properties of the $\overline{X}$ chart with variable sampling intervals were studied by

Reynolds et at. (1988). Their paper has been extended by several others: Reynolds and Arnold (1989); Runger and Pignatiello (1991); Saccucci, Amin, and Lucas (1992); Runger and Montgomery (1993); Amin and Miller (1993); Baxley (1996); Reynolds, and Arnold, and Baik (1996). Tagaras (1998) reviewed the literature on adaptive control charts. However, most work on developing ASI control charts has been down for the problem of monitoring mean of a process. Very little work has been down on VSI control charts for monitoring process mean and variance simultaneously. Chengular, Arnold and Reynolds (1989) detected process mean and variance using VSI $\overline{X}$ and R control charts. Reynolds and Stoumbos (2001) discussed the properties of VSI X and MR control charts. These papers show that most work on developing VSI control charts has been down for the problem of monitoring process mean and/or variance without considering the effects of a process with incorrect adjustment. The VSI control charts used to control the mean and variance of a process with incorrect adjustment has not yet been addressed. Therefore, to study the performance of the VSI control charts on the whole process with incorrect adjustment is reasonable. In this paper, the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts are proposed to control the whole process with incorrect adjustment. In next section, the performance of the proposed $Z_{\overline{X}}$ and Z_{S^2} control charts is measured by the adjusted average time to signal (AATS), which is derived using a Markov chain approach. Finally, we illustrate the application of the proposed control charts using a real example and compare the performance between the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts and FSI $Z_{\overline{X}}$ and Z_{S^2} control chart.

2. DESCRIPTION OF THE VSI $Z_{\overline{X}}$ AND Z_{S^2} CHARTS FOR CONTROLLING A PROCESS WITH INCORRECT ADJUSTMENT

Consider a process controlled by the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts. Let X be the measurable quality variable on the process. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X.

In our study the rational samples of size (n) are taken from the process; the standardized sample mean and variance, $Z_{\overline{X}}$ and Z_{S^2} are

$$Z_{\overline{X}_i} = \frac{\overline{X}_i - \mu_X}{\sigma_X/\sqrt{n}} \sim N(0,1) \qquad \text{and} \quad Z_{s_i^2} = \frac{(n\text{-}1)S_i^2}{\sigma_x^2} \sim \chi^2(\mathrm{n}-1) \tag{2-1}$$

where $\overline{X}_i = \dfrac{\sum_{j=1}^{n} X_{ij}}{n}$ and $S_i^2 = \dfrac{\sum_{j=1}^{n} (X_{ij} - \overline{X}_i)^2}{n\text{-}1}$, $i = 1,2,3,\ldots, j = 1,2,3,\ldots., n.$

Also assume that a special cause or incorrect adjustment may occur in the in-control process. The process is subject to the special cause or incorrect adjustment such that the mean of X shifts from μ_X to $\mu_X + \delta_1\sigma_X$ ($\delta_1 \neq 0$) and the variance shifts from σ_X to $\delta_2\sigma_X$ ($\delta_2 > 1$). The out-of-control distribution of X will be adjusted to in-control state, once at least one true signal is obtained from the proposed control charts. Let T_{sc} be the time until the occurrence of a special cause, and follow an exponential distribution of the form

$$f(t) = \lambda \exp(-\lambda t) \quad t > 0, \tag{2-2}$$

where $1/\lambda$ is the mean time that the process remains in a state of statistical control.

An in-control state analysis for the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts is performed since the shifts in the process mean and variance do not occur when the process is just starting, but occur at some time in the future. The samples $z_{\overline{X}_i}$ and z_{S^2} are plotted on the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts with warning limits of the form $\pm w_1$ and w_2 and control limits of the form $\pm k_1$ and k_2, respectively, where $0 < w_1 < k_1$ and $0 < w_2 < k_2$ (see figure 2.1).

$$\begin{aligned} UCL_{z_{\overline{X}}} &= k_1 \\ UWL_{z_{\overline{X}}} &= w_1 \\ CL_{z_{\overline{X}}} &= 0 \\ LWL_{z_{\overline{X}}} &= -w_1 \\ LCL_{z_{\overline{X}}} &= -k_1 \end{aligned} \qquad \begin{aligned} UCL_{z_{S^2}} &= k_2 \\ UWL_{z_{S^2}} &= w_2 \\ LCL_{z_{S^2}} &= 0 \end{aligned}$$

(1) $Z_{\overline{X}}$ chart (2) Z_{S^2} chart

Figure 2.1 The control limits of VSI $Z_{\overline{X}}$ and Z_{S^2} control charts

The search for the special cause and adjustment in the process is undertaken when the sample $z_{\overline{X}_i}$ falls outside the interval $(-k_1, k_1)$ and/or when the z_{S^2} falls outside the interval $(0, k_2)$, that is when the $Z_{\overline{X}}$ and/or Z_{S^2} charts produce a signal. For a discontinuous process, the process is stopped to search for and eliminate the special cause and adjustment after a signal is obtained from a control chart. The process adjustment may be incorrect when the signal is false, but the adjustment may be corrected when the signal is true and then the process is brought back to an in-control state.

The positions of the current samples in the joint $Z_{\overline{X}}$ and Z_{S^2} charts construct the sampling interval of the next sample.

We divide the proposed VSI $Z_{\overline{X}}$ and Z_{S^2} control charts into the following three regions (2-4), respectively.

$$\begin{aligned} &I_{Z_{\overline{X}1}} = [-w_1, w_1] \quad \text{(central region)} \quad I_{Z_{S_x^2 1}} = (0, w_2) \quad \text{(central region)} \\ &I_{Z_{\overline{X}2}} = (-k_1, -w_1) \cup (w_1, k_1) \quad \text{(warning region)} \quad I_{Z_{S_x^2 2}} = (w_2, k_2) \quad \text{(warning region)} \\ &I_{Z_{\overline{X}3}} = [-k_1, k_1] \quad \text{(control region)} \quad I_{Z_{S_x^2 3}} = (0, k_2) \quad \text{(control region)} \end{aligned} \tag{2-3}$$

The first region, within two warning limits, is called the central region. The second region, within warning limit and control limit, is called the warning region. The third region, within control limits, is called the control region.

Three VSIs are adopted, $0 < t_1 < t_2 < t_3 < \infty$. If both the samples, $z_{\overline{X}_i}$ and z_{S^2}, fall within the central regions, $I_{\overline{X}1}$ and $I_{Z_{S_x^2 1}}$, then the next sampling interval should be long

(t_3). If any one of the samples fall within the central region but the other falls within the warning region, then the next sampling interval should be median (t_2). If both the samples fall within the warning regions, then the next sampling interval should be short (t_1).

The relationship between the next sampling interval $(t_m, m=1,2,3)$ and the position of the current samples is expressed as equation (2-4).

$$t_m = \begin{cases} t_3 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}1}} \\ t_2 & \text{if } (Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}1}}) \text{ or } (Z_{\bar{x}_i} \in I_{Z_{\bar{X}1}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}2}}) \\ t_1 & \text{if } Z_{\bar{x}_i} \in I_{Z_{\bar{X}2}} \cap Z_{S^2_{xi}} \in I_{Z_{S^2_{\bar{x}}2}} \end{cases} \tag{2-4}$$

Following Costa (1997), the first sampling interval taken from the process when it is just starting is chosen randomly. When the process is in control, all sampling intervals, including the first one, should have a probability of p_{01} of being long, a probability of p_{02} of being median, a probability of p_{03} of being short, where $\sum_{i=1}^{3} p_{0i} = 1$, p_{01}, p_{02} and p_{03} are given by

$$\begin{aligned} p_{01} &= \mathrm{P}(Z_{\bar{X}} \in I_{Z_{\bar{X}}1},\ Z_{S^2} \in I_{Z_{S^2 1}} \,\Big|\, \bar{X} \sim N(\mu, \frac{\sigma^2}{n}), -k_1 < Z_{\bar{X}} < k_1, 0 < Z_{S^2} < k_2) \\ &= \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]} \end{aligned} \tag{2-5}$$

$$\begin{aligned} p_{02} &= \mathrm{P}(Z_{\bar{X}} \in I_{Z_{\bar{X}1}},\ Z_{S^2} \in I_{Z_{S^2 2}} \,\Big|\, \bar{X} \sim N(\mu, \frac{\sigma^2}{n}), -k_1 < Z_{\bar{X}} < k_1, 0 < Z_{S^2} < k_2) + \\ &\quad \mathrm{P}(Z_{\bar{X}} \in I_{Z_{\bar{X}2}},\ Z_{S^2} \in I_{Z_{S^2 1}} \,\Big|\, \bar{X} \sim N(\mu, \frac{\sigma^2}{n}), -k_1 < Z_{\bar{X}} < k_1, 0 < Z_{S^2} < k_2) \\ &= \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]} + \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(w_2)]}{[2\Phi(k_{\bar{X}})\text{-}1][F_{\chi^2}(k_2)]} \end{aligned} \tag{2-6}$$

$$\begin{aligned} p_{03} &= \mathrm{P}(Z_{\bar{X}} \in I_{Z_{\bar{X}2}},\ Z_{S^2} \in I_{Z_{S^2 2}} \,\Big|\, \bar{X} \sim N(\mu, \frac{\sigma^2}{n}), -k_1 < Z_{\bar{X}} < k_1, 0 < Z_{S^2} < k_2) \\ &= 1 - p_{01} - p_{02} = \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}. \end{aligned} \tag{2-7}$$

$\Phi(\cdot)$ and $F_{\chi^2}(\cdot)$ are the cumulative probabilities of the standard normal and $\chi^2_{(n\text{-}1)}$ distributions, respectively.

If both w_1=0, w_2=0 and $t_1=t_2=t_3=t_0$, then the VSI $Z_{\overline{X}}$ and Z_{S^2} charts reduce to $Z_{\overline{X}}$ and Z_{S^2} charts with FSI t_0.

3. CONTROL CHARTS COMPARISON

Sampling schemes should be compared under equal conditions; that is, VSI and FSI schemes should demand the same average sampling interval under the in-control period. That is,

$$E\left[t_m \middle| \delta_1 = 0, \delta_2 = 1, |Z_{\overline{X}}| < k_1, 0 < Z_{S^2} < k_2\right] = t_0, \text{ m=1, 2, 3.}$$

or

$$p_{01}t_3 + p_{02}t_2 + p_{02}t_2 + p_{03}t_1 = t_0 \quad (3\text{-}1)$$

Based on the equation (3-1), the warning limit w_1 of the VSI $Z_{\overline{X}}$ control chart is derived as follows (see Appendix 1).

$$w_1 = \Phi^{-1}\left\{\frac{F_{\chi^2}(w_2)[t_3\text{-}t_2\text{-}2\Phi(k_1)(t_2\text{-}t_1)] + F_{\chi^2}(k_2)[t_2\text{-}t_0 + 2\Phi(k_1)(t_0\text{-}t_1)]}{2[F_{\chi^2}(w_2)](t_1\text{-}2t_2 + t_3) + 2[F_{\chi^2}(k_2)](t_2\text{-}t_1)}\right\} \quad (3\text{-}2)$$

However, to obtain w_1 and let $0<w_1<k_1$, the constraints $0<w_2<k_2$ and $0<t_1<t_2<t_3<\infty$ are required. Thus, the warning limit, w_1, can be obtained by using equation (3-2) and choosing a combination of the VSIs, (t_1, t_2, t_3), the fixed sampling interval, t_0, and the constraint $0<w_2<k_2$.

In this paper, the VSI scheme is compared with the FSI scheme and sampling scheme was considered to be better than another when it allowed the $Z_{\overline{X}}$ and Z_{S^2} charts to detect changes in the process mean and variance faster.

4. MEASUREMENT OF PERFORMANCE

The speed with which a control chart detects process shifts measures its statistical efficiency. For a VSI, the detection speed is measured by the average time from either mean or variance or both shifting until either $Z_{\overline{X}}$ and Z_{S^2} charts or both signal, which is known as the adjusted average time to signal (AATS). That is, the AATS is the mean time that the process remains out of control.

Since $T_{SC} \sim \exp(-\lambda t), \quad t > 0$, the occurrence time until the special cause occurs. Hence,

$$AATS = ATC - \frac{1}{\lambda} \tag{4-1}$$

The average time of the cycle (ATC) is the average time from the start of process until a true signal obtained from one of the proposed charts (see Costa (1997)). The Markov chain approach is allowed to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 14 states is assigned based on whether the process step is in or out of control and the position of samples (see Table 4.1 for the 14 states of the process). The status of the process when the $(i+1)^{th}$ sample is taken, and the position of the i^{th} sample on the joint $Z_{\overline{X}}$ and Z_{S^2} charts define the transient states of the Markov chain. The joint VSI $Z_{\overline{X}}$ and Z_{S^2} charts produce a signal when at least one of the samples falls outside the control limits. If the current state is any one of the States 1, 2, 4, 5, 10~13, then there is no signal. If the current state is State 3, it indicates one false signal comes from the Z_{S^2} chart then the process is adjusted unnecessarily and State 3 instantly becomes any one of the States 10~13 with probability $P_{3=j}$, and $\sum_{j=10}^{13} P_{3=j} = 1$, $j = 10 \sim 13$. Any one of the States 10~13 thus transits to any other state after a sampling time interval. States 6, 7, 8 and 9 are same to State 3. If the current state is any one of the States 1~13, then it may transit to any other state, hence States 1~13 are transient states. The absorbing state (State 14) is reached when at least one true signal occurs.

Table 4.1. Possible states for the process

state	Special cause occurs?	$Z_{\overline{X}}$ chart alarm?	Z_{S^2} chart alarm?	process adjustment?
1	No	No, $I_{Z_{\overline{X}1}}$	No, $I_{Z_{S^2 1}}$	No
2	No	No, $I_{Z_{\overline{X}1}}$	No, $I_{Z_{S^2 2}}$	No

3	No	No, $I_{Z_{\overline{X}1}}$	Yes, False alarm	Yes, incorrect adjustment
4	No	No, $I_{Z_{\overline{X}2}}$	No, $I_{Z_{S^2 1}}$	No
5	No	No, $I_{Z_{\overline{X}2}}$	No, $I_{Z_{S^2 2}}$	No
6	No	No, $I_{Z_{\overline{X}2}}$	Yes, False alarm	Yes, incorrect adjustment
7	No	Yes, False alarm	No, $I_{Z_{S^2 1}}$	Yes , incorrect adjustment
8	No	Yes, False alarm	No, $I_{Z_{S^2 2}}$	Yes, incorrect adjustment
9	No	Yes, False alarm	Yes, False alarm	Yes, incorrect adjustment
10	Yes	No, $I_{Z_{\overline{X}1}}$	No, $I_{Z_{S^2 1}}$	No
11	Yes	No, $I_{Z_{\overline{X}1}}$	No, $I_{Z_{S^2 2}}$	No
12	Yes	No, $I_{Z_{\overline{X}1}}$	No, $I_{Z_{S^2 1}}$	No
13	Yes	No, $I_{Z_{\overline{X}2}}$	No, $I_{Z_{S^2 2}}$	No
14	Yes	At least	one true alarm	Yes, correct adjustment

Let **P** be the transition probability matrix, and **P** is a square matrix of order 14. Let $P_{i,j}(t_m)$ to be the transition probability from prior state i to the current state j with sampling interval t_m, where t_m is determined by the prior state i, $\mathrm{i} = 1,2,...,14, \mathrm{j} = 1, 2,...,14, \mathrm{m} = 1, 2, 3$. The transition probability, for example, from state 1 to state 4 with sampling interval t_3 and fixed sample size n is calculated as

$$\begin{aligned} P_{1,4}(t_3) &= P(T_{sc} > t_3)P(Z_{\overline{X}} \in I_{Z_{\overline{X}2}}, Z_{S^2} \in I_{Z_{S^2 1}} \mid \delta_1 = 0, \delta_2 = 1) \\ &= (e^{-\lambda t_3})[2\Phi(k_1) - 2\Phi(w_1)][F_{\chi^2}(w_2)] \end{aligned} \quad (4\text{-}2)$$

The calculation of all transition probabilities is shown in Appendix 2.

From the elementary properties of Markov chains (see, e.g., Cinlar (1975)), the ATC is derived as

$$ATC = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}t + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}M_f + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(A)T_r \quad (4\text{-}3)$$

where $b^{'}$=(p_{01}, $\frac{p_{02}}{2}$, 0, $\frac{p_{02}}{2}$, p_{03}, 0, 0, 0, 0, 0, 0, 0, 0) is a (1×13) is the vector of starting probabilities for state 1, 2, 3,…,13, and the first sampling interval has probability p_{01} of being long (state 1 with probability p_{01}) , the probability p_{02} of being median (state 2 or stat 4 with probability $\frac{p_{02}}{2}$), and the probability p_{03} of being short (state 5 with probability p_{03}); $\mathbf{I}$ is the identity matrix of order 14; $\mathbf{Q}$ is the transition probability matrix where each element represents the transition probability, $P_{i,j}(t_m)$, from transient state i to transient state j with sampling interval t_m , where i=1,…,13, j=1,…,13, m=1, 2, 3; $\mathbf{t}^{'} = (t_3, t_2, t^*, t_2, t_1, t^*, t^*, t^*, t^*, t_3, t_2, t_2, t_1)$ is the vector of the sampling intervals for State 1~State 13, t^* is the average sampling interval for State 3, 6, 7, 8, and 9 (derivation see Appendix 2); and $M_f^{'}$=(0 0 T_{ad} 0 0 T_{ad} T_{ad} T_{ad} T_{ad} 0 0 0 0), T_{ad} is the incorrect adjustment time for States 3, 6, 7, 8 and 9. A is the vector of transition probability, $P_{i,j}(t_m)$, form transition state i, i=1,…,13, to a absorbing state j, j=14; T_r is the time to adjust the process correctly.

5. AN EXAMPLE

An example of process control for producing metallic film thickness of the computer connectors is presented, and the data of the process are measurements of the film thickness. The variable X= film thickness is measured from the end of the process. One machine could fail in the process, and shift the mean and variance on film thickness. Presently, the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts are used to monitor the process per hour (t_0=1h). Information about the state of the process is available only through sampling. When the proposed control charts indicate that the process is out of control, it requires adjustment. Sometimes, the process may be adjusted unnecessarily when at least one false signal occurs. To construct the control charts, thirty samples of size five are collected from historical data under the stable process. The estimated mean and standard deviation of variable X are ($\hat{\mu} = 210.1, \hat{\sigma} = 1.23$),

One machine could fail in the process. From historical data, the estimated failure frequency for the machine is 2 times per hour. The failure machine or the incorrect adjustment of process would shift the mean and variance of X to ($\hat{\mu} + \delta_1 \hat{\sigma}, \delta_2^2 \hat{\sigma}^2$), where $\delta_1 = 0.5$ and $\delta_2 = 2.0$. Hence, for out-of-control process, $\overline{X} \sim N(210.1 + 1.23 \bullet 0.5,\ 4\left(\frac{1.23}{\sqrt{5}}\right)^2)$. The FSI $Z_{\overline{X}}$ and Z_{s^2} charts have control limits placed at ± 3 and 16.25, respectively. Thus, approximately 5.4 false alarms are expected per 1,000 samples have in-control average run length (ARL) of 185 hours and AATS=19.482 hours. The slowness with which the FSI $Z_{\overline{X}}$ and Z_{s^2} charts detect shifts in the process (δ_1 =0.5 and δ_2 =2.0) has led the quality manager to propose building the $Z_{\overline{X}}$ and Z_{s^2} charts with VSIs, (t_1, t_2, t_3). The construction and application of the proposed VSI $Z_{\overline{X}}$ and Z_{s^2} charts is illustrated. The following are the guidelines for using the proposed control charts:

Step 1. Let the control limits $k_1 = 3$ and $k_2 = 16.25$, to maintain the average false alarm rate at around 5.4 per 1000 samples. The reciprocal of 5.4 false alarms is also the ARL, but for the in-control case of $\delta_1 = 0, \delta_2 = 1$.

Step 2. The incorrect adjustment time of the process is 0.5 hours ($T_{\text{ad}} = 0.5$).

Step 3. Since $0 < t_1 < t_2 < t_0 = 1 < t_3 < \infty$ is required. The combination (n=5, t_1=0.1 hours, t_2=0.5 hours, t_3=2) is adopted by engineers.

Step 4. Letting t_1 =0.1, t_2 =0.5, t_3 =2, n =5, $T_{\text{ad}} = 0.5$, $k_1 = 3$, $k_2 = 16.25$, $\lambda = 2$ and $0 < w_2 < 16.25$ in the equation (3-2) leads to $w_2 = 8.500$ and w_1 =0.483 with minimum AATS=3.450 using optimization technique in Fortran IMSL subroutine BCONF (Quasi-Newton approach).

Consequently, the structure of the proposed VSI $Z_{\overline{X}}$ and Z_{s^2} charts are in Figure 5.1.

$$\begin{aligned} \mathrm{UCL}_{z_{\overline{X}}} &= 3 \\ UWL_{z_{\overline{X}}} &= 0.483 \\ \mathrm{CL}_{z_{\overline{X}}} &= 0 \\ \mathrm{LWL}_{z_{\overline{X}}} &= -0.483 \\ \mathrm{LCL}_{z_{\overline{X}}} &= -3 \end{aligned} \qquad \begin{aligned} \mathrm{UCL}_{z_{s^2}} &= 16.25 \\ UWL_{z_{s^2}} &= 8.5 \\ \mathrm{LCL}_{z_{s^2}} &= 0 \end{aligned}$$

Figure 5.1 control limits of the VSI $Z_{\overline{X}}$ and Z_{s^2} charts

With the design parameters determined, the VSI $Z_{\overline{X}}$ and Z_{s^2} control charts can be used for controlling the metallic film thickness of the computer connectors with incorrect adjustment. According to the VSI scheme, if all samples, ($z_{\overline{X}}, z_{s^2}$), fall within warning limits, then the long sampling interval t_3=2.0h should be taken. If one of the samples fall within warning limits but another falls between warning and control limits, then a sampling interval t_2=0.5h should be taken. If both the samples fall between warning and control limits, then a sampling interval t_1=0.1h should be taken. If at least one signal is obtained from the control charts, then the process is stopped and adjusted.

An example using the VSI is introduced. When the process starts, a random procedure decides the first sampling interval t_3=2.0h with sample of size five. The first sample mean and variance are ($\overline{x}_1 = 210, s_1^2 = 0.625$) and the standardized values are ($z_{\overline{X}}$=0.182, z_{S^2} = 1.652). Since both the samples fall within warning limits, the second samples will be observed adopting a sample of size five after t_3=2.0h. The second sample, ($z_{\overline{X}}, z_{S^2}$), is (1.64, 10.576). Since both samples fall within warning and control limits, the third sample will be observed after t_1=0.1h. The process is stopped and adjusted when one signal is obtained.

The AATS is used to measure the performance of the proposed VSI $Z_{\overline{X}}$ and Z_{S^2} control charts. The proposed Markov chain approach is used to obtain the ATC and calculate the AATS. There are 14 possible process states, as presented in Table 4.1. Hence, the AATS is 3.450h according to equation (4-1).

The VSI scheme improves the sensitivity of the FSI $Z_{\overline{X}}$ and Z_{S^2} charts. From the example, in order to detect a shift in the mean and variance of the process, the AATS for the VSI $Z_{\overline{X}}$ and Z_{S^2} charts has been reduced from 19.482 hours to only 3.450 hours, and the saving rate of detecting time is 82.291%.

6. PERFORMANCE COMPARISONS BETWEEN ASI AND FSI SCHEMES

Table 6.1 provides the AATS of VSI and FSI schemes, which are obtained under various combinations of parameters based on orthogonal array $L_{12}(2^{11})$ table, $\lambda = 2.0, 4.0$, $\delta_1 = 0.5, 1.5$, $\delta_2 = 1.0, 2.0$, $t_1 = 0.01, 0.1$, $t_2 = 0.1, 0.5$, $t_3 = 2.0, 4.0$, $t_0 = 1.0$, Tad =0.5,1.0, $k_1 = 3$, $k_2 = 16.25$, and $n = 5$.

Comparing the AATS between the FSI and VSI $Z_{\overline{X}}$ and Z_{S^2} control charts it can be seen that the performance of the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts is much better for detecting small and median shifts ($0.5 \le \delta_1, \delta_2 \le 2.0$) in process mean and variance. The VSI $Z_{\overline{X}}$ and Z_{S^2} control charts save detection time from 21% to 82% compared to the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts. To examine the effects of various parameters on the VSI AATS, the main effects plots show the significant parameters are t_1, t_2, δ_1, δ_2 and λ (see Fig. 6.1).

Insert Table 6.1, 6.2 and Figure 6.1

Furthermore, sometimes quality engineers cannot specify the VSIs. The optimal VSI $Z_{\overline{X}}$ - Z_{S^2} control charts are thus suggested. The optimal VSI of the proposed charts are determined using optimization technique (Fortran IMSL BCONF subroutine) to minimum AATS under the same constraints and parameters as described before. The optimum AATS under various combinations of parameters are illustrated in Table 6.2. We found that the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} charts work much better than the VSI $Z_{\overline{X}}$ - Z_{S^2} charts with specific VSIs and FSI charts (Table 6.2). The optimum VSI $Z_{\overline{X}}$ and Z_{S^2} control charts save detection time more that 90% compared to the FSI and VSI $Z_{\overline{X}}$ and Z_{S^2} control charts, respectively. Since the optimum VSI $Z_{\overline{X}}$ - Z_{S^2} charts almost have the same VSIs and warning limits under various combinations of parameters. We may also call them the robust VSI $Z_{\overline{X}}$ - Z_{S^2} control charts. In reality, we may adopt the warning limits of the optimum control charts on w_1=0.0004 and $w_2 = 8.7$, and the optimum VSIs t_1=t_2=1.0h, t_3=4.1h.

Insert Table 6.2

7. CONCLUSIONS

The proposed VSI scheme controlling a process with incorrect adjustment substantially improves the performance of the FSI scheme by increasing the speed with which small and median shifts in the mean and variance of a process are detected. We have found that the VSI $Z_{\overline{X}}$ and Z_{S^2} control charts always work better (in the cases examined) than the FSI $Z_{\overline{X}}$ and Z_{S^2} control charts for small and median δ_1 and δ_2 values. The optimum VSI scheme controlling the process with incorrect adjustment is also suggested when quality engineers cannot specify the VSIs.

This paper considers a process with incorrect adjustment. However, a study of the variable sample size (VSS), variable sample size and sampling interval (VSSI) or variable parameters (VP) $Z_{\overline{X}}$ and Z_{S^2} control charts under a process with incorrect adjustment is an interesting topic for future research. Other important extensions of the proposed model can also be developed. It is straight forward to extend the proposed model to study VP control charts or other control charts, such as attribute charts, EWMA-charts, CUSUM-charts or multivariate charts.

Table 6.1 AATS for the proposed VSI control charts under various combinations of parameters

No	t_1	t_2	t_3	δ_1	δ_2	λ	T_{ad}	k	t_0	w_1*	w_2*	VSI AATS	FSI AATS	Save Time%
1	0.01	0.1	2	0.5	1	2	0.5	3	1	0.640	12.496	308.18	398.231	22.613
2	0.01	0.1	4	0.5	1	2	1	3	1	0.307	9.695	300.32	376.981	20.336
3	0.1	0.5	2	0.5	2	2	0.5	3	1	0.483	8.500	3.450	19.482	82.291
4	0.1	0.5	2	1.5	1	2	1	3	1	0.484	8.500	46.670	86.492	46.041
5	0.01	0.5	4	1.5	2	2	0.5	3	1	0.205	8.500	1.620	6.324	74.383
6	0.1	0.1	4	1.5	2	2	1	3	1	0.316	8.500	6.370	21.875	70.880
7	0.1	0.1	2	0.5	2	4	1	3	1	0.689	8.500	10.090	26.534	61.973
8	0.01	0.5	4	0.5	2	4	1	3	1	0.205	8.500	4.440	11.987	62.960
9	0.1	0.5	4	0.5	1	4	0.5	3	1	0.180	16.250	96.680	132.563	27.069
10	0.01	0.1	2	1.5	2	4	0.5	3	1	0.692	8.500	3.920	10.869	63.934
11	0.1	0.1	4	1.5	1	4	0.5	3	1	0.316	8.500	10.780	26.421	59.199
12	0.01	0.5	2	1.5	1	4	1	3	1	0.488	8.500	46.720	65.793	28.989

Note: Save Time%=[(VSI AATS-FSI AATS)/FSI AATS]100%

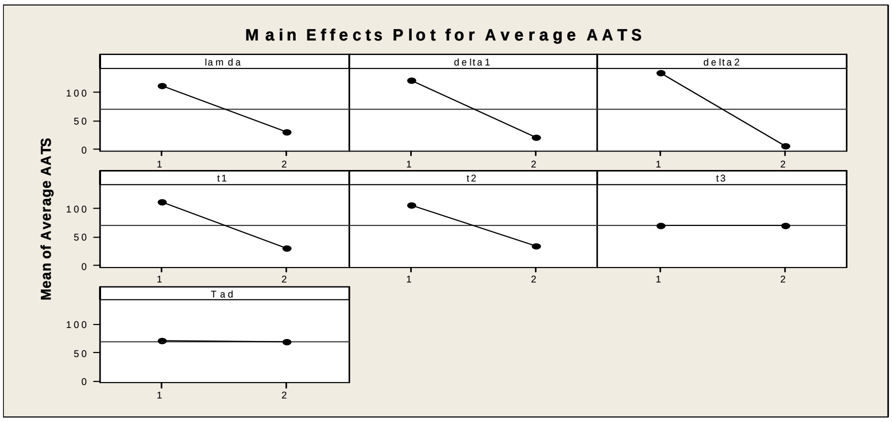

Figure 6.1 Response figures for average AATS

Table 6.2 AATS of the optimum VSI control charts under various combinations of parameters

No	t_1*	t_2*	t_3*	w_1*	w_2*	optimum AATS	FSI AATS	Save Time% (FSI)	VSI AATS	Save Time% (VSI)
1	0.999	0.999	4.077	0.0004	8.587	0.024	398.231	99.99	308.180	99.99
2	0.999	0.999	4.023	0.0004	8.564	0.548	376.981	99.85	300.320	99.82
3	0.999	0.999	4.163	0.0004	8.738	0.006	19.482	99.97	3.450	99.83
4	0.999	0.999	4.023	0.0004	8.564	0.548	86.492	99.37	46.670	99.83
5	0.999	0.999	4.153	0.0004	8.728	0.001	6.324	99.98	1.620	99.96
6	0.999	0.999	4.170	0.0004	8.748	0.501	21.875	97.71	6.370	92.15
7	0.999	0.999	4.182	0.0004	8.757	0.751	26.534	97.17	10.090	92.56
8	0.999	0.999	4.181	0.0004	8.757	0.751	11.987	93.74	4.440	83.11
9	0.999	0.999	4.057	0.0004	8.578	0.274	132.563	99.79	96.680	99.72
10	0.999	0.999	4.155	0.0004	8.726	0.250	10.869	97.70	3.920	93.62
11	0.999	0.999	4.133	0.0004	8.667	0.253	26.421	99.04	10.780	97.65
12	0.999	0.999	4.281	0.0004	8.851	0.752	65.793	98.86	46.720	98.39

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Appendix 1: The Derivation of w_1

$$t_0 = p_{01}t_3 + p_{02}t_2 + p_{02}t_2 + p_{03}t_1$$
$$= \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}t_3 + \frac{[2\Phi(w_1)\text{-}1][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}t_2 + \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}t_2$$
$$+ \frac{[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}t_1$$

$$\Rightarrow \frac{2\Phi(w_1)[F_{\chi^2}(w_2)]t_3 + 2\Phi(w_1)[F_{\chi^2}(k_2)]t_2\text{-}4\Phi(w_1)[F_{\chi^2}(w_2)]t_2\text{-}2\Phi(w_1)[F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]t_1}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}$$
$$= t_0 \text{-} \frac{\text{-}[F_{\chi^2}(w_2)]t_3 + [F_{\chi^2}(w_2)]t_2 + 2\Phi(k_1)F_{\chi^2}(w_2)t_2\text{-}2\Phi(k_1)F_{\chi^2}(w_2)t_1}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]} \text{-} \frac{\text{-}[F_{\chi^2}(k_2)]t_2 + 2\Phi(k_1)F_{\chi^2}(k_2)t_1}{[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)]}$$

$$\Rightarrow 2\Phi(w_1)\{[F_{\chi^2}(w_2)]t_3 + [F_{\chi^2}(k_2)]t_2\text{-}2[F_{\chi^2}(w_2)]t_2 + [F_{\chi^2}(k_2)]t_1\text{-}[F_{\chi^2}(w_2)]t_1\}$$
$$= t_0[2\Phi(k_1)\text{-}1][F_{\chi^2}(k_2)] + [F_{\chi^2}(w_2)]t_3\text{-}[F_{\chi^2}(w_2)]t_2\text{-}2\Phi(k_1)[F_{\chi^2}(w_2)]t_2 + 2\Phi(k_1)[F_{\chi^2}(w_2)]t_1$$

$$\Rightarrow \Phi(w_1) = \frac{F_{\chi^2}(w_2)[t_3\text{-}t_2\text{-}2\Phi(k_1)(t_2\text{-}t_1)] + F_{\chi^2}(k_2)[t_2\text{-}t_0 + 2\Phi(k_1)(t_0\text{-}t_1)]}{2[F_{\chi^2}(w_2)](t_1\text{-}2t_2 + t_3) + 2[F_{\chi^2}(k_2)](t_2\text{-}t_1)}$$
$$\Rightarrow w_1 = \Phi^{\text{-}1}\{\frac{F_{\chi^2}(w_2)[t_3\text{-}t_2\text{-}2\Phi(k_1)(t_2\text{-}t_1)] + F_{\chi^2}(k_2)[t_2\text{-}t_0 + 2\Phi(k_1)(t_0\text{-}t_1)]}{2[F_{\chi^2}(w_2)](t_1\text{-}2t_2 + t_3) + 2[F_{\chi^2}(k_2)](t_2\text{-}t_1)}\}$$

Appendix 2: Calculation of Transition probabilities

Definition of Notation:

$$P_{Z_{\overline{X}1}} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}1}} \mid \overline{X} \sim N(\mu, \frac{\sigma^2}{n})) = 2\Phi(w_1)\text{-}1.$$

$$P_{Z_{\overline{X}2}} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}2}} \mid \overline{X} \sim N(\mu, \frac{\sigma^2}{n})) = 2\Phi(k_1)\text{-}2\Phi(w_1).$$

$$P_{Z_{S^2 1}} = P(Z_{S^2} \in I_{Z_{S^2 1}} \mid Z_{S^2} \sim \frac{\chi^2_{(n\text{-}1)}}{n\text{-}1}) = F_{\chi^2}(w_2(n\text{-}1)) \ .$$

$$P_{Z_{S^2 2}} = P(Z_{S^2} \in I_{Z_{S^2 2}} \mid Z_{S^2} \sim \frac{\chi^2_{(n\text{-}1)}}{n\text{-}1}) = F_{\chi^2}(k_2(n\text{-}1))\text{-}F_{\chi^2}(w_2(n\text{-}1))$$

$$\alpha_{Z_{\overline{X}}} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}3}} \mid \overline{X} \sim N(\mu, \frac{\sigma^2}{n})) = 1\text{-}P_{Z_{\overline{X}1}}\text{-}P_{Z_{\overline{X}2}} = 2\text{-}2\Phi(k_1)$$

$$\alpha_{Z_{S^2}} = 1\text{-}P_{Z_{\bar{X}1}}\text{-}P_{Z_{\bar{X}2}} = 1\text{-}F_{\chi^2}(k_2(n\text{-}1)).$$

$$\beta_{Z_{\bar{X}1}} = P(Z_{\bar{X}} \in I_{Z_{\bar{X}1}})) = P(\left|Z_{\bar{X}}\right| < w_1 \Big| Z_{\bar{X}} \sim N(\delta_1, \delta_2^2))$$
$$= P(\frac{Z_{\bar{X}}\text{-}\delta_1}{\delta_2} < \frac{w_1\text{-}\delta_1}{\delta_2})\text{-}P(\frac{Z_{\bar{X}}\text{-}\delta_1}{\delta_2} < \frac{\text{-}w_1\text{-}\delta_1}{\delta_2}) = \Phi(\frac{w_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{w_1+\delta_1}{\delta_2})\text{-}1.$$

$$\beta_{Z_{\bar{X}2}} = P(Z_{\bar{X}} \in I_{Z_{\bar{X}2}})) = P(\left|Z_{\bar{X}}\right| < k_1 \Big| Z_{\bar{X}} \sim N(\delta_1, \delta_2^2))\text{-}P(\left|Z_{\bar{X}}\right| < w_1 \Big| Z_{\bar{X}} \sim N(\delta_1, \delta_2^2))$$
$$= \Phi(\frac{k_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{k_1+\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1+\delta_1}{\delta_2}).$$

$$\beta_{Z_{S^2 1}} = P(Z_{S^2} \in I_{Z_{S^2 1}} \Bigg| Z_{S^2} \sim \frac{\delta_2^2}{n\text{-}1}\chi^2_{(n\text{-}1)}) = F_{\chi^2}(\frac{w_2(n\text{-}1)}{\delta_2^2}).$$

$$\beta_{Z_{S^2 2}} = P(Z_{S^2} \in I_{Z_{S^2 2}} \Bigg| Z_{S^2} \sim \frac{\delta_2^2}{n\text{-}1}\chi^2_{(n\text{-}1)}) = F_{\chi^2}(\frac{k_2(n\text{-}1)}{\delta_2^2})\text{-}F_{\chi^2}(\frac{w_2(n\text{-}1)}{\delta_2^2}).$$

$$1\text{-}\beta_{Z_{\bar{X}}} = 1\text{-}\beta_{Z_{\bar{X}1}}\text{-}\beta_{Z_{\bar{X}2}} = 2\text{-}\Phi(\frac{k_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{k_1+\delta_1}{\delta_2}).$$
$$1\text{-}\beta_{Z_{S^2}} = 1\text{-}\beta_{Z_{S^2 1}}\text{-}\beta_{Z_{S^2 2}} = 1\text{-}F_{\chi^2}(\frac{k_2(n\text{-}1)}{\delta_2^2}).$$

Transition probability:

$$P_{1,1}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}1}}P_{Z_{S^2 1}} = e^{\text{-}\lambda t_3}[2\Phi(w_1)\text{-}1][F_{\chi^2}(w_2)]$$
$$P_{1,2}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}1}}P_{Z_{S^2 2}} = e^{\text{-}\lambda t_3}[2\Phi(w_1)\text{-}1][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]$$
$$P_{1,3}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}1}}\alpha_{Z_{S^2}} = e^{\text{-}\lambda t_3}[2\Phi(w_1)\text{-}1][1\text{-}F_{\chi^2}(k_2)]$$
$$P_{1,4}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}2}}P_{Z_{S^2 1}} = e^{\text{-}\lambda t_3}[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(w_2)]$$
$$P_{1,5}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}2}}P_{Z_{S^2 2}} = e^{\text{-}\lambda t_3}[2\Phi(k_1)\text{-}2\Phi(w_1)][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]$$
$$P_{1,6}(t_3) = P(T_{sc} > t_3)P_{Z_{\bar{X}2}}\alpha_{Z_{S^2}} = e^{\text{-}\lambda t_3}[2\Phi(k_1)\text{-}2\Phi(w_1)][1\text{-}F_{\chi^2}(k_2)]$$
$$P_{1,7}(t_3) = P(T_{sc} > t_3)\alpha_{Z_{\bar{X}}}P_{Z_{S^2 1}} = e^{\text{-}\lambda t_3}[2\text{-}2\Phi(k_1)][F_{\chi^2}(w_2)]$$
$$P_{1,8}(t_3) = P(T_{sc} > t_3)\alpha_{Z_{\bar{X}}}P_{Z_{S^2 2}} = e^{\text{-}\lambda t_3}[2\text{-}2\Phi(k_1)][F_{\chi^2}(k_2)\text{-}F_{\chi^2}(w_2)]$$
$$P_{1,9}(t_3) = P(T_{sc} > t_3)\alpha_{Z_{\bar{X}}}\alpha_{Z_{S^2}} = e^{\text{-}\lambda t_3}[2\text{-}2\Phi(k_1)][1\text{-}F_{\chi^2}(k_2)]$$

$$P_{1,10}(t_3) = P(T_{sc} < t_3)\beta_{Z_{\bar{X}1}}\beta_{Z_{s^2 1}} = (1\text{-}e^{-\lambda t_3})[\Phi(\frac{w_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{w_1+\delta_1}{\delta_2})\text{-}1][F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{1,11}(t_3) = P(T_{sc} < t_3)\beta_{Z_{\bar{X}1}}\beta_{Z_{s^2 2}} = (1\text{-}e^{-\lambda t_3})[\Phi(\frac{w_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{w_1+\delta_1}{\delta_2})\text{-}1][F_{\chi^2}(\frac{k_2}{\delta_2^2})\text{-}F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{1,12}(t_3) = P(T_{sc} < t_3)\beta_{Z_{\bar{X}2}}\beta_{Z_{s^2 1}} = (1\text{-}e^{-\lambda t_3})[\Phi(\frac{k_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{k_1+\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1+\delta_1}{\delta_2})][F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{1,13}(t_3) = P(T_{sc} < t_3)\beta_{Z_{\bar{X}2}}\beta_{Z_{s^2 2}}$$
$$= (1 - e^{-\lambda t_3})[\Phi(\frac{k_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{k_1+\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1+\delta_1}{\delta_2})][F_{\chi^2}(\frac{k_2}{\delta_2^2})\text{-}F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{1,14}(t_3) = 1 - \sum_{j=1}^{13} P_{1,13}(t_3)$$

$$P_{2,j}(t_2) = P_{1,j}(t_2) \quad , j = 1,2,3,\ldots,14$$

$$P_{4,j}(t_2) = P_{1,j}(t_2) \quad , j = 1,2,3,\ldots,14$$

$$P_{5,j}(t_1) = P_{1,j}(t_1) \quad , j = 1,2,3,\ldots,14$$

$$P_{10,j}(t_3) = 0 \qquad ,j = 1,2,,\ldots,9$$

$$P_{10,10}(t_3) = \beta_{Z_{\bar{X}1}}\beta_{Z_{s^2 1}} = [\Phi(\frac{w_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{w_1+\delta_1}{\delta_2})\text{-}1][F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{10,11}(t_3) = \beta_{Z_{\bar{X}1}}\beta_{Z_{s^2 2}} = [\Phi(\frac{w_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{w_1+\delta_1}{\delta_2})\text{-}1][F_{\chi^2}(\frac{k_2}{\delta_2^2})\text{-}F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{10,12}(t_3) = \beta_{Z_{\bar{X}2}}\beta_{Z_{s^2 1}} = [\Phi(\frac{k_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{k_1+\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1+\delta_1}{\delta_2})][F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{10,13}(t_3) = \beta_{Z_{\bar{X}2}}\beta_{Z_{s^2 2}} = [\Phi(\frac{k_1\text{-}\delta_1}{\delta_2}) + \Phi(\frac{k_1+\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1\text{-}\delta_1}{\delta_2})\text{-}\Phi(\frac{w_1+\delta_1}{\delta_2})][F_{\chi^2}(\frac{k_2}{\delta_2^2})\text{-}F_{\chi^2}(\frac{w_2}{\delta_2^2})]$$

$$P_{10,14}(t_3) = 1 - \sum_{j=1}^{13} P_{10,j}(t_3)$$

$$P_{11,j}(t_2) = P_{10,j}(t_2) \quad ,j = 1,2,3,\ldots,14$$
$$P_{12,j}(t_2) = P_{10,j}(t_2) \quad ,j = 1,2,3,\ldots,14$$
$$P_{13,j}(t_1) = P_{10,j}(t_1) \quad ,j = 1,2,3,\ldots,14$$

$$P_{i,j} = 1 \quad ,i = 14 \ ;j = i$$
$$P_{i,j} = 0 \quad ,i = 14 \ ;j \neq i$$

$$P_{i,j}(t^*) = 0, \quad i = 3,6,7,8,9, j \neq 10 \sim 14$$
$$P_{i,j}(t^*) = P_{i=10}P_{10,j}(t_3) + P_{i=11}P_{11,j}(t_2) + P_{i=12}P_{12,j}(t_2) + P_{i=13}P_{13,j}(t_1), j = 10 \sim 14, i = 3,6,7,8,9$$

where

$$t^* = p_{i=10} \bullet t_3 + p_{i=11} \bullet t_2 + p_{i=12} \bullet t_2 + p_{i=13} \bullet t_1$$

$$p_{i=10} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}_1}}, Z_{S^2} \in I_{Z_{S^2{}_1}} \left| \overline{X} \sim N(\mu + \delta_1 \frac{\sigma}{\sqrt{n}}, \frac{\delta_2^2 \sigma^2}{n}) \right.)$$

$$= \frac{[\Phi(\frac{W_1 - \delta_1}{\delta_2}) + \Phi(\frac{W_1 + \delta_1}{\delta_2}) - 1][F_{\chi^2}(\frac{W_2(n-1)}{\delta_2^2})]}{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - 1]\,[F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2})]}$$

$$p_{i=11} = P(Z_1 \in I_{Z_{\overline{X}_1}}, Z_{S^2} \in I_{Z_{S^2{}_2}} \left| \overline{X} \sim N(\mu + \delta_1 \frac{\sigma}{\sqrt{n}}, \frac{\delta_2^2 \sigma^2}{n}) \right.)$$

$$= \frac{[\Phi(\frac{W_1 - \delta_1}{\delta_2}) + \Phi(\frac{W_1 + \delta_1}{\delta_2}) - 1][F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2}) - F_{\chi^2}(\frac{W_2(n-1)}{\delta_2^2})]}{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - 1][F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2})]}$$

$$p_{i=12} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}_2}}, Z_{S^2} \in I_{Z_{S^2{}_1}} \left| \overline{X} \sim N(\mu + \delta_1 \frac{\sigma}{\sqrt{n}}, \frac{\delta_2^2 \sigma^2}{n}) \right.)$$

$$= \frac{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - \Phi(\frac{W_1 - \delta_1}{\delta_2}) - \Phi(\frac{W_1 + \delta_1}{\delta_2})][F_{\chi^2}(\frac{W_2(n-1)}{\delta_2^2})]}{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - 1][F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2})]}$$

$$p_{i=13} = P(Z_{\overline{X}} \in I_{Z_{\overline{X}_2}}, Z_{S^2} \in I_{Z_{S^2{}_2}} \left| \overline{X} \sim N(\mu + \delta_1 \frac{\sigma}{\sqrt{n}}, \frac{\delta_2^2 \sigma^2}{n}) \right.)$$

$$= \frac{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - \Phi(\frac{W_1 - \delta_1}{\delta_2}) - \Phi(\frac{W_1 + \delta_1}{\delta_2})][F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2}) - F_{\chi^2}(\frac{W_2(n-1)}{\delta_2^2})]}{[\Phi(\frac{K_1 - \delta_1}{\delta_2}) + \Phi(\frac{K_1 + \delta_1}{\delta_2}) - 1][F_{\chi^2}(\frac{K_2(n-1)}{\delta_2^2})]}$$

參加 ISBIS 2007 會議報告　　　楊素芬

此次第六屆商業與工業統計會議於葡萄牙亞宿島(Acores, Portugal)舉行，目的在讓工業統計和商業統計的學者，研究者或實務者能聚在一起，分享新想法和技術。發表的文章廣泛包含統計在各領域的應用，如工業上品質和可靠度上的應用，生物科技的應用，健康，管理，財務，商業上的應用，和資料擷取等。這是本人第四次參加 ISBIS，第一次到葡萄牙，心中既期待又興奮。

會議共 3 天(8 月 18 至 20 日)。第一天一早是 Opening session，由 ISBIS 理事長和會議負責人歡迎與會者及說明會議籌備經過。接著由美國學者發表 text document data mining 的研究情形。接下來每段時間都有六場平行的演講;爲實驗設計、統計製程管制、財務統計問題、商業統計、多變量統計和統計理論等，而我都選擇與研究領域相關的統計製程管制和品質改善的場次聽講。8 月 18 日 11:00~12:30am 統計製程管制場次首由葡萄牙代表 Fernanda Otilia Sousa Figueiredo 報告如何建立 Robust control chart 以追蹤非常態的製程管制問題，並以模擬數據分析和比較其績效。接著由葡萄牙 Galp Energia 公司報告其公司之品質管理過程(distributed control system and automatic control process)，績效非常好。我問報告者數據是否符合常態分配他說系統會自動調整數據。這倒是令我不敢相信其正確性。最後由葡萄牙博士生 Elisabete Carolino 報告非常態分配(exponential and Weibull distributions)之允收抽樣方法和以模擬數據分析比較其績效。這場讓我了解葡萄牙學界和業界在品質管理領域的研究和實用情形。

8 月 19 日早上第一場由日本學者介紹日本最近的 TQM 活動。Dr. Amasaka 提出新 TQM 原理，包含 TMS (total marketing system), TDS (total development system), TPS (total production system), TIS (total intelligence management system) and TJS (total job quality management system) 。這是新解。 接著, Dr Tsubaki 說明統計方法在技術發展上包括 Taguchi method and QFD 的重要,而 QI story 則用到簡單的統計方法。此外行銷研究也越重視統計方法的應用。最後由提出二維品質創新的 Dr Kano 演講 six sigma 的改善步驟 DMADV 來自何處? Dr Kano 認爲 GE company 並未說明 DMADV 出處，而他認爲 DMADV 其實就是 QI story。這點也是在校上品管課時我對學生說明的見解。

8 月 19 日第二場主題是製程管制與改善。主持人是巴西的 Dr. Epprecht。第一個演講者 Dr. Denby 說明如何對工作流程做分析以改善過程的品質與速度。非常實用，這是目前火紅的六標準差所強調的。

接著 Dr. Epprecht 報告多注頭製程的 SPC 在理論和實務的差異。他除了回顧相關文獻外，也說明實務應用上的困難和問題。多年前，我曾對多注頭製程做相關研究，有二篇文章刊登於期刊上，後來沒再繼續。看來多注頭製程管制是還有一些研究題目可做。

最後 Dr. Czitrom 報告工業實驗圖解法，說明如何對工程師以圖形解釋資料分析結果，才能使工程師容易接受以統計方法做品質改善。這在實務上非常有效。

下午，刁錦寰大師受邀演講統計學在商學院領域的發展與應用。刁大師是時

間數列的大師，又服務於芝加哥大學商學院，他說明統計在商學各領域的重要，尤其是財務金融方面。

最後一場的 SPC 由美國健康統計中心的 Dr. Choi 主持。首先由南韓 Dr. Park 介紹如何對非對稱分配的數據建立百分比管制圖以追蹤製程，效果比用 Shewhat Chart 好。

接著 Dr. Lahirl 提出實證貝氏品質量測計畫。最後 Dr. Choi 報告多變量管制圖可以貝氏方法降低其維度爲 2，以提昇多變量管制圖之績效，作者並以半導體製程爲例。

晚上大會準備 Gala dinner 歡迎與會的人。地點在學校附近的大餐廳。晚餐品嚐葡式料理和甜點並有民俗表演，令賓主盡歡。

本人的論文發表在 8 月 20 日早上第一場。第一位是義大利的 Dr. Fichera 報告 S^2-Cusum 管制圖的建立方法。Ficheraa 和 Castagliolu 有幾篇文章是有關 S^2-EWMA 管制圖的探討。這次，他們將此方法應用於 Cusum 圖的建立。接著由我介紹不正確製程調整下 VSI 管制圖的建立。有聽者問到如何調整製程，也有聽者認爲實務上變異數調整並不容易。我接受他們的看法，因爲本文是完全在理論上做推導的，實務上可能碰到的問題都假設可以解決的。事實上，製程調整我認爲是可以由另一種 Engineering Process Control 的技術來完成的，而這並不是 SPC 的功能。接著，由來自巴西的學者報告，多變量製程管制的優缺點，和多變量管制圖在包裝產品封口的應用，非常實用且績效不錯。

大會最後一天下午安排半日 Tour，遊到亞宿島中西部參觀有名的火山溫泉，地熱和製茶廠品茶。聰明的亞宿島人並以地熱煮熟食物，這些豐盛的食物變成我們的午宴，非常美味。大會用心的安排與服務，使得參與者盡興而歸。

ISBIS 3 天的研討會到此結束。此研討會強調統計方法在工業和商業等理論和實務問題的解決、創新和應用，所以偏重應用績效和理論發展，這可以很容易令人了解統計的重要並進一步促進產學合作和多多進用統計人才，讓統計人發揮最大的功效。類似這種研討會應可以多多舉辦。

國內統計界的研討會多偏重學術，甚少辦較應用的研討會，但應多鼓勵以使一部份的統計人能走出外面與實務結合，並使參與學生體會統計方法的應用與績效且能廣泛被業界所招覽而發揮學以致用之功效。

攜回論文集摘要本和研討會議程表一份。

此次台灣來參加研討會的只有 2 人；清大曾勝滄教授和我。亞宿島人普遍不知道亞洲的台灣，但因是觀光島所以英文都很好，所以溝通上問題少。出遊參加旅行團是較好之選擇。會議後，我和曾教授夫婦三人自行到亞宿島市西北邊看火山湖，看溫泉，一路上繡球花奇大鮮艷而黃薑花盛開且整齊排列真是漂亮。據說是此火山島土地肥沃造成。這些景物真是奇美無比。這次所到之處我們都告訴葡萄牙人，我們來自台灣。參加研討會及出遊不僅可促進學術交流，可進行國民外交且增廣見聞，建議國科會能多鼓勵和補助學者到較無邦交的國家參加國際研討會，特別是多鼓勵以團對方式參加國際研討會以更提升台灣的知名度和促進國民外交。