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兩個一致性的模型設定新檢定
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摘要

一般而言, 經濟或計量模型都可以藉由「條件動差限制式」來加以定義。因此, 如何利用條件動差限制式來檢驗模型設定的正確與否一直是文獻上的重要議題。其中, 廣為人知的檢定是 Bierens (1982, *Journal of Econometrics*) 所提出的「條件動差積分型檢定」(Integrated Conditional Moment test, 簡稱為 ICM 檢定)。然而, 這類的 ICM 檢定有一些已知的缺點: 它需要對應於虛無假設下的一致性參數估計式、需要運用數值方法以進行積分運算, 並且其檢定統計量的極限分配會隨著資料的特性而不同。這些特徵都增加了 ICM 檢定於實際操作時的難度。準此, 這個計畫提出了兩個方式來改善此類 ICM 檢定。第一個方法結合了 ICM 檢定及傅利葉分析, 進而提出了一個容易操作的檢定。這個檢定統計量和 ICM 檢定具有相同的極限分配, 但是它不需仰賴數值方法進行積分。再者, 由於此方法自然而然地將條件動差限制下的參數估計及模型檢定相結合, 因此我們也不需要額外的一致性參數估計式。除此之外, 我更擴展了這第一個方法的精神進而得到了一組無窮多條「無條件動差限制式」, 並與一般化的實證概似法 (generalized empirical likelihood) 結合, 建立了一個概似比例型態 (Likelihood-Ratio type) 的檢定統計量。這是我於此計畫中所提出的第二個檢定方法。在一些條件假設下, 此統計量經過適當的標準化後, 其極限分配將會是和原始資料型態無關的標準常態分配。針對這兩種新的檢定, 我們建立了對應的極限性質, 也提供了模擬的結果。

關鍵詞: 一致性檢定, 條件動差限制式, 條件動差積分型檢定, 傅力葉分析, 一般化的實證概似法

Abstract

Economic and econometric models are usually defined by conditional moment restrictions. Hence, checking the validity of models through these conditional moment restrictions is a central issue in the literature. One of the popular consistent tests is the Integrated Conditional Moment (ICM) test proposed by Bierens (1982, *Journal of Econometrics*). This ICM-type test, however, suffers from some drawbacks: it needs a preliminary consistent estimate of model parameters under the null, the numerical method of integrations, and it is not pivotal. These features make the implementation of this ICM-type test cumbersome. This project proposes two approaches to improve the ICM test. The first proposed test statistic is an easy-to-implement version of ICM test by extending the ICM test and the Fourier analysis. It is asymptotic equivalent to the ICM test statistic, but it has an analytic form instead. Moreover, no preliminary consistent estimate is needed because this unified approach relates estimation and diagnostic testing in a rather natural way. Besides, I extend the idea behind the first approach by deriving a new set of infinite many unconditional moment restrictions, and employ generalized empirical likelihood (GEL) method to construct the Likelihood-Ratio type test statistic. After suitable normalization, the asymptotic distribution of the proposed empirical LR test statistic should be standard normal, which is pivotal. For these two proposed tests, we establish the corresponding asymptotics and provide some Monte Carlo simulation results.

Keywords: consistent test, conditional moment restrictions, integrated conditional moment test, Fourier analysis, generalized empirical likelihood method, pivotal

1 Introduction

Economic and econometric models are usually defined by conditional moment restrictions, for example, the Euler equations in various rational expectation models. How to consistently (and efficiently) estimate parameters of models through these moment restrictions is thus an important issue and has been considered by Chamberlain (1987), Donald et al. (2003), Domínguez and Lobato (2004) and Hsu and Kuan (2010) to mention just a few. On the other hand, because a correct model specification implies the certain zero conditional moments, checking the validity of models through these conditional moment restrictions is thus another central issue in the literature. A good test should have power approaching one asymptotically for any deviations from the null. Accordingly, the tests should be constructed to against general alternatives, and they are said to be "consistent". To provide *consistent* model specification tests is the main purpose of this work.

Based on the idea that unconditional moment restrictions can be induced from conditional moment restrictions, many researchers propose the conditional moment tests by testing whether some (finitely) induced unconditional moment are zeros or not, see Newey (1985), Tauchen (1985), among others. It is well known that these tests are in general not consistent because they are "directional". Other than the specified alternatives, they may not be able to detect all deviations from the null. A way to deliver a consistent model specification test is taking all induced unconditional moment restrictions into account, then any deviations from the null will be revealed by some of these unconditional moments. In this framework, we test the conditional moment restrictions "indirectly" by testing induced unconditional ones. On the other hand, some consistent model specification tests are proposed by using nonparametric methods to measure the "distance" between conditional moment restrictions and zero "directly", Hong and White (1995), Zheng (1996) and Fan and Li (2000) are a few examples.

In order to achieve consistency, Bierens (1982, 1990) consider *infinite many* induced unconditional moment restrictions by employing a class of weighting functions indexed by a continuous nuisance parameter. Stinchcombe and White (1998) provide and characterize the features of these weighting functions. A test statistic by integrating these nuisance parameters out is first proposed by Bierens (1982), and Bierens and Ploberger (1997) provide the general asymptotics for this test and name it by the Integrated Conditional Moment (ICM) test. Theoretically, the ICM test has nontrivial local power, and is asymptotically admissible under the normal errors assumption. Boning and Sowell (1999) also show that the ICM test proposed by Bierens (1982) and Bierens and Ploberger (1997) is the best ICM test according to the weighted average power criterion considered by Andrews and Ploberger (1994).

This ICM-type test, however, suffers from three drawbacks. First, the preliminary consistent estimate of model parameters under the null is necessary in forming the statistics. A inconsistent estimate will give the wrong type I error. This point is well known and some examples are provided by Domínguez and Lobato (2006). It means that we may need another estimation method which can deliver a consistent estimate given the conditional moment restrictions under the null before testing. Second, the numerical method of integrations is always needed in computing these test statistics because there are no analytic forms in general. These two features make the implementation of this ICM-type tests cumbersome. Third, this type of test is not pivotal while the asymptotic distribution and critical values depend on the underlying data generating process. Stinchcombe and White (1998) and Bierens and Ploberger (1997) provide data-independent bounds of the critical values; Whang (2001), Domínguez and Lobato (2006) and Hsu and Kuan (2008) infer based on Bootstrap methods instead.

In order to improve the ICM test, this project proposes two approaches. The first proposed test statistic is to present a global methodology for performing consistent statistical inference on model specification by extending the ICM test and the results in Hsu and Kuan (2010). This proposed test statistic is asymptotic equivalent to the ICM test statistic, but it has an analytic form instead. Moreover, because this unified approach relates estimation and diagnostic testing in a rather natural way, we need no preliminary consistent estimate for the parameters of the model under the null. Roughly speaking, I provide an easy-to-implement version of ICM test in this approach. However, because this test is asymptotic equivalent to the ICM test, it is not pivotal, either. Bootstrap methods are employed for inference. Note that the consistent linearity test statistic proposed in Hsu and Kuan (2008) is just a special case of this test.

On the other hand, in order to get a pivotal test, I extend the idea behind the first approach. I derive a new set of infinite many unconditional moment restrictions and employ generalized empirical likelihood (GEL) method to construct the Likelihood-Ratio (LR) type test statistic. In the literature, GEL method has been well established and applied to estimation and diagnostic checking for models, see Imbens (2002), Kitamura et al. (2004), and Donald et al. (2003) among others. After suitable normalization, the asymptotic distribution of the proposed empirical LR test statistic should be standard normal, which is pivotal. Note that this approach also needs no preliminary consistent estimate for the parameters of the model under the null. To my best knowledge, this is the first attempt to link the GEL method to ICM test for model specification testing.

The remainder of this paper is organized as follows. The preliminaries about this issue is given in in Section 2. Section 3 describes two proposed test statistics and their asymptotics. In Section 4, we show some Monte Carlos simulations. Finally, Section 5 concludes.

2 Preliminaries

Assume all random variables are defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and denote $\sigma(\boldsymbol{X}) \subset \mathcal{F}$ the minimal σ-algebra such that $\boldsymbol{X} : \Omega \to \mathbb{R}^k$ measurable. In what follows, I consider a class of models $\mathcal{M} := \left\{ f(\cdot, \theta) : \mathbb{R}^m \to \mathbb{R} \middle| \theta \in \Theta \right\}$, where $\Theta \subset \mathbb{R}^p$, then we say the model is correctly specified if there exists some θ_o such that $f(\boldsymbol{X}, \theta_o)$ is a version of the conditional expectation of Y relative to $\mathcal{F}$. As a result, the null and alternative of this model specification test can be represented as

$$H_0 : \mathbb{P}\left(\mathbb{E}\left[Y|\boldsymbol{X}\right] = f(\boldsymbol{X}, \theta_o)\right) = 1 \text{ for some } \theta_o \in \Theta \subset \mathbb{R}^p, \tag{1}$$

against

$$H_1 : \mathbb{P}\left(\mathbb{E}\left[Y|\boldsymbol{X}\right] = f(\boldsymbol{X}, \theta)\right) < 1 \text{ for all } \theta \in \Theta. \tag{2}$$

Obviously, this test is portmanteau since no particular models are specified in the alternative.

Let $\boldsymbol{Z} = (Y, \boldsymbol{X})$ and $\boldsymbol{z}_t = (y_t, \boldsymbol{x}_t)$ is observable data for $t = 1, \ldots, T$.[1] Denote $\epsilon(\boldsymbol{Z}, \theta) = Y - f(\boldsymbol{X}, \theta)$ the residual function of the model, then the null hypothesis (1) suggests to test the conditional moment restriction

$$\mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) \middle| \boldsymbol{X}\right] = 0, \quad \text{with probability one (w.p.1 henceforth).} \tag{3}$$

As well known, this conditional moment restriction (3) implies $\mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(\boldsymbol{X})\right] = 0$, for any measurable function $w(\boldsymbol{X})$. Since there are infinite many implied unconditional moment restrictions, intuition suggests that any tests based on an arbitrary finite set of them can not detect all deviations from the null. That's why the CM tests in Newey (1985) test and Tauchen (1985) are not consistent.

In order to obtain a consistent CM test, one may systematically consider all these unconditional moment restrictions, by using some indexed functions. Let $w(\boldsymbol{X}, \boldsymbol{\xi})$ be that function with index $\boldsymbol{\xi} \in \boldsymbol{\Xi}$, where $\boldsymbol{\Xi}$ is the nonempty set depended on $w(\cdot)$. A consistent test can then be constructed by testing

$$H_0 : \mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(\boldsymbol{X}, \boldsymbol{\xi})\right] = 0, \forall \boldsymbol{\tau} \in \boldsymbol{\Xi}, \text{ for some } \theta_o \in \Theta, \quad \text{w.p.1.}$$

Given this null hypothesis which involves infinite many unconditional moment restrictions, we may form the tests based on the L_q norm:

$$H_0 : \left[\int_{\boldsymbol{\Xi}} \left| \mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(\boldsymbol{X}, \boldsymbol{\xi})\right] \right|^q d\mu(\boldsymbol{\xi}) \right]^{1/q} = 0, \text{ for some } \theta_o \in \Theta, \quad \text{w.p.1,} \tag{4}$$

[1] Note that $\boldsymbol{x}_t$ may contain a finite number of lagged y_t.

where $1 \leq q < \infty$, and μ is a given probability measure on $\boldsymbol{\Xi}$ which is absolutely continuous with respect to Lebesgue measure on $\boldsymbol{\Xi}$; see Stute (1997), Koul and Stute (1999), Bierens (1982), and Bierens and Ploberger (1997). On the other hand, one may also test the null based on the supremum norm:

$$H_0 : \sup_{\boldsymbol{\xi} \in \boldsymbol{\Xi}} \left| \mathbb{E} \left[\epsilon(\boldsymbol{Z}, \theta_o) w(\boldsymbol{X}, \boldsymbol{\xi}) \right] \right| = 0, \text{ for some } \theta_o \in \Theta, \quad \text{w.p.1.} \tag{5}$$

Bierens (1990) and some Kolmogorov-Smirnov-type tests are based on this null.

3 The Proposed Approaches

Two approaches to testing model specification consistently are proposed and their asymptotics are established in this section. In order to illustrate the idea more easily, we consider the univariate X (and hence a scalar ξ) in what follows. Extensions to multivariate $\boldsymbol{X}$ is rather straightforward.

3.1 The proposed approach (I)

Given the preliminary consistent estimate $\hat{\theta}_T$ of θ_o under the null, the original ICM test statistic of Bierens and Ploberger (1997) takes the form

$$\eta_T(\hat{\theta}_T) = \int_{\Xi} \left| z(\xi, \hat{\theta}_T) \right|^2 d\mu(\xi) = \int_{\Xi} \left| \frac{1}{\sqrt{T}} \sum_{i=1}^{T} \epsilon(z_t, \hat{\theta}_T) w(x_t, \xi) \right|^2 d\mu(\xi), \tag{6}$$

the integration here could be cumbersome. Since $z(\xi, \hat{\theta}_T)$ is a function of ξ given $\hat{\theta}_T$, it has its own Fourier series representation. To be more precise, denote $\{\psi_m(\cdot)\}$ the Fourier series which is orthonormal and complete in the space $C(\Xi)$ of continuous real functions on Ξ as well as on the space $L_2(\mu)$, then

$$z(\xi, \hat{\theta}_T) = \sum_{m=1}^{\infty} \mathcal{C}_m(\hat{\theta}_T) \psi_m(\xi),$$

where $\mathcal{C}_m(\hat{\theta}_T)$ is the corresponding Fourier coefficient

$$\begin{aligned} \mathcal{C}_m(\hat{\theta}_T) &= \int_{\Xi} z(\xi, \hat{\theta}_T) \psi_m(\xi) d\mu(\xi) = \int_{\Xi} \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \epsilon(z_t, \hat{\theta}_T) w(x_t, \xi) \psi_m(\xi) d\mu(\xi) \\ &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \epsilon(z_t, \hat{\theta}_T) \int_{\Xi} w(x_t, \xi) \psi_m(\xi) d\mu(\xi) := \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \epsilon(z_t, \hat{\theta}_T) \varpi_m(x_t), \end{aligned}$$

with $\varpi_m(\cdot) = \int_\Xi w(\cdot, \xi)\psi_m(\xi)d\mu(\xi)$. It shows that each $\varpi_m(\cdot)$ can be viewed as a "weighted average" of all $w(\cdot, \xi)$ for $\xi \in \Xi$. Any deviations from the null detected by some $w(\cdot, \xi)$ can also be revealed by each $\varpi_m(\cdot)$. Besides, the integration in $\varpi_m(x_t)$ is much easy to compute, and it may have closed form if we select matched weighting function w and Fourier series, for example, w is exponential function and ψ_m is exponential Fourier series.

After invoking Paserval's Theorem, the ICM test statistic $\eta_T(\hat{\theta}_T)$, which is the L_2 norm of $z(\xi, \hat{\theta}_T)$, can then be expressed as the summation of the magnitudes of the corresponding Fourier coefficients:

$$\eta_T(\hat{\theta}_T) = \int_\Xi \left| z(\xi, \hat{\theta}_T) \right|^2 d\mu(\xi) = \sum_{m=1}^{\infty} \left| \mathcal{C}_m(\hat{\theta}_T) \right|^2.$$

In this step, we simply the construction of $\eta_T(\hat{\theta}_T)$ from integration to summation. In accordance with Bessel's inequality, $|\mathcal{C}_m|$ is close to zero when m is large enough and hence is not helpful to detect the deviations form the null. Therefore, given $\hat{\theta}_T$, we can consider

$$\eta_T^*(\hat{\theta}_T, m_T) = \sum_{m=1}^{m_T} \left| \mathcal{C}_m(\hat{\theta}_T) \right|^2,$$

where m_T is some positive integer and needs to grow with the sample size T to ensure this test to be consistent. Compared with the original ICM test statistic $\eta_T(\hat{\theta}_T)$ in (6), the computation of this statistic is rather easy in practice. Besides, Paserval's Theorem ensures that $\eta_T^*(\hat{\theta}_T, \infty) = \eta_T(\hat{\theta}_T)$. It means that $\eta_T^*(\hat{\theta}_T, m_T)$ shares the same asymptotics with the ICM statistic $\eta_T(\hat{\theta}_T)$ theoretically, if m_T is allowed to increase to infinity. Notice that the consistent linearity test statistic proposed in Hsu and Kuan (2008) is just a special case of $\eta_T^*(\hat{\theta}_T, m_T)$.

Recall that a preliminary consistent estimate $\hat{\theta}_T$ is crucial in above analysis. In this part of this project, I propose a new test statistic for model specification without any preliminary consistent parameter estimates instead. That is

$$J_T^1(m_T) = \min_{\theta \in \Theta} \eta_T^*(\theta, m_T) = \min_{\theta \in \Theta} \sum_{m=1}^{m_T} |\mathcal{C}_m(\theta)|^2. \tag{7}$$

The asymptotics of this $J_T^1(m_T)$ test statistic can be easily established based on the results of Bierens(1982, 1990), Bierens and Ploberger (1997), Stinchcombe and White (1998) and Hsu and Kuan (2010). To see this, we impose the following conditions.

[A1] The observed data $\boldsymbol{z}_t = (y_t, x_t)'$, $t = 1, \ldots, T$, are independent realizations of $\boldsymbol{Z} = (Y, X)'$.

[A2] For each $\theta \in \Theta$, $\epsilon(\cdot, \theta)$ is measurable, and for each $\boldsymbol{z}$, $\epsilon(\boldsymbol{Z}, \cdot)$ is continuous on Θ, where Θ is a compact subset in $\mathbb{R}^p$. Also, θ_o in Θ is the unique solution to $\mathbb{E}[\epsilon(\boldsymbol{Z}, \theta)|X] = 0$ under H_0.

[A3] $\mathbb{E}[\sup_{\theta\in\Theta}|\epsilon(\boldsymbol{Z},\theta)|^4|X] < \infty$; $\epsilon(\boldsymbol{Z},\theta)$ is twice continuously differentiable in a neighborhood of θ_o, the corresponding first and second derivatives are bounded, and the second moment of the first derivative is nonsingular.

[A4] The function $w(\cdot)$ is generically comprehensive revealing.

Given the local alternative of the form

$$H_1^L : \mathbb{E}[Y|X] = f(\boldsymbol{X},\theta_o) + \frac{g(\boldsymbol{X})}{\sqrt{T}} \quad \text{for some } \theta_o \in \Theta, \quad \text{w.p.1}, \tag{8}$$

where function g is measurable with respect to $\mathcal{F}$. Notice that when g is a zero function, the local alternative degenerates to the null of interest. Define

$$\phi_t(\boldsymbol{\xi}) = w(\boldsymbol{x}_t,\boldsymbol{\xi}) - B(\theta_o,\boldsymbol{\xi})'A(\theta_o)^{-1}\nabla_{\theta'} f(\boldsymbol{x}_t,\theta_o)$$

with $A(\theta_o) = \text{plim}_{T\to\infty}\frac{1}{T}\sum_{t=1}^T [\nabla_{\theta'} f(\boldsymbol{x}_t,\theta_o)][\nabla_{\theta'} f(\boldsymbol{x}_t,\theta_o)]'$ and $B(\theta_o,\boldsymbol{\xi}) = \text{plim}_{T\to\infty}\frac{1}{T}\sum_{t=1}^T \nabla_{\theta'} f(\boldsymbol{x}_t,\theta_o)w(\boldsymbol{x}_t,\boldsymbol{\xi})$. Under H_1^L in (8), a Taylor expansion of $z(\boldsymbol{\xi},\hat{\theta}_T)$ around θ_o and laws of large numbers yield

$$\begin{aligned} z(\boldsymbol{\xi},\hat{\theta}_T) &= \frac{1}{\sqrt{T}}\sum_{t=1}^T \epsilon(\boldsymbol{Z},\theta_o)\phi_t(\boldsymbol{\xi}) + \frac{1}{T}\sum_{t=1}^T g(\boldsymbol{x}_t)\phi_t(\boldsymbol{\xi}) + o_{\mathbf{P}}(1) \\ &:= \mathcal{Z}_T(\boldsymbol{\xi}) + \mathcal{Z}_T^g(\boldsymbol{\xi}) + o_{\mathbf{P}}(1). \end{aligned}$$

Under either the null or (local) alternative, the asymptotics of test statistic J_T^1 are established based on the limiting processes of $\mathcal{Z}_T(\boldsymbol{\xi})$ and $\mathcal{Z}_T^g(\boldsymbol{\xi})$.

Theorem 3.1 *(Asymptotics of J_T^1 test)*

Given conditions [A1]–[A4], and If $m_T = o(T^{1/2})$ as $T \to \infty$, we have

(a) For all $\theta \in \Theta$,

$$\eta_T^*(\theta,m_T) \xrightarrow{P} \int_\Xi \left|\mathbb{E}[\epsilon(\boldsymbol{Z},\theta)w(X,\xi)]\right|^2 d\mu(\xi).$$

(b) $\arg\min_{\theta\in\Theta}\eta_T^*(\theta,m_T) \xrightarrow{P} \theta_o$ under H_0 in (1).

(c) under H_0 in (1),

$$J_T^1(m_T) \xrightarrow{d} \int_{\boldsymbol{\Xi}} |\mathcal{Z}(\boldsymbol{\xi})|^2\, d\boldsymbol{\xi},$$

where $\mathcal{Z}(\boldsymbol{\xi})$ the limiting function of $\mathcal{Z}_T(\boldsymbol{\xi})$, is a Gaussian process with zero mean and covariance function $\Gamma(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2) = \text{plim}_{T\to\infty}\sum_{t=1}^T \epsilon(z_t,\theta_o)^2\phi_t(\boldsymbol{\xi}_1)\phi_t(\boldsymbol{\xi}_2)$.

(d) under the alternative H_1 in (2), $J_T^1(m_T)$ diverges.

(e) under the local alternative H_1^L in (8),

$$J_T^1(m_T) \xrightarrow{d} \int_{\Xi} \left|\mathcal{Z}(\boldsymbol{\xi}) + \mathcal{Z}^g(\boldsymbol{\xi})\right|^2 d\boldsymbol{\xi},$$

where $\mathcal{Z}(\boldsymbol{\xi})$ is defined in (c) and $\mathcal{Z}^g(\boldsymbol{\xi})$ is the limiting function of $\mathcal{Z}_T^g(\boldsymbol{\xi})$.

There are some remarks. First, from Theorem 3.1(b), we know that the θ which minimizes the $\eta_T^*(\theta, m_T)$ is the consistent estimate of θ_o under the null if m_T grows with T at the rate $o(T^{1/2})$. Second, even though the J_T^1 statistic shares the same asymptotics with the original ICM test, the proposed J_T^1 statistic improves the original ICM test in two directions: one is that the construction of proposed statistic J_T^1 is more easier because no numerical integration is needed; the other is that the J_T^1 statistic does not depend on the preliminary consistent parameter estimates. This is a unified approach which links the estimation and diagnostic testing in a natural way by extending the ICM test and the work in Hsu and Kuan (2010). Besides, because the asymptotic properties of J_T^1 is equivalent to the original ICM test, it is not pivotal, the bootstrap or simulation methods should be further imposed to obtain critical values.[2]

3.2 The proposed approach (II)

In this part, we propose a pivotal test by extending the proposed approach (I). We rewrite the null hypothesis in the ICM test approach as

$$H_0 : \left[\int_{\Xi} \left|\mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(X, \xi)\right]\right|^2 d\mu(\xi)\right]^{1/2} = 0, \text{ for some } \theta_o \in \Theta, \quad \text{w.p.1.}$$

By having Fourier representation of $\mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(X, \xi)\right]$ with respect to an orthonomal Fourier series $\{\psi_m(\cdot)\}$, under the null, we have

$$\left[\int_{\Xi} \left|\mathbb{E}\left[\epsilon(\boldsymbol{Z}, \theta_o) w(X, \xi)\right]\right|^2 d\mu(\xi)\right]^{1/2} = \left(\sum_{m=1}^{\infty} \left|\mathbb{E}[\epsilon(\boldsymbol{Z}, \theta_o) \varpi_m(X)]\right|^2\right)^{1/2} = 0,$$

where $\varpi_m(X)$ is defined above. This result immediately suggests that we can test a set of infinite many unconditional moment restrictions instead. That is,

$$H_0 : \mathbb{E}[\epsilon(\boldsymbol{Z}, \theta) \varpi_m(X)] = 0, \quad m = 1, 2, \ldots, m_T, \; m_T \to \infty. \tag{9}$$

[2]It the simulations below, wild bootstrap method is used for obtaining the corresponding critical values.

The $\epsilon(\boldsymbol{Z}, \theta)\varpi_m(X)$, $m = 1, 2, \ldots, m_T$ are stacked into an $m_T \times 1$ vector $\rho(\boldsymbol{Z}, \theta, m_T)$. Given this set of unconditional moment restrictions and the sample counterparts, the second test statistic based on Donald et al. (2003) is constructed by

$$J_T^2(m_T) = 2\left\{\min_{\theta}\max_{\lambda}\sum_{t=1}^{T}\Phi\big(\lambda'\rho(z_t, \theta, m_T)\big) - T\Phi(0)\right\}, \tag{10}$$

where Φ is C^2 in a neighborhood of 0, concave on an open interval of the real line containing 0, and λ is a $m_T \times 1$ vector of Lagrange multipliers. $J_T^2(m_T)$ in (10) is nothing but objective function of the GEL estimation method while having m_T unconditional moment restrictions. Similar to the conventional likelihood-ratio test statistic, the asymptotic distribution of $J_T^2(m_T)$ under H_0 is approximated by $\chi^2(m_T - p)$ which has mean $(m_T - p)$ and variance $2(m_T - p)$; more discussions may refer to Donald et al. (2003) for example. The asymptotics of $J_T^2(m_T)$ follows.

Theorem 3.2 *(Asymptotics of J_T^2 test)*
Given conditions [A1]–[A4], and If $m_T = o(T^{1/3})$ as $T \to \infty$, under the null, we have

$$\frac{J_T^2(m_T) - (m_T - p)}{\sqrt{2(m_T - p)}} \xrightarrow{d} N(0, 1).$$

There are some remarks. First, the base stone of the proposed test $J_T^2(m_T)$, $\varpi_m(\cdot)$, is quite different from the some particular basis functions used in Donald et al. (2003). Each $\varpi_m(\cdot)$ can be viewed as a "weighted average" of all basis functions $w(\cdot, \xi)$ for $\xi \in \Xi$. Second, unlike the ICM-type tests (including $J_T^1(m_T)$ test), this test is pivotal instead. Last, but not least, as well as $J_T^1(m_T)$, this test based on $J_T^2(m_T)$ needs no preliminary consistent estimate of θ_o.

4 Monte Carlo Simulations

In this section, we consider two experiments to evaluating the performance of the Cramer-von Mises type test statistic: CM_T in (6), the Kolmogorov-Smirnov type test statistic: KS_T in (5) and the proposed statistics $J_T^1(m_T)$ and $J_T^2(m_T)$. The null hypothesis of both experiments is that the model is linear, and the nominal size is 5% for all cases.

In the first experiment, like what in Hsu and Kuan (2008), we specify the model as

$$y_t = x_t + \frac{ax_t}{1 + \exp(-x_t)} + \epsilon_t,$$

where x_t follows a standard normal distribution. Various values of a, $a = -1.5, -1, -0.5, 0, 0.5,$ 1 and 1.5, are considered in this experiments. When $a = 0$, we evaluate the sizes of the tests;

Table 1: Rejection probabilities.

Test	$T=100$				$T=200$			
	a				a			
	0	0.5	1	1.5	0	0.5	1	1.5
$J_T^1(1)$	0.052	0.201	0.566	0.870	0.063	0.362	0.869	0.995
$J_T^1(2)$	0.050	0.199	0.574	0.873	0.059	0.365	0.874	0.995
$J_T^1(3)$	0.051	0.203	0.577	0.877	0.064	0.369	0.874	0.995
$J_T^2(1)$	0.258	0.543	0.889	0.993	0.254	0.694	0.983	1.000
$J_T^2(2)$	0.402	0.628	0.902	0.991	0.388	0.736	0.986	1.000
$J_T^2(3)$	0.573	0.741	0.935	0.995	0.535	0.819	0.989	1.000
CM_T	0.049	0.134	0.409	0.744	0.062	0.250	0.716	0.972
KS_T	0.049	0.144	0.439	0.769	0.060	0.266	0.744	0.976

otherwise, we compare the powers. In this setting, the number of Monte Carlo replications and of bootstrap replications are 3000 and 3000, respectively. The results are reported in Table 1. Roughly speaking, the test J_T^1 and J_T^2 have better power performances than CM_T and KS_T. However, the test J_T^2 suffers serious size-distortion problem when we focus on the case with $a = 0$. This shortcoming of test J_T^2 needs further investigation in the future work.

In the second experiment, we consider another nonlinear model as

$$y_t = \begin{cases} 1 + \epsilon_t & \text{if } y_{t-1} > 0; \\ 0 & \text{if } y_{t-1} = 0; \\ -1 + \epsilon_t & \text{if } y_{t-1} < 0. \end{cases}$$

Four sample size are considered, $T = 50, 100, 200, 500$. The results are reported in Table 2. Based on these results, the power performance of J_T^1, CM_T and KS_T are quite similar in all cases.

Table 2: Rejection probabilities.

Test	T			
	50	100	200	500
$J_T^1(1)$	0.848	0.994	1.000	1.000
$J_T^1(2)$	0.839	0.994	1.000	1.000
$J_T^1(3)$	0.850	0.994	1.000	1.000
$J_T^1(4)$	0.840	0.992	1.000	1.000
$J_T^1(5)$	0.848	0.994	1.000	1.000
CM_T	0.845	0.990	1.000	1.000
KS_T	0.876	0.995	1.000	1.000

5 Concluding Remarks

In this project, I construct two consistent model specification tests. The first proposed approach is the unified approach which links the estimation and diagnostic testing in a natural way by extending the ICM test of Bierens (1982) and Bierens and Ploberger (1997) and the work in Hsu and Kuan (2010). This test improves the original ICM test in two directions: it is more easier to implement and it does not depend on the preliminary consistent parameter estimates. The second proposed approach further extend the first proposed approach by using GEL methods. For this test, it needs no preliminary consistent parameter estimates, its implementation is not hard, and it is pivotal. To my best knowledge, this is the first attempt to link the GEL method to ICM tests for model specifications. The corresponding asymptotics of these two tests are established, and some interesting Monte Carlo simulations are considered. Based on limited experiments, the finite sample performance of J_T^1 are not worse than the common used Cramer-von Mises type and Kolmogorov-Smirnov type tests; the J_T^2 test has good power performance but with serious size-distortion problem. The size-distortion problem of this pivotal test is interesting and is worth further investigations.

References

Andrews, D.W., and W. Ploberger (1994). Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative, *Econometrica*, **62**, 1383–1414.

Bierens, H. J. (1982). Consistent model specification tests, *Journal of Econometrics*, **20**, 105–134.

Bierens, H. J. (1990). A consistent conditional moment test of functional form, *Econometrica*, **58**, 1443–1458.

Bierens, H. J. and W. Ploberger (1997). Asymptotic theory of integrated conditional mean test, *Econometrica*, **65**, 1127–1151.

Boning, B. and F. Sowell (1999). Optimality for the Integrated Conditional Moment Test, *Econometric Theory*, **15**, 710–719.

Chamberlain, G. (1987). Asymptotic efficiency in estimation with conditional moment restrictions, *Journal of Econometrics*, **34**, 305–334.

Domínguez, M. A. and I. N. Lobato (2003). Testing the martingale difference hypothesis, *Econometric Reviews*, **22**, 351–377.

Domínguez, M. A. and I. N. Lobato (2004). Consistent estimation of models defined by conditional moment restrictions, *Econometrica*, **72**, 1601–1615.

Domínguez, M. A. and I. N. Lobato (2006). A consistent specification test for models defined by conditional moment restrictions, working paper.

Donald, S. G., G. W. Imbens, and W. K. Newey (2003). Empirical likelihood estimation and consistent tests with conditional moment restrictions, *Journal of Econometrics*, **117**, 55-93.

Fan, Y.Q., and Q., Li (2000). Consistent Model Specification Tests: Kernel Based Tests Versus Bierens' ICM tests, *Econometric Theory*, **16**, 1016–1041.

Hong, Y. and H. White (1995). Consistent Specification Testing Via Non- parametric Series Regression, *Econometrica*, **63**, 1133–1159.

Hsu, S-H. and C-M. Kuan (2010). Estimation of conditional moment restrictions without assuming parameter identifiability in the implied unconditional moments, working paper.

Hsu, S-H. and C-M. Kuan (2008). Consistent test of linearity in conditional mean function, working paper.

Imbens, G. W. (2002). Generalized method of moments and empirical likelihood, *Journal of Business and Economic Statistics*, **20**, 493–506.

Kitamura, Y., G. Tripathi, and H. Ahn (2004). Empirical likelihood-based inference in conditional moment restriction models, *Econometrica*, **72**, 1167–1714.

Koul, H. L. and W. Stute (1999). Nonparametric model checks for times series, *The Annals of Statistics*, **27**, 204–236.

Lobato, I. N. (2003). Testing for nonlinear autoregression, *Journal of Business and Economic Statistics*, **21**, 164–173.

Newey, W.K. (1985). Maximum Likelihood Specification Testing and Conditional Moment Tests, *Econometrica*, **53**, 1047–1070.

Qin, J. and J. Lawless (1994). Empirical likelihood and generalized estimating equations, *The Annals of Statistics*, **22**, 300–325.

Stinchcombe, M. B. and H. White (1998). Consistent specification testing with nuisance parameters present only under the alternative, *Econometric Theory*, **14**, 295–325.

Stute, W. (1997). Nonparametric model checks for regression, *The Annals of Statistics*, **25**, 613–641.

Tauchen, G. (1985). Diagnostic Testing and Evaluation of Maximum Likelihood Models, *Journal of Econometrics*, **30**, 415–443.

Whang, Y. (2001). Consistent specification testing for conditional moment restrictions, *Economics Letters*, **71**, 299–306.

Zheng, X. (1996). A Consistent Test of Functional Form via Nonparametric Estimation Techniques, *Journal of Econometrics*, **75**, 263–289.

結果自評

在這研究中, 我提出了兩種方式來改善傳統的 ICM 檢定。第一個方法結合了 ICM 檢定及傅利葉分析, 進而提出了一個容易操作的檢定。這個檢定統計量和 ICM 檢定具有相同的極限分配, 但是它不需仰賴數值方法進行積分。再者, 由於此方法自然而然地將條件動差限制下的參數估計及模型檢定相結合, 因此我們也不需要額外的一致性參數估計式。因此, 此檢定方法在學術上有一定的參考及使用價值, 文稿加以修飾雕琢後, 預期可以投稿至 SSCI 的期刊中。除此之外, 我更擴展了第一個方法的精神進而與一般化的實證概似法 (generalized empirical likelihood) 結合, 建立了一個概似比例型態 (LikelihoodRatio type) 的檢定統計量。這也是文獻中的一項新嘗試。我也提供了這兩種檢定的相關理論性質及模擬。然而, 第二種檢定在小樣本時遇到 size-distortion 的問題, 這值得進一步探討。若能對此第二種檢定的小樣本性質有更深入更完整的探討, 我預期在學術上仍能提供一定的貢獻, 並預期能投稿至國際期刊中。簡而言之, 藉由此計畫的進行, 我除了對於此相關檢定問題有更深入的瞭解外, 也提供了不亞於於現行檢定的另一種解決方法。

無研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：徐士勛	計畫編號：98-2410-H-004-057-
計畫名稱：兩個一致性的模型設定新檢定	

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	1	1	100%		根據該計畫的實行進度，完成了初步的研究成果報告。
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	1	100%	人次	原申請一名碩士班兼任助理，後來改聘一位博士班兼任助理。
		博士生	3	2	100%		原申請兩名博士班兼任助理，後來共聘 3 名。
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以 100 字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：□已發表 □未發表之文稿 ■撰寫中 □無 專利：□已獲得 □申請中 ■無 技轉：□已技轉 □洽談中 ■無 其他：（以 100 字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限） 在這研究中，我提出了兩種方式來改善傳統的 ICM 檢定。第一個方法結合了 ICM 檢定及傅利葉分析,進而提出了一個容易操作的檢定。這個檢定統計量和 ICM 檢定具有相同的極限分配，但是它不需仰賴數值方法進行積分。 再者，由於此方法自然而然地將條件動差限制下的參數估計及模型檢定相結合，因此我們也不需要額外的一致性參數估計式。因此，此檢定方法在學術上有一定的參考及使用價值。文稿加以修飾雕琢後，預期可以投稿至 SSCI 的期刊中。除此之外，我更擴展了這第一個方法的精神進而與一般化的實證概似法（generalized empirical likelihood）結合,建立了一個概似比例型態(LikelihoodRatio type）的檢定統計量。 這也是文獻中的一項新嘗試。我也提供了這兩種檢定的相關理論性質及模擬。然而，第二種檢定在小樣本時遇到 size-distortion 的問題，這值得進一步探討。若能對此第二種檢定的小樣本性質有更深入更完整的探討，我預期在學術上仍能提供一定的貢獻，並預期能投稿至國際期刊中。簡而言之，藉由此計畫的進行，我除了對於此相關檢定問題有更深入的瞭解外，也提供了不亞於於現行檢定的另一種解決方法。