行政院國家科學委員會專題研究計畫 成果報告

隨機數位網路上最佳路徑選擇之研究(II)
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 95-2221-E-004-007-
執 行 期 間 ：95 年 08 月 01 日至 96 年 07 月 31 日
執 行 單 位 ：國立政治大學應用數學學系

計畫主持人：陸行

計畫參與人員：博士班研究生-兼任助理：王嘉宏
碩士班研究生-兼任助理：余文政、蘇愛玲、陳薇
大學生-兼任助理：張芝瑜

報 告 附 件 ：出席國際會議研究心得報告及發表論文

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 96 年 08 月 28 日

行政院國家科學委員會補助專題研究計畫成果報告

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本成果報告包括以下應繳交之附件：
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□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位：政治大學應用數學系

中　華　民　國　96　年　8　月　9　日

行政院國家科學委員會專題研究計畫成果報告
隨機數位網路上最佳路徑選擇之研究(II)
A Near-Optimal Routing for Heterogeneous Networks under Budget Constraints

計畫編號: **NSC 95-2221-E-004-007**
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Abstract. We present a bandwidth allocation scheme offering optimal solutions to the network optimization problem. The bandwidth allocation policy in class-based networks can be defined with the proportionally fair rule expressed by piecewise linear objective functions. This scheme is formulated as a mixed-integer linear programming (MILP) model, preparing a database identifying suitable paths upon each connection request. A branch and bound algorithm is proposed for solving the optimization problem.

1 Introduction

QoS routing concerns the selection of a path satisfying the QoS requirements of a connection [3], [6]. The path selection process involves the knowledge of the connection's QoS requirements and information on the availability of bandwidth. QoS routing poses major challenges in terms of algorithmic design [4]. On one hand, the path selection process is a complex task, due to the need to concurrently deal with the connection's QoS requirements, as well as with the global utilization of network resources [1]; on the other hand, connection requests need to be handled promptly upon their arrival. Depending on the specifics and the number of QoS metrics involved, computation in real time required for path selection can become prohibitively expensive as the network size grows.

We deal with the problem of dimensioning bandwidth for elastic data applications in packet-switched communication networks of multiple classes, which can be considered as a multiple-objective optimization model. Each user is allowed to request more than one type of service, and users' satisfaction is summarized by means of their achievement functions. We focus on allocating resources with proportional fairness and finding a routing scheme on communication networks. An approach is presented for the fair resource allocation problem and QoS routing in networks offering multiple services to users. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the sum of the users' satisfaction.

2 Problem Definition

Consider a directed network topology $G = (V, E)$, where V and E denote the set of nodes and the set of links in the network respectively. There are m (different) QoS classes in this network. Let $E_o \subseteq E$ and $E_d \subseteq E$ be subsets of links connected with the source o and destination d respectively. Each connection is delivered between the same source o and destination d in the core network. We denote $E_\nu^{in} \subseteq E$ a subset of incoming links to the node $\nu \in V$, and we also denote $E_\nu^{out} \subseteq E$ a subset of outgoing links from the node $\nu \in V$. The maximal link capacity is U_e on each link $e \in E$. For each link $e \in E$, we use d_e and κ_e to represent average delay and the purchasing cost of bandwidth respectively. Let $A_{i,j}(e)$ represent the bandwidth allocated to link $e \in E$ for connection j in class i. We use $\chi_{i,j}(e)$ to denote the binary variable which determines whether the link e is chosen for connection j in class i.

The decision variable θ_i is the bandwidth allocated to each connection in class i. In each class i, every connection is allocated the same bandwidth θ_i and has the same QoS requirement. The specific QoS requirements include minimal bandwidth requirement b_i and maximal end-to-end delay constraint D_i for each class i. Assume every connection in class i has the same aspiration level and reservation level of bandwidth, a_i and r_i, and assume that the average number of connections in class i is K_i.

The purpose of this work is to show that a methodology that allows the decision maker to explore a set of solutions could satisfy preferences with fairness and to choose the solution optimally.

3 A Precomputation Scheme for Network Optimization

This paper uses the bandwidth and budget as constraints of requirements for feasible path computations. Due to the limited budget on network planning, there exists the budget constraint (1).

$$\sum_{e \in E} \sum_{i=1}^{m} \sum_{j=1}^{K_i} \kappa_e A_{i,j}(e) \leq B \tag{1}$$

Because the aggregate bandwidth of all connections at any link does not exceed the capacity, we have constraint (2).

$$\sum_{i=1}^{m} \sum_{j=1}^{K_i} A_{i,j}(e) \leq U_e, \ \forall e \in E \tag{2}$$

Constraints (3), (4), (5), and (6) show that every connection in the same class uses the same bandwidth and has the same bandwidth requirement. For each $e \in E$, $j = 1, \ldots, K_i$, and $i = 1, \ldots, m$,

$$A_{i,j}(e) - M \cdot \chi_{i,j}(e) \leq 0, \tag{3}$$

$$\theta_i - A_{i,j}(e) \leq M(1 - \chi_{i,j}(e)), \tag{4}$$

$$A_{i,j}(e) - \theta_i \leq M(1 - \chi_{i,j}(e)), \tag{5}$$

and

$$\theta_i \geq b_i, \tag{6}$$

where M is a sufficiently large number. Constraints (7), (9), and (8) express the node conservation relations indicating that flow in equals flow out for every connection j in class i. For each $j = 1, \ldots, K_i$, and $i = 1, \ldots, m$,

$$\sum_{e \in E_o} A_{i,j}(e) = \theta_i, \tag{7}$$

$$\sum_{e \in E_d} A_{i,j}(e) = \theta_i, \tag{8}$$

and

$$\sum_{e \in E_\nu^{in}} A_{i,j}(e) = \sum_{e \in E_\nu^{out}} A_{i,j}(e), \tag{9}$$

for all $\nu \in V \setminus \{o, d\}$. Although $A_{i,j}(e)$ are continuous variables, constraints (7)-(8) are flow conservation constraints. Continuous decision variables and binary variables must be nonnegative, shown in constraints (10)-(12). For $e \in E$, $j = 1, \ldots, K_i$, $i = 1, \ldots, m$,

$$A_{i,j}(e) \geq 0, \tag{10}$$

$$\theta_i \geq 0, \tag{11}$$

and

$$\chi_{i,j}(e) \in \{0, 1\}. \tag{12}$$

Using the achievement function interpreted as a measure of QoS [5], we can formulate the mathematical model of the fair bandwidth allocation. Depending on the specified aspiration and reservation levels, a_i and r_i, respectively, Wang and Luh [6], [7] transformed the different QoS measurements onto a normalized scale by using achievement functions. At the first phase, a precomputation-based scheme for network optimization, Model 1, is executed:

$$\begin{array}{ll} \text{Maximize} & \displaystyle\sum_{i=1}^{m} w_i \log_{\alpha_i} \frac{\theta_i}{r_i} \\ \text{subject to} & \text{constraints } (1) - (12), \end{array} \tag{13}$$

where w_i is a fixed weight and $\alpha_i = a_i / r_i$.

4 An LP-Based Algorithm

Branch and bound algorithms combine a partial or subset enumeration strategy with the linear programming (LP) relaxations. The relaxation constraints

$$0 \leq z_{i,j}(e) \leq 1, \ \forall \ e \in \mathbb{E}, \ \forall \ j \in \mathbb{J}_i, \ i \in \mathbb{I} \tag{14}$$

are obtained by dropping the integer constraint on (12) in Model 1. Constraints (3)-(5) are always satisfied if constraint (14) holds.

Let $\digamma = \{x_i, y_{i,j}(e), z_{i,j}(e) | j \in \lambda_i, \ i \in \mathbb{I}, \ e \in \mathbb{E}\}$ be the set of all feasible solutions to Relaxation Model. the LP-based branch and bound algorithm branches by fixing the fractional decision variable $0 < z_{i,j}(e) < 1$. When more than one integer-restricted solutions in $\digamma$ are fractional, the LP-based branch and bound algorithm branch by fixing the one closest to 0.5. Branch and bound searches stop when every solution in $\digamma$ has been branched or terminated. The incumbent solution at any stage in a search of a discrete model is the best feasible solution known so far. We denote the incumbent solution $\tilde{\mathbf{X}} = \{\tilde{x}_i, \tilde{A}_{i,j}(e), \tilde{z}_{i,j}(e) | \forall j \in \lambda_i, \ i \in \mathbb{I}, \ e \in \mathbb{E}\}$ and its objective function value $\tilde{f} = \sum_{i \in \mathbb{I}} w_i f_i(\tilde{x}_i)$. If all the solutions have been either branched or fathom, then the final incumbent solution is the optimum. The following is the LP-based branch and bound algorithm, which is implemented to solve Model 1.

Subprogram 1:

Step 1. (Relaxation.) Solve the LP relaxation of Model 1. Let $\digamma$ be the collection of all optimal solutions, and let $\bar{f} = \sum_{i \in \mathbb{I}} w_i \bar{f}_i$ be the optimal value of Relaxation Model. Suppose $f_i(k_{i,l-1}) \leq f_i^* \leq f_i(k_{i,l})$, where $k_{i,l-1}$ and $k_{i,l}$ are two of break points of the achievement function. Proceed to Step 2.

Step 2. (Initialization.) Set $t = 0$,

$$\varepsilon = \frac{1}{2} \min_{i \in \mathbb{I}} \{\bar{f}_i - f_i(k_{i,l-1}), f_i(k_{i,l}) - \bar{f}_i\},$$

$\tilde{\mathbf{X}} = \{x_i^0, y_{i,j}^0(e), z_{i,j}^0(e) | x_i^0 = l_i, y_{i,j}^0(e) = l_i z(e), z_{i,j}^0(e) = z(e), \ \forall j \in \lambda_i, \ i \in \mathbb{I}, \ e \in \mathbb{E}\}$, and $\tilde{f} = \sum_{i \in \mathbb{I}} w_i f_i(x_i^0)$. Proceed to Step 3.

Step 3. (Branching.) Set $t \leftarrow t+1$, and select one solution

$\mathbf{X}^t = \{x_i^t, y_{i,j}^t(e), z_{i,j}^t(e) | j \in \lambda_i, \ i \in \mathbb{I}, e \in \mathbb{E}\} \in \digamma$.

Choose an $z_{i,j}^t(e)$ which is a fractional part of the solution $\mathbf{X}^t$ node, create two new active nodes one of which has $z_{i,j}^t(e) = 0$ or $z_{i,j}^t(e) = 1$, and add both of them into $\digamma$. Proceed to Step 4.

Step 4. (Termination by Bound.) If

$$\sum_{i \in \mathbb{I}} w_i f_i(x_i^t) < \tilde{f},$$

then set $\digamma \leftarrow \digamma \setminus \{\mathbf{X}^t\}$ and go to Step 6. Otherwise, proceed to Step 5.

Step 5. (Termination by Solving.) If

$$\sum_{i \in \mathbb{I}} w_i f_i(x_i^t) \geq \tilde{f}$$

and

$$\{z_{i,j}^t(e) | j \in \lambda_i, \ i \in \mathbb{I}, \ e \in \mathbb{E}\}$$

are integer solutions, then update $\tilde{f} \leftarrow \sum_{i \in \mathbb{I}} w_i f_i(x_i^t)$, $\tilde{\mathbf{X}} \leftarrow \mathbf{X}^t$, $\digamma \leftarrow \digamma \setminus \{\mathbf{X}^t\}$, and proceed to Step 6. Otherwise, go to Step 3.

Step 6. (Optimal Criteria.) If $\digamma = \emptyset$ or

$$|\tilde{f} - \bar{f}| < \varepsilon,$$

then the procedure stops, and go to Subprogram 2. The incumbent solution $\tilde{\mathbf{X}}$ is called the ε-optimal solution. Otherwise, go to Step 3.

From the output of Subprogram 1, $\tilde{\mathbf{X}}$, we know which interval $\tilde{x}_i$ lies in. Suppose $\tilde{x}_i$ lies in $[k_{i,l_i-1}, k_{i,l_i})$ for each $i \in \mathbb{I}$. The piecewise-linear achievement function is reduced to a linear function

$$f_i(x_i) = \rho_{l_i} \cdot (x_i - k_{i,l_i}) + \mu_i(k_{i,l_i}), \tag{15}$$

where $\rho_{l_i} = (n \log_{\alpha_i} k_{i,l_i}) / k_{i,l-1}(a_i - r_i)$ and $\mu_i(k_{i,l_i}) = \log_{\alpha_i}(k_{i,l_i}/r_i)$ are constants.

Subprogram 2:

Step 1. (Choice of Interval) Set $l_i \leftarrow l$ if $\tilde{x}_i$ lies in $[k_{i,l-1}, k_{i,l})$ for some $l = 1, \ldots, n$.

Step 2. (Restriction.) Set $f_i(x_i) \leftarrow \rho_{l_i} \cdot (x_i - k_{i,l_i}) + \mu_i(k_{i,l_i})$ and $z_{i,j}(e) \leftarrow \tilde{z}_{i,j}(e)$.

Step 3. (Solving.) Solve the LP model, Relaxation Model.

With the help of (15) and $z_{i,j}(e) = \tilde{z}_{i,j}(e)$, Model 1 is simplified as the following LP model, Relaxation Model:

$$\begin{aligned} \max \quad & \sum_{i \in \mathbb{I}} w_i[\rho_{l_i}(x_i - k_{i,l_i}) + \mu_i(k_{i,l_i})] \\ \text{s. t.} \quad & \text{constraints } (1) - (11) \\ & z_{i,j}(e) = \tilde{z}_{i,j}(e), \ \forall\, e \in \mathbb{E}, \ \forall\, j \in \mathbb{J}_i, \ i \in \mathbb{I}, \end{aligned}$$

where $w_i \in (0,1)$ is the weight assigned to each class i and $\sum_{i \in \mathbb{I}} w_i = 1$.

Theorem 1 *The difference of optimal values between Model 1 and Relaxation Model is no greater than ε.*

5 Optimal Solutions

We determine the optimal choices of links, $z^*_{i,j}(e)$, the optimal bandwidth allocation for each link e and for each connection of class i, $y^*_{i,j}(e)$ and x^*_i. The optimal solution x^*_i is unique, and it can provide the proportional fairness to every connection in all classes. That is, this allocation can provide the fair satisfaction to each user in all classes. We also find the total bandwidth allocated to each class i, $\lambda_i x^*_i$.

Definition 2 *The ratio $\sum_{j=1}^{\lambda_i} \sum_{e \in \mathbb{E}} c_e y^*_{i,j}(e)/B$ is called a **budget ratio** allocated to class i.*

Each class is given a percentage, budget ratio, of the total budget B.

Proposition 3 *If $p_{i,j} = \{e \in \mathbb{E} |\ z^*_{i,j}(e) = 1\}$ for connection j in class i, then path $p_{i,j}$ is the optimal path from o to d for connection j in class i.*

Proposition 4 *The end-to-end unit cost for bandwidth on the optimal path $p_{i,j}$ is*

$$\sum_{e \in p_{i,j}} c_e z^*_{i,j}(e)$$

for connection j in class i.

Proposition 5 *If link e belongs to the optimal path $p_{i,j}$, then the bandwidth by which the link e can offer for connection j in class i is the same. That is, $y^*_{i,j}(e) = y^*_{i,j}(e')$ for all $e,\ e' \in p_{i,j}$.*

Proposition 6 *A link e is the bottleneck link if $\sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}_i} y^*_{i,j}(e) = U_e$.*

6 Conclusions

We present an approach for the bandwidth allocation and QoS routing in class-based networks. This scheme determines QoS routing under the network constraints. Users' utility functions are summarized by means of achievement functions. We can find an optimal allocation of bandwidth on the network under a limited budget, and this allocation can provide the proportional fairness to every class. Our approach is executed in advance, and its purpose is to establish a database, selecting one of the solutions when connections arrive. The on-line algorithm may select a path with the maximum reservable bandwidth among all feasible paths in this database.

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行政院國家科學委員會補助國內專家學者出席國際學術會議報告

2006　年 8 月 20 日

附件三

報告人姓名	陸　　行	服務機構及職稱	政大應數系教授
會議 時間 地點	August 8-12, 2006 Xinjiang, China	本會核定補助文號	
會議名稱	（中文）國際作業研究第六屆學術會議 （英文）the 6th International Symposium on Operations Research and Its Applications		
發表論文題目	（中文）　利用成就函數描述網路設計與評估問題 （英文）　Network Dimensioning Problems of Applying Achievement Functions		

報告內容應包括下列各項：

一、參加會議經過

因為筆者是台灣作業研究學會的理事長，接受大會邀請，擔任論文發表與評審。

二、與會心得

這是亞太地區作業研究重要的會議，每二年舉辦一次。與會人員包括來自日本、韓國、科威特、台灣和中國的學者專家，都是有重要影響力的意見領袖。

以後值得作推廣和舉辦是項會議。

三、考察參觀活動(無是項活動者省略)

參觀吐魯番盆地。

四、建議

亞太地區發展作業研究突飛猛進。值得我們學習。特別是做跨領域的合作。

五、攜回資料名稱及內容

大會手冊

六、其他

ISORA'2006
Tentative Programs*

The 6th International Symposium on Operations Research and Its Applications
August 8-12, 2006, Xinjiang, China

August 7 (Monday): Participants arrive Urumqi, check in Yilite Hotel, and Registration package pick up.

August 8 (Tuesday): Technical sessions. (Xinjiang time = Beijing time +2 hours.)

10:00-10:20 Opening Session
Welcome address from ISORA2006 co-chairs: Prof. Xiangsun Zhang, Prof. Tatsuo Oyama.

10:20-12:05 Plenary Session I (Session Chair: Xiangsun Zhang)
10:20-11:05 "*Newsvendor Bounds and Heuristics for Optimal Policy of Serial Supply Chains with and without Expedited Shippings*", Xiuli Chao, North Carolina State University, USA.
11:05-11:50 "*Applying Network Flow Optimization Techniques for Measuring the Robustness of Water Supply Network System in Tokyo*", Hiroshi Ashida (Tokyo Metropolitan Government), Hozumi Morohoshi, and Tatsuo Oyama (National Graduate Institute for Policy Studies).
11:50-12:10 Coffee Break

12:10-13:40 Plenary Session II (Session Chair: Tatsuo Oyama)
12:10-12:55 *"Network Dimensioning Problems by Applying Achievement Functions"*, Hsing Luh, Taiwan Zhengchi University, Taiwan
12:55-13:40 "*ILOG Optimization Software and Industrial Solutions"* Kiat Shi Tan and and Lily Deng, ILOG Beijing. (ILOG Software Demo is available during the symposium)

14:00-15:30 Lunch

16:00-17:40 Parallel Session A1 (Session Chair: Prof. Wuyi Yue)
16:00-16:25 "Algorithmic Solutions for the Stationary Distribution of M/M/c/K Retrial Queue" Yang Woo Shin, Dug Hee Moon, Changwon National University, Korea
16:25-16:50 "Analytical Network Process of Group Choice for a Foreign Market Entry Mode", Su-Chuan Shih, Taiwan Providence University, Taiwan
16:50-17:15 "Global logistics road planning: a genetic algorithm approach" Wu Yue, University of Southampton, UK
17:15-17:40 "Stochastic Optimal Control Problems with a Bounded Memory", Tao Pang, North Carolina State University, USA

16:00-17:40 Parallel Session B1 (Session Chair: Prof. Masanori Fushimi)
16:00-16:25 "Dual Scaling Using Mathematical Programming and Its Application", Tohru Ueda,

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Network Dimensioning Problems by Applying Achievement Functions

Hsing Luh

Department of Mathematical Sciences
National Chengchi University

August 8, 2006

Outline

- Introduction
- Achievement functions with proportional fairness
- Fair bandwidth allocation on network dimensioning problems
- Numerical examples
- Conclusions

Introduction

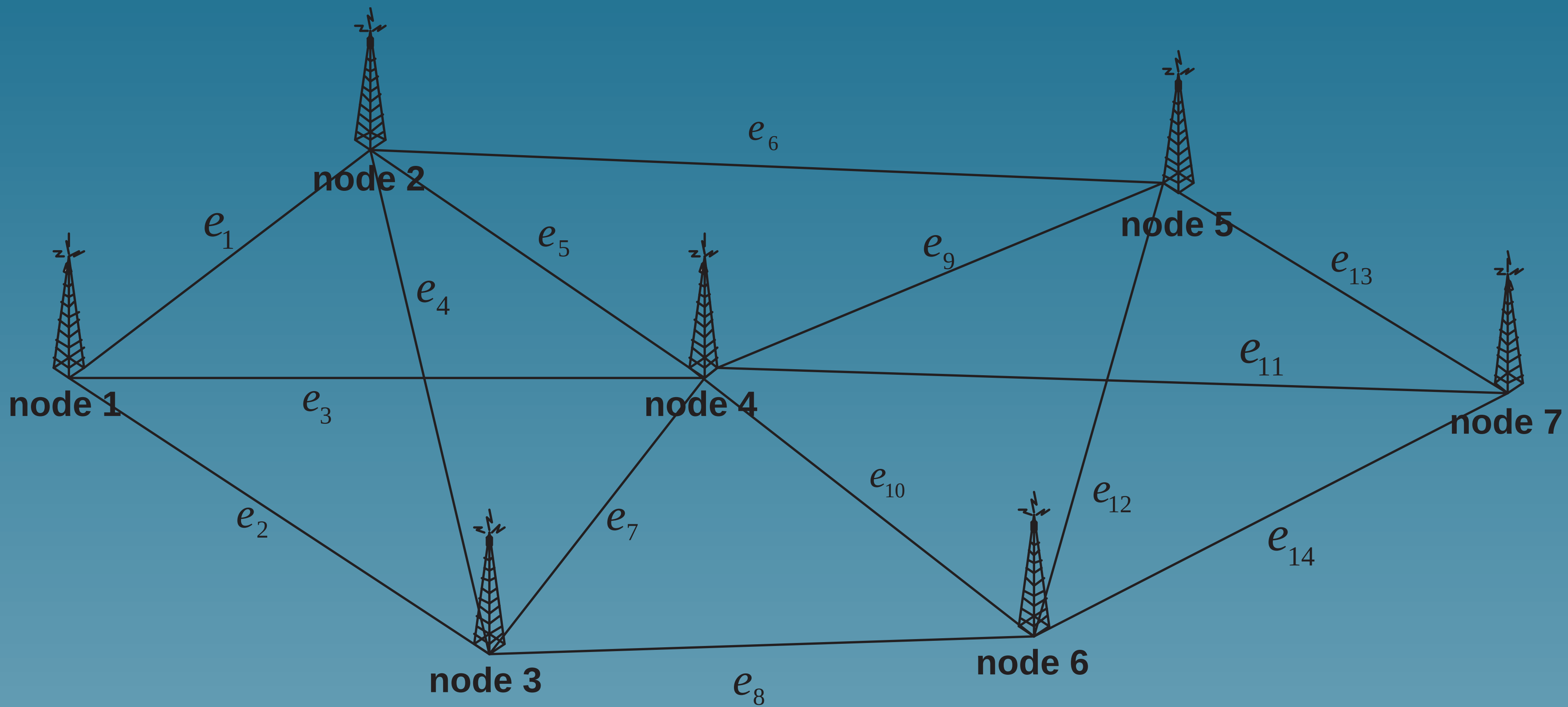


Fig. 1. A Sample Network Topology

Introduction

UMTS network services have different QoS classes for four types of traffic:

- Conversational class (voice, video telephony, video gaming)
- Streaming class (multimedia, video on demand, webcast)
- Interactive class (web browsing, network gaming, database access)
- Background class (email, SMS, downloading)

Table 1. The Characteristics of UMTS Service Classes

Traffic Classes	Sensitivity to Jitter	Sensitivity to Delay	Sensitivity to Packet Loss
Conversational	high	high	low
Streaming	high	high	low
Interactive class	low	low	high
Background class	very low	low	high

Proportional Fairness

Kelly et al. (1998, 2001, 2003, 2004) advocated proportional fairness characterized by $\log(\theta_i)$. This log utility function is strictly concave. The proportional fair bandwidth allocation is determined by the following objective function:

$$\max \qquad \sum_{i \in I} K_i \log(K_i \theta_i).$$

Mo and Walrand (2000) characterized the class of (w, α)- proportionally fair bandwidth allocation, for any given number α ($\alpha > 0$, $\alpha \neq 1$), as the following objective function:

$$\max \qquad \sum_{i \in I} w_i K_i^{\alpha} \frac{(K_i \theta_i)^{1-\alpha}}{1-\alpha},$$

where w_i is a fixed parameter.

Achievement functions with proportional fairness

Proportionally fair bandwidth allocation problems are considered by Pioro et al. (2002), Ogryczak et al. (2003), Park and Choi (2004), Sarkar and Tassiulas (2004), Ye and Qu (2005), Wang and Luh (2005), etc.

Depending on the specified aspiration and reservation levels, a_i and r_i, respectively, we construct our achievement function of θ_i as follows:

$$\mu_i(\theta_i) = \log_{\alpha_i} \frac{\theta_i}{r_i},\ where \alpha_i = \frac{a_i}{r_i}. \tag{1}$$

Proposition 1. *The achievement function $\mu_i(\theta_i)$ is continuous, increasing, and concave.*

Achievement functions with proportional fairness

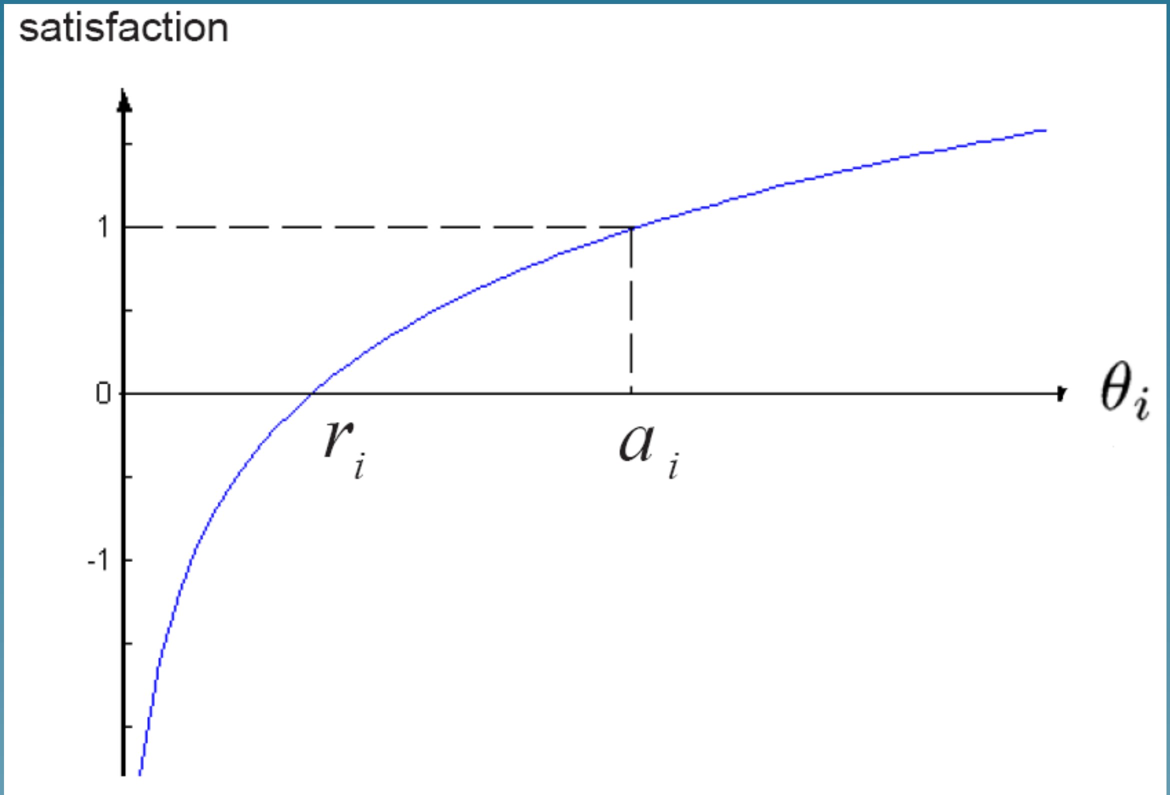


Fig. 2. The Graph of an Achievement Function $\mu_i(z_i)$

Achievement functions with proportional fairness

Lemma 2. *Let κ be the cheapest cost per unit bandwidth given in an end-to-end path. Suppose the total budget is B. There exists a finite number $M_i \leq B/\kappa K_i$ such that $\theta_i \leq M_i$, $\forall\ i$, where K_i is the number of connections in class i.*

$$\hat{\mu}_i(\theta_i) = \begin{cases} -M & if\ 0 \leq \theta_i < b_i \\ \rho_0 \cdot (\theta_i - k_{i,0}) & if\ b_i \leq \theta_i < r_i \\ \rho_1 \cdot (\theta_i - k_{i,1}) + \mu_i(k_{i,1}) & if\ r_i \leq \theta_i < k_{i,1} \\ \rho_2 \cdot (\theta_i - k_{i,2}) + \mu_i(k_{i,2}) & if\ k_{i,1} \leq \theta_i < k_{i,2} \\ \vdots & \\ \rho_{n-1} \cdot (\theta_i - k_{i,n-1}) + \mu_i(k_{i,n-1}) & if\ k_{i,n-2} \leq \theta_i < k_{i,n-1} \\ \rho_n \cdot (\theta_i - k_{i,n}) + 1 & if\ k_{i,n-1} \leq \theta_i < a_i \\ \rho_M \cdot (\theta_i - M_i) + \mu_i(M_i) & if\ a_i \leq \theta_i \leq M_i. \end{cases} \tag{2}$$

Proposition 3. *The achievement function $\hat{\mu}_i(\theta_i)$ is continuous, increasing, and concave.*

Achievement functions with proportional fairness

Lemma 4. *Let $\hat{\mu}_i^{(n)}(\theta_i) : [r_i, a_i] \longrightarrow [0, 1]$, where n means the number of break points, be defined as the achievement function (2) restricted on $[r_i, a_i]$. Then the sequence of functions $\{\hat{\mu}_i^{(n)}(\theta_i)\}_{n=1}^{\infty}$ converges uniformly to $\mu_i(\theta_i) = \log_{\alpha_i}(\theta_i/r_i)$ on $[r_i, a_i]$.*

Theorem 5. *If $r_i \leq \theta_i \leq a_i$, then the ε-proportionally fair bandwidth allocation obtained by using (2) as objective function approaches to proportional fairness as $n \longrightarrow \infty$.*

Fair bandwidth allocation on network dimensioning problems

Given a network topology $G =< V, E >$, where V and E denote the set of nodes and the set of links in the network respectively. There is given a set S of m classes, i.e., $|S| = m$. We denote by S^i a set of sessions in class i. There is also given the maximal possible number K_i in each class i, that is $|S^i| = K_i$.

Denote x_e and θ_i be the bandwidth allocated to the link e and the connection j of class i respectively. We also let $\chi_{i,j}(e)$ be a binary variable which determines whether the link e is chosen for connection j in class i.

Given the total available budget B and the marginal cost κ_e of bandwidths for each link $e \in E$, we want to allocate the bandwidths in order to provide each class with maximal possible QoS.

Fair bandwidth allocation on network dimensioning problems

Let $\mathbf{x}$ denote the vector of decision variables and Q denote the feasible set. We consider a resource allocation problem defined as an optimization problem with m objective functions $f_i(\mathbf{x})$:

$$\max\{\, \mathbf{f}(\mathbf{x}) : \mathbf{x} \in Q \,\}, \tag{3}$$

where $\mathbf{f}(\mathbf{x})$ is a vector-function that maps the decision space $\mathbf{R}^n$ into the criterion space $\mathbf{R}^m$.

Majorization

For the n-dimensional decision vector $\mathbf{x}$=($x_1, \ldots, x_n$) of reals, let $x_{(1)} \leq \ldots \leq x_{(n)}$ denote the components of $\mathbf{x}$ in increasing order.

Definition 6. *For $\mathbf{x}$ and $\mathbf{y}$ in $\mathbf{R}^n$, $\mathbf{x} \leq_M \mathbf{y}$ if $\sum_{i=1}^n x_{(i)} = \sum_{i=1}^n y_{(i)}$ and $\sum_{i=1}^k x_{(i)} \geq \sum_{i=1}^k y_{(i)}$, for $k = 1, \ldots, n-1$. When $\mathbf{x} \leq_M \mathbf{y}$ then $\mathbf{x}$ is said to be **majorized** by $\mathbf{y}$.*

Definition 7. *A function $g : \mathbf{R}^n \to \mathbf{R}$ is called **Schur-concave**, if $\mathbf{x} \leq_M \mathbf{y}$ implies $g(\mathbf{x}) \geq g(\mathbf{y})$.*

Theorem 8. *Let h be an arbitrary real function and define $g(\mathbf{x})=\sum_{i=1}^n h(x_i)$ for $\mathbf{x} \in \mathbf{R}^n$, then g is Schur-concave if and only if h is concave.*

Majorization

Typical solution concepts for multiple criteria problems are defined by aggregation functions $g : \mathbf{R}^m \to \mathbf{R}$ to be maximized. Thus, (3) $\Rightarrow$

$$\max\{g(\mathbf{f}(\mathbf{x})) : \mathbf{x} \in Q\}. \tag{4}$$

An aggregation (4) is fair if it is defined by a strictly increasing and strictly Schur-concave function g.

Theorem 9. *For a strictly concave, increasing function $\mu_i : \mathbf{R} \to \mathbf{R}$, the function $g(\mathbf{f}(\mathbf{x})) = \sum_{i=1}^m w_i f_i(\mathbf{x})$ is a strictly monotonic and strictly Schur-concave function.*

Theorem 10. *For a strictly concave, increasing function $\mu_i : \mathbf{R} \to \mathbf{R}$, the optimal solution of the problem $\max\{\sum_{i=1}^m w_i f_i(\mathbf{x}) : \mathbf{x} \in Q\}$ is a fair solution for resource allocation problem (3).*

Mathematical Model

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{m} w_i f_i(\mathbf{x}) \\
s.t. \quad & \sum_{e \in E} \kappa_e x_e \leq B \\
& \sum_{i} \sum_{j} \chi_{i,j}(e)\theta_i = x_e, \ \forall e \in E \\
& \sum_{i} K_i \cdot c_i \leq B \\
& \theta_i \cdot \sum_{e} \kappa_e \chi_{i,j}(e) = c_i, \ \forall j \in S_i, \ \forall i = 1, \ldots, m \\
& x_e \leq U_e, \ \forall e \in E
\end{aligned}
$$

$$\theta_i \geq b_i, \ \forall i = 1, \ldots, m$$

$$\sum_{e \in E_o} \chi_{i,j}(e) = 1, \ \forall j \in S_i, \ \forall i = 1, \ldots, m$$

$$\sum_{e \in E_\nu} \chi_{i,j}(e) = \sum_{e \in E'_\nu} \chi_{i,j}(e), \ \forall \nu \in V \setminus \{o, d\}, \ \forall j \in S_i, \ \forall i = 1, \ldots, m$$

$$\sum_{e \in E_d} \chi_{i,j}(e) = 1, \ \forall j \in S_i, \ \forall i = 1, \ldots, m$$

$$x_e \geq 0, \ \forall e \in E$$

$$\theta_i \geq 0, \quad \forall i = 1, \ldots, m$$

$$\chi_{i,j}(e) = 0 \ or \ 1, \ \forall e \in E, \ \forall j \in S_i, \ \forall i = 1, \ldots, m,$$

where $w_i \in (0, 1)$ is given for each i and $\sum_{i=1}^{m} w_i = 1$.

Numerical Example 1

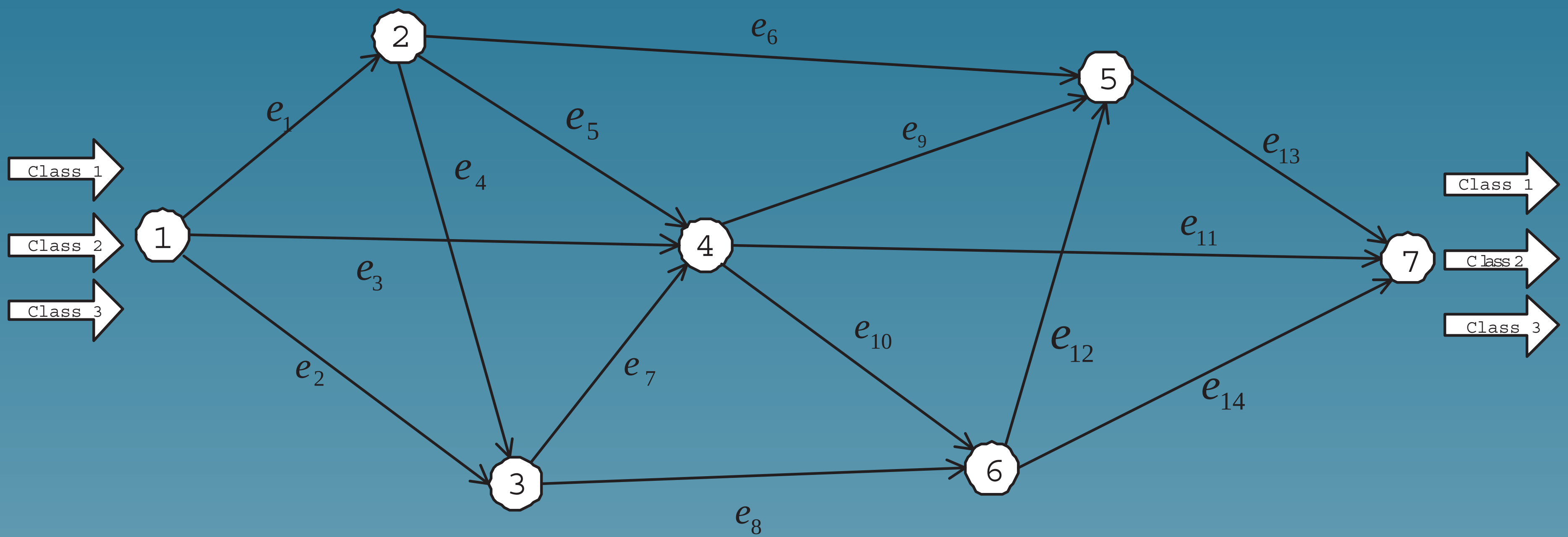


Fig. 3. Sample Network 1

Numerical Example 1

Table 2. The Characteristics of Each Class

Class	Bandwidth Requirement	Aspiration Level	Reservation Level
1	160 kbps	334 kbps	167 kbps
2	80 kbps	166 kbps	83 kbps
3	25 kbps	56 kbps	28 kbps

Suppose the number of connections in each class i is K_i for $i = 1, 2, 3$. Under the total available budget $B = \$1,000,000$, we want to allocate the bandwidths in order to provide each class with maximal possible QoS defined via the achievement function (1).

Numerical Example 1

Given $(K_1, K_2, K_3) = (80, 120, 150)$ and $B = 1,000,000$. We change the weight assigned to each class, and the computational result is shown in Table 3.

Table 3. Change in the Weight for Example 1

weight (w_1, w_2, w_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*optimal value* (*satisfaction*)	*ratio* $(\frac{K_1c_1}{B}, \frac{K_2c_2}{B}, \frac{K_3c_3}{B})$
$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	$e_1 - e_5 - e_9 - e_{13}$	(334,159,28)	(6680,3180,560)	0.442	(0.534,0.382,0.084)
$(0.4, 0.3, 0.3)$	$e_2 - e_8 - e_{14}$	(334,159,28)	(6680,3180,560)	0.482	(0.534,0.382,0.084)
$(0.4, 0.4, 0.2)$	$e_1 - e_5 - e_9 - e_{13}$	(334,162.75,25)	(6680,3255,500)	0.514	(0.534,0.391,0.075)
$(0.5, 0.3, 0.2)$	$e_2 - e_8 - e_{14}$	(453.625,83,25)	(9072.5,1660,500)	0.561	(0.726,0.199,0.075)
$(0.5, 0.4, 0.1)$	$e_2 - e_8 - e_{14}$	(334,162.75,25)	(6680,3255,500)	0.589	(0.534,0.391,0.075)
$(0.6, 0.2, 0.2)$	$e_1 - e_5 - e_9 - e_{13}$	(458.125,80,25)	(9162.5,1600,500)	0.628	(0.733,0.192,0.075)
$(0.6, 0.3, 0.1)$	$e_2 - e_8 - e_{14}$	(453.625,83,25)	(9072.5,1660,500)	0.646	(0.726,0.199,0.075)
$(0.7, 0.2, 0.1)$	$e_2 - e_8 - e_{14}$	(458.125,80,25)	(9162.5,1600,500)	0.713	(0.733,0.192,0.075)
$(0.8, 0.1, 0.1)$	$e_2 - e_8 - e_{14}$	(458.125,80,25)	(9162.5,1600,500)	0.781	(0.733,0.192,0.075)

Numerical Example 1

Given $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$ and $B = 1,000,000$. We change the numbers of connections in each class, and the computational results are shown in Table 4, Table 5, and Table 6.

Table 4. Change in the Number of Connections in Class 1 for Example 1

number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* (\$) (c_1, c_2, c_3)	*optimal value* (*satisfaction*)	*ratio* $(\frac{K_1c_1}{B}, \frac{K_2c_2}{B}, \frac{K_3c_3}{B})$
$(150, 100, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(261.333,83,25)	(5226.667,1660,500)	0.510	(0.784,0.166,0.05)
$(140, 100, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(280,83,25)	(5600,1660,500)	0.530	(0.784,0.166,0.05)
$(130, 100, 100)$	$e_2 - e_8 - e_{14}$	(301.539,83,25)	(6030.769,1660,500)	0.5536	(0.784,0.166,0.05)
$(120, 100, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(326.667,83,25)	(6533.333,1660,500)	0.5807	(0.784,0.166,0.05)
$(110, 100, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(334,107.6,25)	(6680,2152,500)	0.6019	(0.735,0.215,0.05)
$(100, 100, 100)$	$e_2 - e_8 - e_{14}$	(334,141,25)	(6680,2820,500)	0.62	(0.668,0.282,0.05)
$(90, 100, 100)$	$e_1 - e_5 - e_{11}$	(334,150.591,25)	(7014,3162.4,525)	0.6251	(0.631,0.316,0.053)
$(80, 100, 100)$	$e_1 - e_5 - e_{11}$	(460.238,83,25)	(9665,1743,525)	0.6492	(0.773,0.174,0.053)
$(70, 100, 100)$	$e_2 - e_8 - e_{14}$	(560,83,25)	(11200,1660,500)	0.6971	(0.784,0.166,0.05)
$(60, 100, 100)$	$e_2 - e_8 - e_{14}$	(653.333,83,25)	(13066.67,1660,500)	0.7419	(0.784,0.166,0.05)
$(50, 100, 100)$	$e_2 - e_8 - e_{14}$	(790,80,25)	(15800,1600,500)	0.8046	(0.79,0.16,0.05)

Numerical Example 1

Table 5. Change in the Number of Connections in Class 2 for Example 1

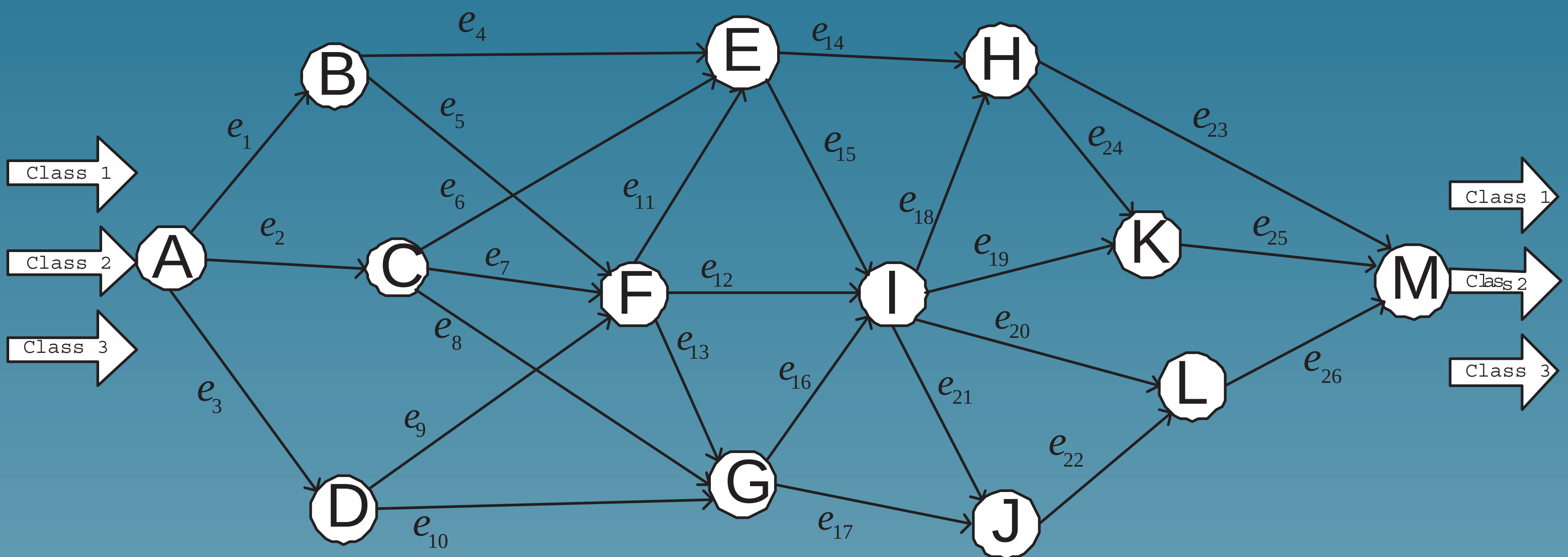
number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*optimal value* (*satisfaction*)	*ratio* $(\frac{K_1c_1}{B}, \frac{K_2c_2}{B}, \frac{K_3c_3}{B})$
$(100, 150, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(350.5,83,25)	(7010,1660,500)	0.597	(0.701,0.249,0.05)
$(100, 140, 100)$	$e_2 - e_8 - e_{14}$	(358.8,83,25)	(7176,1660,500)	0.601	(0.718,0.232,0.05)
$(100, 130, 100)$	$e_2 - e_8 - e_{14}$	(367.1,83,25)	(7342,1660,500)	0.605	(0.734,0.216,0.05)
$(100, 120, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(375.4,83,25)	(7508,1660,500)	0.609	(0.751,0.199,0.05)
$(100, 110, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(334,128.181,25)	(6680,2563.636,500)	0.613	(0.668,0.282,0.05)
$(100, 100, 100)$	$e_2 - e_8 - e_{14}$	(334,141,25)	(6680,2820,500)	0.620	(0.668,0.282,0.05)
$(100, 90, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(334,156.667,25)	(6680,3133.333,500)	0.628	(0.668,0.282,0.05)
$(100, 80, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(342.2,166,25)	(6844,3320,500)	0.637	(0.684,0.266,0.05)
$(100, 70, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(358.8,166,25)	(7176,3320,500)	0.645	(0.718,0.232,0.05)
$(100, 60, 100)$	$e_1 - e_5 - e_9 - e_{13}$	(375.4,166,25)	(7508,3320,500)	0.653	(0.751,0.199,0.05)
$(100, 50, 100)$	$e_2 - e_8 - e_{14}$	(392,166,25)	(7840,3320,500)	0.661	(0.784,0.166,0.05)

Numerical Example 1

Table 6. Change in the Number of Connections in Class 3 for Example 1

number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* (\$) (c_1, c_2, c_3)	*optimal value* (*satisfaction*)	*ratio* $(\frac{K_1c_1}{B}, \frac{K_2c_2}{B}, \frac{K_3c_3}{B})$
$(100, 100, 150)$	$e_1 - e_5 - e_9 - e_{13}$	(334,128.5,25)	(6680,2570,500)	0.613	(0.668,0.257,0.075)
$(100, 100, 140)$	$e_1 - e_5 - e_9 - e_{13}$	(334,131,25)	(6680,2620,500)	0.615	(0.668,0.262,0.07)
$(100, 100, 130)$	$e_2 - e_8 - e_{14}$	(334,133.5,25)	(6680,2670,500)	0.616	(0.668,0.267,0.065)
$(100, 100, 120)$	$e_1 - e_5 - e_9 - e_{13}$	(334,136,25)	(6680,2720,500)	0.617	(0.668,0.272,0.06)
$(100, 100, 110)$	$e_1 - e_5 - e_9 - e_{13}$	(334,138.5,25)	(6680,2770,500)	0.619	(0.668,0.277,0.055)
$(100, 100, 100)$	$e_2 - e_8 - e_{14}$	(334,141,25)	(6680,2820,500)	0.620	(0.668,0.282,0.05)
$(100, 100, 90)$	$e_1 - e_5 - e_9 - e_{13}$	(334,143.5,25)	(6680,2870,500)	0.621	(0.668,0.287,0.045)
$(100, 100, 80)$	$e_1 - e_5 - e_9 - e_{13}$	(334,146,25)	(6680,2920,500)	0.623	(0.668,0.292,0.04)
$(100, 100, 70)$	$e_2 - e_8 - e_{14}$	(334,148.5,25)	(6680,2970,500)	0.624	(0.668,0.297,0.035)
$(100, 100, 60)$	$e_1 - e_5 - e_9 - e_{13}$	(334,151,25)	(6680,3020,500)	0.625	(0.668,0.302,0.03)
$(100, 100, 50)$	$e_1 - e_5 - e_9 - e_{13}$	(334,152,28)	(6680,3040,560)	0.627	(0.668,0.304,0.028)

Numerical Example 2



Numerical Example 2

Table 7. Change in the Weight for Example 2

weight (w_1, w_2, w_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*total flow* (kbps)	*optimal value* (*satisfaction*)
$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(334,166,39.94)	(12692,6308,1517.87)	52631.58	0.454
$(0.4, 0.3, 0.3)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(356.39,166,28)	(13543,6308,1064)	52631.58	0.493
$(0.4, 0.4, 0.2)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(356.39,166,28)	(13543,6308,1064)	52631.58	0.525
$(0.5, 0.3, 0.2)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(486.52,83,25)	(18487.75,3154,950)	52631.58	0.574
$(0.5, 0.4, 0.1)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(362.02,166,25)	(13756.75,6308,950)	52631.58	0.603
$(0.6, 0.2, 0.2)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(491.02,80,25)	(18658.75,3040,950)	52631.58	0.644
$(0.6, 0.3, 0.1)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(486.52,83,25)	(18487.75,3154,950)	52631.58	0.662
$(0.7, 0.2, 0.1)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(491.02,80,25)	(18658.75,3040,950)	52631.58	0.732
$(0.8, 0.1, 0.1)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(491.02,80,25)	(18658.75,3040,950)	52631.58	0.802

Numerical Example 2

Table 8. Change in the Number of Connections in Class 1 for Example 2

number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*total flow* (kbps)	*optimal value* (*satisfaction*)
$(150, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(278.878,83,25)	(10597.333,3154,950)	52631.58	0.529
$(140, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(298.797,83,25)	(11354.286,3154,950)	52631.58	0.551
$(130, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(321.781,83,25)	(12227.692,3154,950)	52631.58	0.575
$(120, 100, 100)$	$(e_2, e_6, e_{14}, e_{23})$	(334,100.516,25)	(12692,3819.6,950)	52631.58	0.598
$(110, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(334,133.916,25)	(12692,5088.8,950)	52631.58	0.616
$(100, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(335.316,166,25)	(12742,6308,950)	52631.58	0.634
$(90, 100, 100)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(372.573,166,25)	(14157.778,6308,950)	52631.58	0.652
$(80, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(522.895,83,25)	(19870,3154,950)	52631.58	0.679
$(70, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(597.594,83,25)	(22708.571,3154,950)	52631.58	0.715
$(60, 100, 100)$	$(e_2, e_7, e_{11}, e_{14}, e_{23})$	(697.193,83,25)	(26493.333,3154,950)	52631.58	0.763
$(50, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(842.632,80,25)	(32020,3040,950)	52631.58	0.830

Numerical Example 2

Table 9. Change in the Number of Connections in Class 2 for Example 2

number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*total flow* (kbps)	*optimal value* (*satisfaction*)
$(100, 150, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(376.816,83,25)	(14319,3154,950)	52631.58	0.609
$(100, 140, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(385.116,83,25)	(14634.4,3154,950)	52631.58	0.613
$(100, 130, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(393.416,83,25)	(14949.8,3154,950)	52631.58	0.617
$(100, 120, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(401.716,83,25)	(15265.2,3154,950)	52631.58	0.621
$(100, 110, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(334,152.105,25)	(12692,5780,950)	52631.58	0.626
$(100, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(335.316,166,25)	(12742,6308,950)	52631.58	0.634
$(100, 90, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(351.916,166,25)	(13372.8,6308,950)	52631.58	0.642
$(100, 80, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(368.516,166,25)	(14003.6,6308,950)	52631.58	0.650
$(100, 70, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(385.116,166,25)	(14634.4,6308,950)	52631.58	0.658
$(100, 60, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(401.716,166,25)	(15265.2,6308,950)	52631.58	0.666
$(100, 50, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(334,334.632,25)	(12692,12716,950)	52631.58	0.674

Numerical Example 2

Table 10. Change in the Number of Connections in Class 3 for Example 2

number of connections (K_1, K_2, K_3)	*selected path*	*bandwidth* (kbps) $(\theta_1, \theta_2, \theta_3)$	*budget* ($) (c_1, c_2, c_3)	*total flow* (kbps)	*optimal value* (*satisfaction*)	
$(100, 100, 150)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(334,154.816,25)	(12692,5883,950)	52631.58	0.627	
$(100, 100, 140)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(334,157.316,25)	(12692,5978,950)	52631.58	0.629	
$(100, 100, 130)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(334,159.816,25)	(12692,6073,950)	52631.58	0.630	
$(100, 100, 120)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(334,162.316,25)	(12692,6168,950)	52631.58	0.632	
$(100, 100, 110)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(334,164.816,25)	(12692,6263,950)	52631.58	0.633	
$(100, 100, 100)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(335.316,166,25)	(12742,6308,950)	52631.58	0.634	
$(100, 100, 90)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(337.816,166,25)	(12837,6308,950)	52631.58	0.635	
$(100, 100, 80)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(340.316,166,25)	(12932,6308,950)	52631.58	0.637	
$(100, 100, 70)$	$(e_2, e_7, e_{13}, e_{16}, e_{20}, e_{26})$	(342.816,166,25)	(13027,6308,950)	52631.58	0.638	
$(100, 100, 60)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(343.516,166,28)	(13053.6,6308,1064)	52631.58	0.639	
$(100, 100, 50)$	$(e_2, e_7, e_{13}, e_{16}, e_{19}, e_{25})$	(346.316,166,28)	(13160,6308,1064)	52631.58	0.640	(0.6

Conclusions

- We present an approach for the fair resource allocation problem in All-IP networks that offer multiple services to users.
- Users' utility functions are summarized by means of achievement functions. We find that the achievement function can map different criteria onto a normalized scale.
- The achievement function also can work in the Ordered Weighted Averaging method. Moreover, it may be interpreted as a measure of QoS on All-IP networks.
- Using the bandwidth allocation model, we can find a Pareto optimal allocation of bandwidth on the network under a limited available budget.
- Numerical results show that this scheme can provide each connection with its fair share of the bandwidth, which is proportional to the user's preferences.