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Numerical Methods for Solving Fuzzy System of Linear Equations

解線性模糊等式系統的數值方法

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Abstract

In this report, we propose a parallel iterative method for finding the solution of fuzzy systems of linear equations (FSLE). Conditions for the convergence of the parallel iterative method are derived and the convergence rate for the iterative sequence is also presented. The numerical results indicate that the proposed iterative method is accurate and efficient.

Key words: iterative methods, fuzzy system of linear equations, parallel methods, fuzzy number

中文摘要

本報告中，我們提出一個平行疊代解模糊等式系統的方法。報告中還導出平行疊代法收斂的條件與收斂速率。數值計算的例子顯示所提出的疊代法，計算正確且有效率。

關鍵字：疊代方法、模糊等式系統、平行方法、模糊數

1. Introduction

The problem of system modeling and identification has attracted considerable attention during the past twenty years mostly because of a large number of attentions in diverse fields like portfolio selection, biomedical system, socioeconomic system, transportation, electric power systems, hydrology, and aeronautics. In each of the cases, a model consists with a series of mathematical equations which can be used for understanding the behavior of the system, and whenever possible, for prediction and control.

Many papers have been presented on the study of parameter estimation and model identification. Nonetheless, we would like to point out that in dealing with parameter estimation, we should take the vague case as they belong to two or more classes simultaneously into account. Because many patterns of grouping in parameter estimation really are ambiguous, those phenomena should not be assigned to certain specific classes inflexibly. Hence, the linear systems of equations are not crisp and may be solved by using a fuzzy arithmetic.

This paper investigates the solution of $n \times n$ fuzzy systems of linear equations (FSLE) whose coefficient matrix is crisp and the right hand side is an arbitrary fuzzy vector. To find the solution of FSLE, Friendman et al. (1998) transform the FSLE into a $2n \times 2n$ crisp linear system utilizing the embedding approach. So far, a number of papers have been proposed for finding the numerical solution of this kind of crisp linear system. For example, the numerical algorithms for solving FSLE based on the Jacobi iterative method and the Gauss-Seidel iterative method have been proposed by Allahviranloo (2004) as well as Allahviranloo et al. (2006); the numerical algorithm based on the block SOR iterative method is adapted by Miao et al. (2008); and other iterative methods are developed by Dehghan and Hashemi (2006) as well as Liu et al. (2008).

In this paper, we will propose a parallel iterative method for finding the numerical solution of the crisp linear system and investigate the convergence condition of the proposed method. Moreover, we also derive the convergence rate for the iterative sequence produced by the proposed iterative method.

To compare with the other method, we consider the examples which are cited in Allahviranloo et al. (2006). After 2 iterations, the absolute error of the numerical solution produced by our parallel iterative method is less than 10^{-4}; after 10 iterations, the absolute error is less than 10^{-14}. This indicates that the proposed method is very efficient and accuracy for finding the numerical solution of FSLE.

The rest of the paper is organized as follows. Basic definitions on the fuzzy set theory and some results about the FSLE are introduced in Section 2. The parallel iterative method and the convergence conditions are developed in Section 3. The numerical results for solving the problems cited from the literatures are illustrated on the Section 4. Finally, a concise conclusion is provided in Section 5.

2. Fuzzy Linear Systems

A *fuzzy number* (Cong-Xin and Ming, 1991) is an ordered paired of functions

$(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, which satisfies the following requirements:

(1) $\underline{u}(r)$ is a bounded left continuous nondecreasing function over $[0,1]$.

(2) $\overline{u}(r)$ is a bounded left continuous nonincreasing function over $[0,1]$.

(3) $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

The arithmetic operations of arbitrary fuzzy numbers $x = (\underline{x}(r), \overline{x}(r))$, $y = (\underline{y}(r), \overline{y}(r))$, $0 \le r \le 1$, and real number k is given as

(1) $x + y = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$.

(2) $kx = \begin{cases} (k\underline{x}(r), k\overline{x}(r)) & \text{if } k \ge 0 \\ (k\overline{x}(r), k\underline{x}(r)) & \text{if } k \le 0 \end{cases}$.

Moreover, we say $x = y$ if $\underline{x}(r) = \underline{y}(r)$ and $\overline{x}(r) = \overline{y}(r)$, $0 \le r \le 1$.

Following Friendman et al. (1998), the definitions of fuzzy linear system of equation and their solutions are as follows:

Definition 2.1: The $n \times n$ linear system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= y_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= y_n \end{aligned} \quad (1)$$

is called a *fuzzy linear system of equations*, where the coefficient matrix $A = (a_{ij})$, $1 \le i, j \le n$ is a crisp $n \times n$ matrix and $y = (y_i)$, $i = 1, 2, \cdots, n$ is a fuzzy vector.

The solution $x = (x_1, x_2, \cdots, x_n)^T$ for the FLSE (1) is given as follows.

Definition 2.2: A fuzzy number vector $(x_1, x_2, \cdots, x_n)^T$ given by

$$x_i = (\underline{x}_i(r), \overline{x}_i(r)), \quad i = 1, 2, \cdots, n, \quad 0 \le r \le 1$$

is called a solution of the FLSE if

$$\underline{\sum_{j=1}^{n} a_{ij} x_j} = \sum_{j=1}^{n} a_{ij} \underline{x}_j = \underline{y}_i$$

$$\overline{\sum_{j=1}^{n} a_{ij} x_j} = \sum_{j=1}^{n} a_{ij} \overline{x}_j = \overline{y}_i$$

Friendman et al. (1998) extend FLSE to the following $2n \times 2n$ crisp linear system as follows:

$$\begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} \underline{Y} \\ \overline{Y} \end{bmatrix} \qquad (2)$$

or

$$SX = Y$$

where $X = (\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_n, \overline{x}_1, \overline{x}_2, \cdots, \overline{x}_n)$, $Y = (\underline{y}_1, \underline{y}_2, \cdots, \underline{y}_n, \overline{y}_1, \overline{y}_2, \cdots, \overline{y}_n)$ and

$S = (s_{ij})$, $1 \le i,\ j \le 2n$ are determined as

$$s_{ij} = s_{i+n,\ j+n} = a_{ij} \text{ , if } a_{ij} \ge 0$$

$$s_{i+n,\ j} = s_{i,\ j+n} = a_{ij} \text{ , if } a_{ij} \le 0$$

and any s_{ij} which is not determined is zero. Hence we have

$$s_{ij} \underline{x}_j + s_{i,\ j+n} \overline{x}_j = \begin{cases} s_{ij} \underline{x}_j & \text{if } a_{ij} \ge 0 \\ s_{i,\ j+n} \overline{x}_j & \text{if } a_{ij} < 0 \end{cases} \qquad (3)$$

3. Main Results

The original $n \times n$ fuzzy system of linear equations have been rearranged as an $2n \times 2n$ crisp linear system $SX = Y$. This linear can be uniquely solved for X if and only if S is nonsingular.

Theorem 3.1. (Friendman et al., 1998) *The matrix S is nonsingular if and only if the matrices $A = S_1 - S_2$ and $B = S_1 + S_2$ are both nonsingular.*

Instead of calculating S^{-1}, we develop an iteration process for solving the crisp linear system $SX = Y$. Adapting the splitting matrix

$$Q = \begin{bmatrix} S_1 & \mathbf{0} \\ \mathbf{0} & S_1 \end{bmatrix}$$

we obtain the equivalent form

$$QX = (Q - S)X + Y$$

which suggests an iterative process as follows:

$$QX^{(m+1)} = (Q - S)X^{(m)} + Y, \quad m = 0, 1, 2, \cdots \tag{4}$$

More precisely, (4) yields the following equations

$$\begin{bmatrix} \underline{X}^{(m+1)} \\ \overline{X}^{(m+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -S_1^{-1}S_2 \\ -S_1^{-1}S_2 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \underline{X}^{(m)} \\ \overline{X}^{(m)} \end{bmatrix} + \begin{bmatrix} S_1^{-1}\underline{Y} \\ S_1^{-1}\overline{Y} \end{bmatrix}$$

or

$$\underline{X}^{(m+1)} = -S_1^{-1}S_2\overline{X}^{(m)} + S_1^{-1}\underline{Y} \tag{5}$$

$$\overline{X}^{(m+1)} = -S_1^{-1}S_2\underline{X}^{(m)} + S_1^{-1}\overline{Y} \tag{6}$$

providing that S_1 is nonsingular. Substituting (6) into (5) yields

$$\begin{aligned} \underline{X}^{(m+1)} &= -S_1^{-1}S_2[-S_1^{-1}S_2\underline{X}^{(m)} + S_1^{-1}\overline{Y}] + S_1^{-1}\underline{Y} \\ &= (S_1^{-1}S_2)(S_1^{-1}S_2)\underline{X}^{(m)} - (S_1^{-1}S_2S_1^{-1})\overline{Y} + S_1^{-1}\underline{Y} \end{aligned} \tag{7}$$

Given the initial vector $(\underline{X}^{(0)}, \underline{X}^{(1)})$, (7) can be written as the following two equations:

$$\underline{X}^{(2m+2)} = (S_1^{-1}S_2)(S_1^{-1}S_2)\underline{X}^{(2m)} + f \tag{8}$$

and

$$\underline{X}^{(2m+3)} = (S_1^{-1}S_2)(S_1^{-1}S_2)\underline{X}^{(2m+1)} + f \tag{9}$$

where $f = -(S_1^{-1}S_2S_1^{-1})\overline{Y} + S_1^{-1}\underline{Y}$.

By using the similarly argument, we obtain the iterative process for $\overline{X}$ as follows:

$$\overline{X}^{(2m+2)} = (S_1^{-1}S_2)(S_1^{-1}S_2)\overline{X}^{(2m)} + g \tag{10}$$

$$\overline{X}^{(2m+3)} = (S_1^{-1}S_2)(S_1^{-1}S_2)\overline{X}^{(2m+1)} + g \tag{11}$$

where $g = -(S_1^{-1} S_2 S_1^{-1})\underline{Y} + S_1^{-1}\overline{Y}$.

Since the iteration procedures in generating two sequences $\{\underline{X}^{(m)}\}$ and $\{\overline{X}^{(m)}\}$ do not make use the information between them, both sequence $\{\underline{X}^{(m)}\}$ and $\{\overline{X}^{(m)}\}$ can be computed simultaneously in difference machines. Therefore, we have proposed a parallel iterative method for solving the $2n \times 2n$ crisp linear systems numerically.

The following theorems provide the conditions that two sequence $\{X^{(2m+1)}\}$ and $\{X^{(2m)}\}$ are converging to the same limit vector, where $X^{(m)} = (\underline{X}^{(m)}, \overline{X}^{(m)})$. Let $\rho(A)$ denote the spectral radius of A which will be used in the proof of the theorem.

Theorem 3.2. *The subsequences* $\{\underline{X}^{(2m+1)}\}$ *and* $\{\underline{X}^{(2m)}\}$ *obtained by* (8) *and* (9) *converge for any initial starting vectors if and only if* $\rho(G^2) < 1$, *where* $G = S_1^{-1} S_2$.

Proof: Assume that $(G^2) < 1$. Then $I - G^2$ is nonsingular. Given any initial vector $\underline{X}^{(0)}$ and $\underline{X}^{(1)}$. From (8) and (9), we can iteratively obtain the following relations:

$$\underline{X}^{(2m+2)} - (I - G^2)^{-1} f = G^{2m}(\underline{X}^{(0)} - (I - G^2)^{-1} f)$$

$$\underline{X}^{(2m+3)} - (I - G^2)^{-1} f = G^{2m+1}(\underline{X}^{(1)} - (I - G^2)^{-1} f)$$

Let $\underline{X}^{(*)} = (I - G^2)^{-1} f$, the above equations can be written as follows:

$$\underline{X}^{(2m+2)} - \underline{X}^{(*)} = G^{2m}(\underline{X}^{(0)} - \underline{X}^{(*)})$$

$$\underline{X}^{(2m+3)} - \underline{X}^{(*)} = G^{2m+1}(\underline{X}^{(1)} - \underline{X}^{(*)})$$

It is clear that both sequences converges to $\underline{X}^{(*)}$. On the other hand, assume both $\{\underline{X}^{(2m+2)}\}$ and $\{\underline{X}^{(2m+3)}\}$ converge to the same vector for any initial starting vectors. By iteratively substituting (8) and (9), we obtain

$$\underline{X}^{(2m+2)} - \underline{X}^{(2m)} = G^{2m+2}(f - (I - G^2)\underline{X}^{(0)})$$

$$\underline{X}^{(2m+3)} - \underline{X}^{(2m+1)} = G^{2m+3}(f - (I - G^2)\underline{X}^{(1)})$$

These implies that both $G^{2m+2}(f - (I - G^2)\underline{X}^{(0)})$ and $G^{2m+3}(f - (I - G^2)\underline{X}^{(1)})$ converge to zero. Hence we have proved that $\rho(G^2) < 1$. ❑

Remark 3.3: *The results in Theorem 3.2 also hold for replacing* $\{\underline{X}^{(2m)}, \underline{X}^{(2m+1)}\}$ *with* $\{\overline{X}^{(2m)}, \overline{X}^{(2m+1)}\}$.

By using Theorem 3.2 and Remark 3.3, we have the following theorem.

Theorem 3.4: *The sequence* $\{\underline{X}^{(m)}, \overline{X}^{(m)}\}$ *generated either by* (8) *and* (10) *or by* (9) *and* (11) *converge to the solution of* (1) *if and only if* $\rho(G^2) < 1$.

For the sequence $\{\underline{X}^{(2m)}\}$, the convergence rate can be obtained by calculating the logarithm of the inverse of the spectral radius of the iteration matrix (Saad, 2000).

4. Numerical Results

In order to compare with the other method, we consider the following examples which are cited from Allahvirbloo (2004).

Example 4.1: Consider the 5×5 fuzzy linear system

$$\begin{cases} 6x_1 + x_2 + 3x_3 - x_4 + 6x_5 = (1+r, 3-r) \\ 5x_1 + 9x_2 + x_3 + 2x_4 + 3x_5 = (6+r, 8-r) \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 + 3x_5 = (5+r, 7-r) \\ -x_1 + x_2 + 3x_3 + 8x_4 + 3x_5 = (3+r, 5-r) \\ x_1 + 2x_2 + 2x_3 + x_4 + 9x_5 = (2+r, 4-r) \end{cases} \tag{12}$$

The extended 10×10 matrix is

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix}$$

where

$$S_1 = \begin{bmatrix} 6 & 1 & 3 & -1 & 6 \\ 5 & 9 & 1 & 2 & 3 \\ 6 & 3 & 9 & 2 & 3 \\ -1 & 1 & 3 & 8 & 3 \\ 1 & 2 & 2 & 1 & 9 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The extended linear system becomes $SX = Y$ where

$$X = (\underline{X}, \overline{X})^T = (\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_5, \overline{x}_1, \overline{x}_2, \cdots, \overline{x}_5)^T$$

and

$$Y = (\underline{Y}, \overline{Y})^T = (1, 6, 5, 3, 2, 3, 8, 7, 5, 4)^T$$

The exact solutions are list in Table 1. Since $\rho(G^2) \approx 0.01$, we can apply theorem 2 to find the numerical solution of the fuzzy linear systems (12). The exact and numerical solutions are compared for $r = 0$, $r = 0.5$, and $r = 1$ shown in Table 2, Table 3, and Table 4, respectively, with 2, 5, and 10 iterations. By observing Table 2, 3, and 4, we found that the error of the numerical solution is after 2, 5, and 10 iterations is less that 10^{-4}, 10^{-8}, and 10^{-13}, respectively. This implies that the proposed iterative method is very efficient for finding the solution.

Table 1: The exact solution of fuzzy system (12)

	$\underline{x}_i(r)$	$\overline{x}_i(r)$
x_1	$-0.040903681493213 + 0.046214355948870\,r$	$0.051525030404525 - 0.046214355948869\,r$
x_2	$0.613045564751325 + 0.038839724680432\,r$	$0.690725014112190 - 0.038839724680433\,r$
x_3	$0.319445293003538 + 0.046460176991150\,r$	$0.412365646985839 - 0.046460176991150\,r$
x_4	$0.185413425823663 + 0.067109144542773\,r$	$0.319631714909209 - 0.067109144542773\,r$
x_5	$-0.001054606648909 + 0.079564077351688\,r$	$0.158073548054467 - 0.079564077351688\,r$

Table 2: The exact solution and numerical solution of fuzzy system (12) when $r = 0$

$r = 0$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	−0.040941834095869	−0.040941834095869	−0.040903684127214	−0.040903681493214
$\underline{x}_2$	0.613090741522656	0.613090741522656	0.613045566803353	0.613045564751325
$\underline{x}_3$	0.319464868356041	0.319464868356041	0.319445293554809	0.319445293003538
$\underline{x}_4$	0.185290454183941	0.185290454183941	0.185413422828470	0.185413425823663
$\underline{x}_5$	−0.001051093316163	−0.001051093316163	−0.001054606601954	−0.001054606648909
$\overline{x}_1$	0.051369508336993	0.051369508336993	0.051525023447644	0.051525030404525
$\overline{x}_2$	0.690859538171552	0.690859538171552	0.690725019328586	0.690725014112190
$\overline{x}_3$	0.412408219255036	0.412408219255036	0.412365648289306	0.412365646985839
$\overline{x}_4$	0.319386041947648	0.319386041947648	0.319631708045801	0.319631714909209
$\overline{x}_5$	0.158078770540242	0.158078770540242	0.158073548141196	0.158073548054467

Table 3: The exact solution and numerical solution of fuzzy system (12) when $r = 0.5$

$r = 0.5$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	−0.017796503518779	−0.017863998487653	−0.017796507233499	−0.017796503518779
$\underline{x}_2$	0.632465427091541	0.632532940684880	0.632465429934661	0.632465427091541
$\underline{x}_3$	0.342675381499114	0.342700706080790	0.342675382238434	0.342675381499114
$\underline{x}_4$	0.218967998095050	0.218814351124868	0.218967994132803	0.218967998095050
$\underline{x}_5$	0.038727432026935	0.038731372647938	0.038727432083834	0.038727432026935
$\overline{x}_1$	0.028417852430090	0.028291672728778	0.028417846553930	0.028417852430090
$\overline{x}_2$	0.671305151771974	0.671417339009328	0.671305156197278	0.671305151771974
$\overline{x}_3$	0.389135558490264	0.389172381530288	0.389135559605682	0.389135558490264
$\overline{x}_4$	0.286077142637822	0.285862145006722	0.286077136741468	0.286077142637822
$\overline{x}_5$	0.118291509378623	0.118296304576140	0.118291509455408	0.118291509378623

Table 4: The exact solution and numerical solution of fuzzy system (12) when $r = 1.0$

$r = 1.0$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	0.005310674455656	0.005213837120562	0.005310669660216	0.005310669660216
$\underline{x}_2$	0.651885289431757	0.651975139847104	0.651885293065969	0.651885293065969
$\underline{x}_3$	0.365905469994689	0.365936543805539	0.365905470922058	0.365905470922058
$\underline{x}_4$	0.252522570366436	0.252338248065795	0.252522565437136	0.252522565437136
$\underline{x}_5$	0.078509470702779	0.078513838612039	0.078509470769621	0.078509470769621
$\overline{x}_1$	0.005310674455656	0.005213837120562	0.005310669660216	0.005310669660216
$\overline{x}_2$	0.651885289431757	0.651975139847104	0.651885293065969	0.651885293065969
$\overline{x}_3$	0.365905469994689	0.365936543805539	0.365905470922058	0.365905470922058
$\overline{x}_4$	0.252522570366436	0.252338248065795	0.252522565437136	0.252522565437136
$\overline{x}_5$	0.078509470702779	0.078513838612039	0.078509470769621	0.078509470769621

Example 4.2: Consider the 6×6 fuzzy linear system

$$\begin{cases} 7x_1 + 2x_2 + 3x_3 + 4x_4 + 3x_5 - x_6 = (1+r, 3-r) \\ 7x_1 + 8x_2 + 2x_3 + 2x_4 + 3x_5 + 5x_6 = (4+r, 6-r) \\ 2x_1 + 2x_2 + 8x_3 + 2x_4 + x_5 + 2x_6 = (5+r, 7-r) \\ -x_1 + x_2 + 2x_3 + 9x_4 + 3x_5 + x_6 = (3+r, 5-r) \\ x_1 + 4x_2 + 4x_3 - x_4 + 9x_5 + 3x_6 = (7+r, 9-r) \\ 4x_1 + 2x_2 + 2x_3 + 2x_4 + 3x_5 + 9x_6 = (2+r, 4-r) \end{cases} \tag{13}$$

The extended 12×12 matrix is

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix}$$

where

$$S_1 = \begin{bmatrix} 7 & 2 & 3 & 4 & 3 & 0 \\ 4 & 8 & 2 & 2 & 3 & 5 \\ 2 & 2 & 8 & 2 & 1 & 2 \\ 0 & 1 & 2 & 9 & 3 & 1 \\ 1 & 4 & 4 & 0 & 9 & 3 \\ 4 & 1 & 2 & 2 & 3 & 9 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The exact solution is list in Table 5. Since $\rho(G^2) \approx 0.01$, we can apply theorem 2 to find the numerical solution of the fuzzy linear systems (13). The exact and numerical solutions are computed for $r = 0$, $r = 0.5$, and $r = 1$ shown in Table 6,

Table 7, and Table 8, respectively, with 2, 5, and 10 iterations. By observing Table 6, 7, and 8, we found that the absolute error of the numerical solution after 2, 5, and 10 iterations is less than 10^{-4}, 10^{-8}, and 10^{-14}, respectively.

Table 5: The exact solution of fuzzy system (13)

	$\underline{x}_i(r)$	$\overline{x}_i(r)$
x_1	$-0.341645712871096 + 0.040724944427309\,r$	$-0.260195824016478 - 0.040724944427309\,r$
x_2	$0.329657879973863 + 0.025781474351474\,r$	$0.381220828676812 - 0.025781474351474\,r$
x_3	$0.557832562945562 + 0.074951340657938\,r$	$0.707735244261439 - 0.074951340657938\,r$
x_4	$-0.001652622266682 + 0.069016496314653\,r$	$0.136380370362624 - 0.069016496314653\,r$
x_5	$0.410817826854803 + 0.039108284319460\,r$	$0.489034395493724 - 0.039108284319460\,r$
x_6	$0.076901289954261 + 0.045117580115081\,r$	$0.167136450184422 - 0.045117580115081\,r$

Table 6: The exact solution and numerical solution of fuzzy system (13) when $r = 0$

$r = 0$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	−0.341645712871096	−0.341623667890102	−0.341645712815931	−0.341645712871096
$\underline{x}_2$	0.329657879973863	0.329646617347775	0.329657879953306	0.329657879973863
$\underline{x}_3$	0.557832562945562	0.557835508251683	0.557832562954117	0.557832562945563
$\underline{x}_4$	−0.001652622266682	−0.001680547854974	−0.001652622342600	−0.001652622266682
$\underline{x}_5$	0.410817826854803	0.410867970063916	0.410817826940933	0.410817826854803
$\underline{x}_6$	0.076901289954261	0.076881580358607	0.076901289918287	0.076901289954261
$\overline{x}_1$	−0.260195824016478	−0.260162837431500	−0.260195823938397	−0.260195824016478
$\overline{x}_2$	0.381220828676812	0.381205739955361	0.381220828647091	0.381220828676812
$\overline{x}_3$	0.707735244261439	0.707741730317979	0.707735244274350	0.707735244261439
$\overline{x}_4$	0.136380370362624	0.136327260714072	0.136380370250718	0.136380370362624
$\overline{x}_5$	0.489034395493724	0.489105741136527	0.489034395621554	0.489034395493724
$\overline{x}_6$	0.167136450184422	0.167110044921883	0.167136450132411	0.167136450184422

Table 7: The exact solution and numerical solution of fuzzy system (13) when $r = 0.5$

$r = 0.5$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	−0.341645712871096	−0.321258460275452	−0.321283240596548	−0.321283240657442
$\underline{x}_2$	0.329657879973863	0.342536397999672	0.342548617126753	0.342548617149600
$\underline{x}_3$	0.557832562945562	0.595312063768257	0.595308233284175	0.595308233274532
$\underline{x}_4$	−0.001652622266682	0.032821404287288	0.032855625805729	0.032855625890644
$\underline{x}_5$	0.410817826854803	0.430427412832069	0.430371969111088	0.430371969014533
$\underline{x}_6$	0.076901289954261	0.099438696499426	0.099460079971818	0.099460080011802
$\overline{x}_1$	−0.260195824016478	−0.280528045046150	−0.280558296157781	−0.280558296230133
$\overline{x}_2$	0.381220828676812	0.368315959303465	0.368330091473645	0.368330091501075
$\overline{x}_3$	0.707735244261439	0.670265174801405	0.670259573944291	0.670259573932470
$\overline{x}_4$	0.136380370362624	0.101825308571811	0.101872122102388	0.101872122205297
$\overline{x}_5$	0.489034395493724	0.469546298368375	0.469480253451399	0.469480253333994
$\overline{x}_6$	0.167136450184422	0.144552928781064	0.144577660078880	0.144577660126882

Table 8: The exact solution and numerical solution of fuzzy system (13) when $r = 1.0$

$r = 1.0$	Exact solution	2 iterations	5 iterations	10 iterations
$\underline{x}_1$	−0.341645712871096	−0.321258460275452	−0.321283240596548	−0.321283240657442
$\underline{x}_2$	0.329657879973863	0.342536397999672	0.342548617126753	0.342548617149600
$\underline{x}_3$	0.557832562945562	0.595312063768257	0.595308233284175	0.595308233274532
$\underline{x}_4$	−0.001652622266682	0.032821404287288	0.032855625805729	0.032855625890644
$\underline{x}_5$	0.410817826854803	0.430427412832069	0.430371969111088	0.430371969014533
$\underline{x}_6$	0.076901289954261	0.099438696499426	0.099460079971818	0.099460080011802
$\overline{x}_1$	−0.260195824016478	−0.280528045046150	−0.280558296157781	−0.280558296230133
$\overline{x}_2$	0.381220828676812	0.368315959303465	0.368330091473645	0.368330091501075
$\overline{x}_3$	0.707735244261439	0.670265174801405	0.670259573944291	0.670259573932470
$\overline{x}_4$	0.136380370362624	0.101825308571811	0.101872122102388	0.101872122205297
$\overline{x}_5$	0.489034395493724	0.469546298368375	0.469480253451399	0.469480253333994
$\overline{x}_6$	0.167136450184422	0.144552928781064	0.144577660078880	0.144577660126882

5. Conclusion

For finding the solution of FLSE, we have proposed a parallel iteration method when S_1 is an invertible matrix. The convergence condition and convergence rate are also determined. When converges rate is small enough, i.e., $\rho((S_1^{-1}S_2)^2) << 1$, the iterative sequence produced by the parallel iterative method converges rapidly. Finally, the numerical results indicate that the iterative method is accurate and efficient.

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國科會補助專題研究計畫項下出席國際學術會議心得報告

日期：98年12月10日

<table>
<tr><td>計畫編號</td><td colspan="3">NSC98-2115-M-004-005</td></tr>
<tr><td>計畫名稱</td><td colspan="3">解線性模糊等式系統的數值方法</td></tr>
<tr><td>出國人員姓名</td><td>劉明郎</td><td>服務機構及職稱</td><td>政大應用數學系副教授</td></tr>
<tr><td>會議時間</td><td>98 年 12 月 1 日至
98 年 12 月 3 日</td><td>會議地點</td><td>中國廣西桂林</td></tr>
<tr><td>會議名稱</td><td colspan="3">(中文)第 10 屆國際智慧科技學術研討會
(英文)10th Intelligent Technologies International Conference</td></tr>
<tr><td>發表論文題目</td><td colspan="3">(中文）平行疊代法解線性模糊等式系統
(英文)A Parallel Iteration Method for Solving Fuzzy System of Linear Equations</td></tr>
</table>

一、參加會議經過:主持其中一個 section 的會議並發表一篇論文。

二、與會心得：與相關領域的國際學者交流可增近感情，加强未來的合作可能性。

三、考察參觀活動(無是項活動者略)：無

四、建議：核定之經費只够參加一次國際學術活動，應增加參予國際學術活動的次數(即經費)。

五、攜回資料名稱及內容:大會論文集

六、其他

無衍生研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：劉明郎			計畫編號：98-2115-M-004-005-				
計畫名稱：解線性模糊等式系統的數值方法							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	1	50%	篇	論文「Solving system of lineat equations based on modified homotopy perturbation method」投稿審查中
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		論文「A Parallel Iteration Method for Solving Fuzzy System of Linear Equations」發表於 InTech 12th December 1-3, 2009, Gualin, China
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	1	1	100%		
		博士後研究員	0	0	100%		
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國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
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	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	

		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)		藉由此計畫得以參加 InTech 12th December 1-3, 2009, Gualin, China 研討會，能與國外相&#38306；學者交流。					

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以 100 字為限） □實驗失敗 □因故實驗中斷 □其他原因 說明：
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3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限） 完成以 Parallel Iteration Method 解 Fuzzy System of Linear Equations 的數值演算法，求解的速度快且準確。建立演算法的過程可以提供類似的 Linear Linear Equations 發展數值演算法的參考，若有其他 Fuzzy System of Linear Equations 的應用問題，可以直接使用本演算法獲得準確的解答。