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行政院國家科學委員會補助專題研究計畫 ■成果報告
□期中進度報告

隨機函數及其應用之研究

A study of the random functional and its applications

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行政院國家科學委員會專題研究計畫成果報告

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中文摘要

Jiang, Dickey, and Kuo (2004) 提出多變量 c-特徵函數並且證明它有一些類似多變量 Fourier 轉換的性質，這個新的轉換可以有用地解決一些難以用 Fourier 轉換或傳統特徵函數來處理的問題。在本文中，我們首先提出定義在單位球上的 Ferguson-Dirichlet 過程，其隨機函數的多變量 c-特徵函數，接著透過多變量 c-特徵函數的性質，我們可以求得其機率密度函數，這個三維的新結果推廣了 Jiang (1991) 的二維結果。
關鍵詞：Ferguson-Dirichlet 過程；c-特徵函數；對稱分配；Fourier 轉換

Abstract

Jiang, Dickey, and Kuo (2004) give the multivariate c-characteristic function and show that it has properties similar to those of the multivariate Fourier transformation. This new transformation can be useful when a distribution is difficult to deal with using Fourier transformation or traditional characteristic function. In this paper, we first give the multivariate c-characteristic function of the random functional of a Ferguson-Dirichlet process on the unit ball. We then find out its probability density function using properties of the multivariate c-characteristic function. This new result in three-dimension would generalize the two-dimensional result given by Jiang (1991).
Keywords: Ferguson-Dirichlet process; c-characteristic function; spherical distribution; Fourier transformation

1 Introduction

The distribution of random functional of a Ferguson-Dirichlet process has drawn the interest of many researchers for decades. A partial list of papers in this area are Hannum, Hollander, and Langberg (1981), Yamato (1984), Jiang (1991), Cifarelli and Regazzini (1990), Diaconis and Kemperman (1996), Regazzini, Guglielmi, and Di Nunno (2002), Jiang, Dickey, and Kuo (2004), Lijoi and Regazzini (2004), and Hjort and Ongaro (2005). In particular, Jiang

(1991) gave the distribution of random functional of a Ferguson-Dirichlet process on the unit disk. In this paper, we shall use the multivariate c-characteristic function, a tool given by Jiang, Dickey, and Kuo (2004), to generalize the result to the case on the unit ball in three dimension.

In Section 2, we first review the definition of the multivariate c-characteristic function and some of its properties. We then compute a multivariate c-characteristic function of an interesting distribution. The multivariate c-characteristic function of the random mean of a Ferguson-Dirichlet process on the unit ball is then given in Section 3. Using the uniqueness property of the multivariate c-characteristic function, we then determine the distribution of the random mean of a Ferguson-Dirichlet process on the unit ball. Conclusions are given in Section 4.

2 Multivariate c-characteristic function

Jiang (1988) first gave a univariate c-characteristic function. Jiang, Dickey, and Kuo (2004) generalized it to a multivariate c-characteristic function, which can be very useful when a distribution is difficult to deal with by traditional characteristic function. First, we shall review the definition and some properties of the multivariate c-characteristic function.

Definition 1 (Jiang, Dickey, and Kuo, 2004) *If $\boldsymbol{u} = (u_1, \ldots, u_L)'$ is a random vector on a subset S of $A = [-a_1, a_1] \times \cdots \times [-a_L, a_L]$, its multivariate c-characteristic function is defined as*

$$g(\boldsymbol{t}; \boldsymbol{u}, c) = \underset{\boldsymbol{u}}{E}[(1 - i\boldsymbol{t} \cdot \boldsymbol{u})^{-c}], \ |\boldsymbol{t}| < a^{-1}, \tag{1}$$

where c is a positive real number, $a = \sqrt{\sum_{i=1}^{L} a_i^2}$, $\boldsymbol{t}' = (t_1, \ldots, t_L)$, $|\boldsymbol{t}| = \sqrt{\sum_{i=1}^{L} t_i^2}$, and $\boldsymbol{t} \cdot \boldsymbol{u}$ is the inner product of two vectors (i.e., $\boldsymbol{t} \cdot \boldsymbol{u} = \sum_{i=1}^{L} t_i u_i$).

With the above definition, Jiang, Dickey, and Kuo (2004) showed the one-to-one correspondence between $g(\boldsymbol{t}; \boldsymbol{u}, c)$ and the random vector $\boldsymbol{u}$.

Lemma 2 (Jiang, Dickey, and Kuo, 2004) *For any two random vectors $\boldsymbol{u} = (u_1, \ldots, u_L)'$ and $\boldsymbol{v} = (v_1, \ldots, v_L)'$ on a subset S of $A = [-a_1, a_1] \times \cdots \times [-a_L, a_L]$ and any positive real number c, if we have*

$$g(\boldsymbol{t}; \boldsymbol{u}, c) = g(\boldsymbol{t}; \boldsymbol{v}, c), \tag{2}$$

for all $|\boldsymbol{t}| < a^{-1}$, where $a = \sqrt{\sum_{i=1}^{L} a_i^2}$, then $\boldsymbol{u} \sim \boldsymbol{v}$.

In addition, the important convergence theorem was also established by Jiang, Dickey, and Kuo (2004)

Lemma 3 (Jiang, Dickey, and Kuo, 2004) *Assume $\boldsymbol{u}$, and $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots$ are random vectors on a subset S of $A = [-a_1, a_1] \times \cdots \times [-a_L, a_L]$ and their corresponding multivariate c-characteristic functions are $g(\boldsymbol{t}; \boldsymbol{u}, c)$, $g(\boldsymbol{t}; \boldsymbol{u}_1, c)$, $g(\boldsymbol{t}; \boldsymbol{u}_2, c), \ldots$, respectively. Then, for a given $c > 0$, the following statements are equivalent:*

$$\boldsymbol{u}_n \to \boldsymbol{u} \text{ in distribution as } n \to \infty, \tag{3}$$

$$g(\boldsymbol{t}; \boldsymbol{u}_n, c) \to g(\boldsymbol{t}; \boldsymbol{u}, c), \text{ as } n \to \infty, \text{ for all } |\boldsymbol{t}| < a^{-1}. \tag{4}$$

Next, we shall give the corresponding multivariate c-characteristic function of an interesting distribution in the next lemma.

Lemma 4 *Let $\boldsymbol{u} = (u_1, u_2, u_3)'$ be a three-dimensional distribution on the inside of a unit ball ($\{(u_1, u_2, u_3) \mid u_1^2 + u_2^2 + u_3^2 < 1\}$) with the probability density function*

$$f(u_1, u_2, u_3) = \frac{-e}{4\pi^2 r}(1+r)^{-(1+r)/2}(1-r)^{-(1-r)/2}\left(-\pi \sin\frac{\pi r}{2} + \ln\frac{1-r}{1+r}\cos\frac{\pi r}{2}\right), \quad (5)$$

where $r = \sqrt{u_1^2 + u_2^2 + u_3^2}$. Then the multivariate 1-characteristic function of $\boldsymbol{u}$ is

$$g(\boldsymbol{t}; \boldsymbol{u}, c) = \exp\left(\sum_{n=1}^{\infty} \frac{(-t_1^2 - t_2^2 - t_3^2)^n}{2n(2n+1)}\right). \quad (6)$$

Proof. Let $X = \{(u_1, u_2, u_3) \mid u_1^2 + u_2^2 + u_3^2 < 1\}$. We shall claim the following identity:

$$\int_X (1 - it_1u_1 - it_2u_2 - it_3u_3)^{-1} f(u_1, u_2, u_3)\, du_1\, du_2\, du_3 = \exp\left(\sum_{n=1}^{\infty} \frac{(-t_1^2 - t_2^2 - t_3^2)^n}{2n(2n+1)}\right).$$

Using the spherical coordinate transformation, we have

$$\begin{aligned}
&\int_X (1 - it_1u_1 - it_2u_2 - it_3u_3)^{-1} f(u_1, u_2, u_3)\, du_1\, du_2\, du_3 \\
&= \int_0^1 \int_0^{2\pi} \int_0^{\pi} (1 - it_1 r\cos\theta\sin\phi - it_2 r\sin\theta\sin\phi - it_3 r\cos\phi)^{-1} \\
&\quad \times \frac{-er\sin\phi}{4\pi^2}(1+r)^{-(1+r)/2}(1-r)^{-(1-r)/2}\left(-\pi\sin\frac{\pi r}{2} + \ln\frac{1-r}{1+r}\cos\frac{\pi r}{2}\right) d\phi\, d\theta\, dr.
\end{aligned}$$

First, we shall determine the following integration:

$$\int_0^{2\pi}\int_0^{\pi} (1 - it_1 r\cos\theta\sin\phi - it_2 r\sin\theta\sin\phi - it_3 r\cos\phi)^{-1}\sin\phi\, d\phi\, d\theta. \quad (7)$$

Since $(1-x)^{-1} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$, Eq. (7) can be rewritten as

$$\begin{aligned}
\text{Eq. (7)} &= \sum_{n=0}^{\infty}(ir)^n \int_0^{2\pi}\int_0^{\pi} (t_1\cos\theta\sin\phi + t_2\sin\theta\sin\phi + t_3\cos\phi)^n \sin\phi\, d\phi\, d\theta \\
&= \sum_{n=0}^{\infty}(ir)^n \int_0^{2\pi}\int_0^{\pi} \sum_{k=0}^{n}\binom{n}{k}(t_1\cos\theta + t_2\sin\theta)^k t_3^{n-k}\sin^{k+1}\phi\cos^{n-k}\phi\, d\phi\, d\theta \\
&= \sum_{n=0}^{\infty}(-r^2)^n \sum_{k=0}^{n}\binom{2n}{2k}(t_1^2a_1^2 + t_2^2a_2^2)^k t_3^{2n-2k}\frac{(1/2,k)2\pi}{k!}B(k+1, n-k+1/2) \\
&= \sum_{n=0}^{\infty}\frac{4\pi(-t_1^2 - t_2^2 - t_3^2)^n r^{2n}}{2n+1}.
\end{aligned}$$

The third identity is obtained by the following Eqs. (8)-(10). Eq. (8) is from p. 105 of Gröbner and Hofreiter (1973),

$$\int_0^{2\pi}(a\cos\alpha + b\sin\alpha)^n\, d\alpha = \begin{cases} \frac{(1/2,n/2)2(a^2+b^2)^{n/2}\pi}{(n/2)!}, & n \text{ is even}, \\ 0, & n \text{ is odd}, \end{cases} \quad (8)$$

where a and b are real numbers and $(a,k) = a(a+1)\cdots(a+k-1)$.

$$\int_0^{\pi/2} \sin^{a-1} x \cos^{b-1} x \, dx = \frac{B(a/2, b/2)}{2}, \ \operatorname{Re} a > 0, \operatorname{Re} b > 0, \tag{9}$$

$$\int_{\pi/2}^{\pi} \sin^{a-1} x \cos^{b-1} x \, dx = \begin{cases} B(a/2, b/2)/2, & \text{if } b \text{ is odd}, \\ -B(a/2, b/2)/2, & \text{if } b \text{ is even}. \end{cases} \tag{10}$$

Eq. (9) is from formula 3.621.5 of Gradshteyn and Ryzhik (2000). Eq. (10) can be obtained easily by Eq. (9). Therefore,

$$\begin{aligned} &\int_X (1 - it_1u_1 - it_2u_2 - it_3u_3)^{-1} f(u_1, u_2, u_3) \, du_1 \, du_2 \, du_3 \\ &= \frac{-e}{\pi} \sum_{n=0}^{\infty} \frac{(-t_1^2 - t_2^2 - t_3^2)^n}{2n+1} \int_0^1 r^{2n+1} (1+r)^{-(1+r)/2} (1-r)^{-(1-r)/2} \left(-\pi \sin \frac{\pi r}{2} + \ln \frac{1-r}{1+r} \cos \frac{\pi r}{2} \right) dr \\ &= \frac{2e}{\pi} \sum_{n=0}^{\infty} (-t_1^2 - t_2^2 - t_3^2)^n \int_0^1 r^{2n} (1+r)^{-(1+r)/2} (1-r)^{-(1-r)/2} \cos \frac{\pi r}{2} \, dr. \end{aligned} \tag{11}$$

The second identity follows by using the integration by parts. Using Lemma 8 and Example 2 of Jiang and Kuo (2006), the following identity holds,

$$\exp\left(- \int_{-1}^{1} \ln(1 - itx) \frac{1}{2} \, dx \right) = \int_{-1}^{1} (1 - itx)^{-1} \frac{e}{\pi} (x+1)^{-(x+1)/2} (1-x)^{-(1-x)/2} \cos \frac{\pi x}{2} \, dx.$$

Equivalently,

$$\begin{aligned} \exp\left(\sum_{n=1}^{\infty} \frac{(-t^2)^n}{2n(2n+1)} \right) &= \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{e i^n t^n}{\pi} x^n (x+1)^{-(x+1)/2} (1-x)^{-(1-x)/2} \cos \frac{\pi x}{2} \, dx \\ &= \frac{2e}{\pi} \sum_{n=0}^{\infty} (-t^2)^n \int_0^1 x^{2n} (x+1)^{-(x+1)/2} (1-x)^{-(1-x)/2} \cos \frac{\pi x}{2} \, dx. \end{aligned}$$

The last identity can be obtained by the fact that the function $(x+1)^{-(x+1)/2}(1-x)^{-(1-x)/2} \cos \frac{\pi x}{2}$ is symmetric at $x = 0$. Therefore, Eq. (11) can be rewritten as

$$\exp\left(\sum_{n=1}^{\infty} \frac{(-t_1^2 - t_2^2 - t_3^2)^n}{2n(2n+1)} \right). \qquad \blacksquare$$

3 Distribution of a random functional of a Ferguson-Dirichlet process on the unit ball

Ferguson (1973) first defined the Ferguson-Dirichlet process. Let μ be a finite non-null measure on (X, A), where A is the σ-field of Borel subsets of Euclidean space X, and let U be a stochastic process indexed by elements of A. We say that U is a Ferguson-Dirichlet process with parameter μ, if for every finite measurable partition $\{B_1, \ldots, B_m\}$ of X, the random vector $(U(B_1), \ldots, U(B_m))$ has a Dirichlet distribution with parameter $(\mu(B_1), \ldots, \mu(B_m))$. Here, we shall study the random functional $\boldsymbol{u} = \int_X \boldsymbol{x} \, dU(x)$, where X is the unit ball. First, we give a trivariate c-characteristic function expression of any random functional in the next lemma.

Lemma 5 *Let* $\boldsymbol{w} = \int_X \boldsymbol{h}(x)\, dU(x)$ *be a three-dimensional random functional, where* U *is a Ferguson-Dirichlet process with parameter* μ *on* (X, A)*,* A *is the* σ*-field of Borel subsets of finite Euclidean space* X*, and* $\boldsymbol{h}(x) = (h_1(x), h_2(x), h_3(x))'$ *is a trivariate measurable function. Then the multivariate* c*-characteristic function of* $\boldsymbol{w} = (w_1, w_2, w_3)'$

$$g(\boldsymbol{t}; \boldsymbol{w}, c) = \exp\left(-\int_X \ln(1 - i\boldsymbol{t} \cdot \boldsymbol{h}(x))\, d\mu(x)\right), \tag{12}$$

where $c = \mu(X)$ *and* $\boldsymbol{t} = (t_1, t_2, t_3)'$.

Proof. For any $k \geq 2$, let $\{B_{k1}, B_{k2}, \dots, B_{kk}\}$ be a partition of X, $b_{kj} \in B_{kj}$, for all $j = 1, 2, \dots, k$, $v_k = \max\{\text{volume}(B_{kj}) \mid j = 1, \dots, k\}$, and $\lim_{k\to\infty} v_k = 0$. Define $\boldsymbol{h}_k(x) = (h_{1k}(x), h_{2k}(x), h_{3k}(x)) = \sum_{j=1}^k \boldsymbol{h}(b_{kj})\delta_{B_{kj}}(x)$, and $w_{ik} = \int_X h_{ik}(x)\, dU(x)$, then $\lim_{k\to\infty} \boldsymbol{h}_k(x) = \boldsymbol{h}(x)$, for all $x \in X$, and $w_{ik} = \sum_{j=1}^k h_{ik}(b_{kj})U(B_{kj})$, for all $i = 1, 2, 3$. The trivariate c-characteristic function of $\boldsymbol{w}_k = (w_{1k}, w_{2k}, w_{3k})'$ can be expressed as

$$\begin{aligned}
&g[(\boldsymbol{t}; \boldsymbol{w}_k, c)] \\
&\quad = E(1 - i\boldsymbol{t} \cdot \boldsymbol{w}_k)^{-c} \\
&\quad = E\left[\left(1 - it_1 \sum_{j=1}^k h_{1k}(b_{kj})U(B_{kj}) - it_2 \sum_{j=1}^k h_{2k}(b_{kj})U(B_{kj}) - it_3 \sum_{j=1}^k h_{3k}(b_{kj})U(B_{kj})\right)^{-c}\right] \\
&\quad = E\left[\left(\sum_{j=1}^k U(B_{kj})(1 - it_1h_{1k}(b_{kj}) - it_2h_{2k}(b_{kj}) - it_3h_{3k}(b_{kj})\right)^{-c}\right] \\
&\quad = \mathcal{R}_{-c}(\mu(B_{k1}), \dots, \mu(B_{kk}); 1 - it_1h_{1k}(b_{k1}) - it_2h_{2k}(b_{k1}) - it_3h_{3k}(b_{k1}), \\
&\qquad 1 - it_1h_{1k}(b_{k2}) - it_2h_{2k}(b_{k2}) - it_3h_{3k}(b_{k2}), \dots, 1 - it_1h_{1k}(b_{kk}) - it_2h_{2k}(b_{kk}) - it_3h_{3k}(b_{kk})),
\end{aligned}$$

where $\mathcal{R}$ is a Carlson's multiple hypergeometric function (Carlson, 1977). By the formula (6.6.5) in Carlson (1977), we have

$$g(\boldsymbol{t}; \boldsymbol{w}_k, c) = \prod_{j=1}^k (1 - it_1h_{1k}(b_{kj}) - it_2h_{2k}(b_{kj}) - it_3h_{3k}(b_{kj}))^{-\mu(B_{kj})}.$$

The limit of the trivariate c-characteristic function of $\boldsymbol{w}_k$'s, as k approaches to ∞, is

$$\begin{aligned}
\lim_{k\to\infty} g(\boldsymbol{t}; \boldsymbol{w}_k, c) &= \exp\left(\lim_{k\to\infty} \sum_{j=1}^k -\mu(B_{kj}) \ln(1 - it_1h_{1k}(b_{kj}) - it_2h_{2k}(b_{kj}) - it_3h_{3k}(b_{kj}))\right) \\
&= \exp\left(-\int_X \ln(1 - it_1h_1(x) - it_2h_2(x) - it_3h_3(x))\, d\mu(x)\right).
\end{aligned}$$

In addition, by the Dominated Convergence Theorem, we have $\lim_{k\to\infty} \boldsymbol{w}_k = \boldsymbol{w}$. By Lemma 3, we have $g(\boldsymbol{t}; \boldsymbol{w}, c) = \exp\left(-\int_X \ln(1 - i\boldsymbol{t} \cdot \boldsymbol{h}(x))\, d\mu(x)\right)$. ■

With the above Lemma 5, we can establish the multivariate c-characteristic function of a random functional of a Ferguson-Dirichlet process on the unit ball in the following theorem.

Theorem 6 *Let $X = \{(x_1, x_2, x_3) \mid x_1^2+x_2^2+x_3^2 = 1\}$, and U be a Ferguson-Dirichlet process on X with uniform parameter μ, where $\mu(X) = c$. Then the trivariate c-characteristic function of the random functional*

$$\boldsymbol{v} = \int_X x\, dU(x) \tag{13}$$

can be expressed as

$$g(\boldsymbol{t}; \boldsymbol{v}, c) = \exp\left(\sum_{k=1}^{\infty} \frac{c}{2k(2k+1)}(-t_1^2 - t_2^2 - t_3^2)^k\right), \tag{14}$$

where $\boldsymbol{t} = (t_1, t_2, t_3)'$.

Proof. By Lemma 5, we have

$$\begin{aligned}
&g(\boldsymbol{t}; \boldsymbol{v}, c)\\
&= \exp\left(\frac{-c}{4\pi}\int_X \ln(1 - it_1x_1 - it_2x_2 - it_3x_3)\, dx_1\, dx_2\, dx_3\right)\\
&= \exp\left(\frac{-c}{4\pi}\int_0^{\pi}\int_0^{2\pi} \ln(1 - it_1\cos\theta_1 - it_2\sin\theta_1\cos\theta_2 - it_3\sin\theta_1\sin\theta_2)\sin\theta_1\, d\theta_2\, d\theta_1\right)\\
&= \exp\left(\frac{c}{4\pi}\sum_{k=1}^{\infty}\frac{i^k}{k}\int_0^{\pi}\int_0^{2\pi}(t_1\cos\theta_1 + t_2\sin\theta_1\cos\theta_2 + t_3\sin\theta_1\sin\theta_2)^k\sin\theta_1\, d\theta_2\, d\theta_1\right)\\
&= \exp\left(\frac{c}{4\pi}\sum_{k=1}^{\infty}\frac{i^k}{k}\sum_{n=0}^{k}\binom{k}{n}\int_0^{\pi}\int_0^{2\pi}(t_1\cos\theta_1)^n\sin\theta_1^{k-n}(t_2\cos\theta_2 + t_3\sin\theta_2)^{k-n}\sin\theta_1\, d\theta_2\, d\theta_1\right)\\
&= \exp\left(\frac{c}{2}\sum_{k=1}^{\infty}\frac{i^k}{k}\sum_{\substack{n=0\\ k-n \text{ is even}}}^{k}\binom{k}{n}\frac{(1/2,(k-n)/2)(t_2^2+t_3^2)^{(k-n)/2}}{((k-n)/2)!}\int_0^{\pi}(t_1\cos\theta_1)^n\sin\theta_1^{k-n+1}\, d\theta_1\right)\\
&= \exp\left(\frac{c}{2}\sum_{k=1}^{\infty}\frac{(-1)^k}{2k}\sum_{n=0}^{k}\binom{2k}{2n}\frac{(1/2,k-n)(t_2^2+t_3^2)^{k-n}}{(k-n)!}t_1^{2n}B(k-n+1, n+1/2)\right)\\
&= \exp\left(\sum_{k=1}^{\infty}\frac{c}{2k(2k+1)}(-t_1^2 - t_2^2 - t_3^2)^k\right).
\end{aligned}$$

The fifth identity can be obtained by Eq. (8). The sixth identity follows from Eqs. (9) and (10). We complete the proof. ■

Using Lemma 4 and Theorem 6, we can obtain the following corollary.

Corollary 7 *The probability density function of $\boldsymbol{u} = \int_X x\, dU(x)$, where U is a Ferguson-Dirichlet process with uniform probability measure parameter on the unit ball X is*

$$f_{\boldsymbol{u}}(u_1, u_2, u_3) = \frac{-e}{4\pi^2 r}(1+r)^{-(1+r)/2}(1-r)^{-(1-r)/2}\left(-\pi\sin\frac{\pi r}{2} + \ln\frac{1-r}{1+r}\cos\frac{\pi r}{2}\right),$$

where $u_1^2 + u_2^2 + u_3^2 < 1$ and $r = \sqrt{u_1^2 + u_2^2 + u_3^2}$.

4 Conclusions

In this paper, we obtain the trivariate c-characteristic function expression for a random functional of a Ferguson-Dirichlet process over any finite three-dimensional space. We also obtain the probability density function of the random functional of a Ferguson-Dirichlet process with uniform probability measure parameter on the unit ball. This generalizes the previous result on two-dimension.

References

Carlson, B.C. (1977) *Special Functions of Applied Mathematics.* New York: Academic Press.

Cifarelli, D.M. and Regazzini, E. (1990) Distribution functions of means of a Dirichlet process. *Ann. Statist.*, **18**, 429–442. Correction (1994): *Ann. Statist.*, **22**, 1633–1634.

Diaconis, P. and Kemperman, J. (1996) Some new tools for Dirichlet priors. In J.M. Bernardo, J.O. Berger, A.P. Dawid, and A.F.M. Smith (eds.), *Bayesian Statistics 5*, pp. 97–106. Oxford University Press.

Ferguson, T.S. (1973) A Bayesian analysis of some nonparametric problems. *Ann. Statist.*, **1**, 209–230.

Gradshteyn, I.S. and Ryzhik, I.M. (2000) *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press.

Gröbner, W., Hofreiter, W. (1973) *Integraltafel*, 5th ed., Vol. 2. New York: Springer-Verlag.

Hannum, R.C., Hollander, M., and Langberg, N.A. (1981) Distributional results for random functionals of a Dirichlet process. *Ann. Probab.*, **9**, 665–670.

Hjort, N.L. and Ongaro, A. (2005) Exact inference for random Dirichlet means. *Stat. Inference Stoch. Process.*, **8**, 227–254.

Jiang, J. (1988) Starlike functions and linear functions of a Dirichlet distributed vector. *SIAM J. Math. Anal.*, **19**, 390–397.

Jiang, T.J. (1991) Distribution of random functional of a Dirichlet process on the unit disk. *Statist. Probab. Lett.*, **12**, 263–265.

Jiang, T.J., Dickey, J.M., and Kuo, K.-L. (2004) A new multivariate transform and the distribution of a random functional of a Ferguson-Dirichlet process. *Stochastic Process. Appl.*, **111**, 77–95.

Jiang, T.J. and Kuo, K-L (2006) On the random functional of the Ferguson-Dirichlet process. 2006 Proceeding of the Section on Bayesian Statistical Science of the American Statistical Association, pp. 52–59.

Lijoi, A. and Regazzini, E. (2004) Means of a Dirichlet process and multiple hypergeometric functions. *Ann. Probab.*, **32**, 1469–1495.

Regazzini, E., Guglielmi, A., and Di Nunno, G. (2002) Theory and numerical analysis for exact distributions of functionals of a Dirichlet process. *Ann. Statist.*, **30**, 1376–1411.

Yamato, H. (1984) Characteristic functions of means of distributions chosen from a Dirichlet process. *Ann. Probab.*, **12**, 262-267.

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

2006 年 9 月 8 日

附件三

報告人姓名	姜志銘	服務機構及職稱	國立政治大學應用數學系教授
會議時間 會議地點	2006 年 8 月 6-10 日 美國西雅圖(Seattle)	本會核定補助文號	NSC 95-2118-M-004-006
會議名稱	(中文)2006 聯合統計會議 (英文)2006 Joint Statistical Meetings		
發表論文題目	(中文) (英文)On the random functional of the Ferguson-Dirichlet process		

報告內容應包括下列各項：

一、參加會議經過

由美國統計學會及其他多個國際知名的統計學會，如 IMS, ENAR, WNAR 及 SSC 聯合主辦的 2006 年聯合統計會議，依過去經驗，總會吸引幾千位來自全球各地的統計專家學者等，為追求及交換新的統計理論與方法來參加，本人亦抱持著這種態度，希望能藉由這個機會與其他的統計學者專家作學術上的交流。

二、與會心得

2006 年聯合統計會議含蓋幾十種領域，且各種領域的會議都排得很緊湊。

我的演講題目為” On the random functional of the Ferguson-Dirichlet process” 傳統特微函數的應用在某些問題上有它的困難，因而必須利用 c 特微函數，雖然 c 特微函數的一些特性本人過去曾提出過，但仍缺少一般性的 c 特微函數的反轉公式，同時本文更進一步利用此反轉公式，証得 Ferguson-Dirichlet 過程的隨機動差。事實上，本場會議結束後，會議主席也特地過來向本人表示對本文相當有興趣。當然會議間也遇到一些其他的學者、專家，大家互相交換最近一些研究領域方向的看法，最後，謝謝國科會給予這次機會參加這個有意義的會議。

三、考察參觀活動(無是項活動者省略)

四、建議

無

五、攜回資料名稱及內容

JSM 2006 Abstracts(CD) - 研討會摘要

JSM 2006 Program - 研討會程序

六、其他