行政院國家科學委員會補助專題研究計畫成果報告

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Abstract

To analyze the risk of life insurance reserves, we extend the literature by incorporating the risk of early surrender in addition to the risks of stochastic mortality and interest rate. We first employ the cointegrated vector autoregression technique to estimate an empirical relationship between lapse rate and interest rate and discover a significant cointegrated vector between them. Based on this empirical model, we then simulate the distribution of policy reserves under the consideration of stochastic mortality, interest rate, and lapse rate. Interestingly, we find that the emergence of early surrender may reduce the expected value as well as the risk of policy reserves due to the surrenders in the low interest rate era. Early surrender therefore could benefit the life insurance company.

Keywords: cointegration analysis, lapse rate risk, reserve distribution

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I. Introduction

Risk analyses of policy reserves are important to life insurance companies. Policy reserves have consistently been the largest liability item in the balance sheet of life insurers. It is calculated by summarizing the associated expected discounted cash flows, where the expected cash flows are dependent on factors such as mortality, disability, lapse rate, and other decrements and the discount rate is chosen to reflect the undiversifiable risk of the expected cash flows. The summarization into a single number masks the dynamic nature of policy reserves, however. Policy reserves indeed are subject to various risks since the associated cash flows and discount rates are contingent upon uncertain future micro- and macroeconomic events. Because of the immense size of policy reserves, the uncertainty embedded in the estimation of policy reserves could have significant impact on the solvency of insurers. It is therefore important to quantify the risk associated with policy reserves.

Quantifying the risk of policy reserves is equivalent to estimating the distribution of policy reserves. To achieve this purpose, one must explicitly model the stochastic cash flow and stochastic discount rate. The reserving method developed in classical actuarial mathematics textbooks such as Jordan (1967) and Bowers et al. (1986; 1997) allows probabilistic future lifetime, but the discount rate is assumed to be deterministic. In other words, the classical reserving method takes account of the mortality risk only without considering the interest rate risk. Further generalizations in the literature, for instance, Panjer and Bellhouse (1980), Bellhouse and Panjer (1981), Giaccotto (1986), Beekman and Fuelling (1990; 1991; 1993), and De Schepper and Goovaerts (1992), is to incorporate stochastic interest rate into the classical methodology. These papers derive either the first two moments or the whole distribution of policy reserves for one insurance policy under certain assumptions of interest rate dynamics. More recent papers (Frees, 1990; Parker, 1994a, 1994b, 1994c, 1996, 1997; Marceau and Gaillardetz, 1999) extend previous analyses from a single policy to a pool of policies. Such extension is another break-through because the limit theorems for approximating the behavior of sums of policies are not available under a common stochastic interest rate environment. The literature so far provides us with good understanding about the risk of policy reserves in an environment with stochastic mortality and stochastic interest rate.

We contribute to the literature by incorporating another stochastic element, lapse rate, into the estimation of the risk of policy reserves. Most insurers include in their contracts a provision that grants the policyholder who elects to terminate the policy the right to a cash surrender value. If the lapse behavior of policyholders were independent of interest rate, we could simply treat lapse as another decrement in addition to mortality and use typical multiple-decrement models to deal with the lapse rate risk. However, we can observe that lapse rate is in fact related to interest rate. During the high interest rate period of the 1980s, record high numbers of policies were surrendered. Several recent actuarial studies in the

Transactions of Society of Actuaries Reports also show that policyholders tend to surrender policies as interest rate rises. The right to surrender policy early, also called the surrender option, embedded in a wide range of life insurance products thus may cause the cash flows associated with life insurance policies to be sensitive to interest rate.

The fact that lapse rate and the resulting cash flows of life insurance policies are sensitive to interest rate could significantly alter the distribution of policy reserves and have profound impact on corresponding risk management. Albizzati and Geman (1994) and Grosen and Jorgensen (2000) demonstrate that the surrender option could account for a substantial portion of the present value of all future premiums. In particular, if the exercise of the option is implemented rationally with the change of interest rate, the surrender option could account for more than fifty percent of the contract value. Furthermore, Babbel (1995), Briys and de Varenne (1997), and Santomero and Babbel (1997) find that the interest rate sensitivity of policy's cash flow is critical to the duration and convexity of the insurance policy. They show that mis-specifying the interest rate sensitivity of lapses could cause large errors in the estimates of effective duration and even greater errors in the estimates of convexity. The disintermediation that happened to the U.S. life insurers during the 1980s also demonstrated the adverse effect of interest-rate-sensitive lapses. Many life insurers experience negative cash flows for the first time since the 1930s' depression and are forced to liquidate assets at depressed prices (Black and Skipper, 2000, p. 111). Incorporating lapse rate thus is crucial.

To integrate interest-rate-sensitive surrender behaviors into the estimation of the policy reserve distribution, we need to determine the relationship between lapse rate and interest rate. We use the cointegrated vector autoregression (VAR) model to construct an empirical model for this relationship. Statistical tests show that there does exist a significant cointegrated vector between lapse rate and interest rate. We therefore should incorporate such interest-rate-sensitive feature of lapse rate into the estimation of policy reserve distribution to better manage the risk of policy reserves.

After establishing the empirical lapse rate model, we employ Monte Carlo simulation methodology to generate the distribution of policy reserves for a pool of level-premium endowment policies[1] with cash-value schedules fixed at policy inception in the environment with stochastic mortality, interest rate, and lapse rate. Our simulation results show that the mortality risk is ignorable compared to the interest rate risk, as expected. We also find, somewhat surprisingly, that the emergence of early surrender may actually reduce the mean, the standard deviation, and the 95th percentile of policy reserve distribution. In other words, the surrender option offered by the life insurance company could indeed benefit the insurance

[1] Endowment policies play an important role in many areas such as Europe, Japan, Taiwan, and South-East Asia. The endowment polices analyzed here have an important common feature: level premiums. Previous studies (except Parker, 1996) consider single-premium contracts only, leaving the extension to annual premiums implicit. When cash flows are independent of interest rates, extending previous results to the case of annual premiums is easy (Parker, 1996; 1997). However, when lapse rate is a function of interest rate, both cash outflows resulting from early surrenders and cash inflows determined by the number of people left in the pool are contingent on interest rate. Generalization from single-premium contracts to level-premium policies is then no longer straightforward and the results of single-premium cases may not hold for level-premium ones. Since most insurance policies are sold with level premiums, our analysis on level-premium policies can offer further practical insights into the risk of policy reserves.

company. The driving force for such positive effect comes from the lapses occurring during the low interest rate era, a seemingly irrational phenomenon that can however be observed from the history of United States and Japan. If a sufficiently large portion of policyholders "irrationally" surrender the valuable credit rate guarantee[2] embedded in their policies when market interest rate is low, the surrender option might turn out to be beneficial to the insurance company. Such surprising result confirms our argument at the beginning that a robust lapse rate model is essential to the correct valuation and sound risk management of policy reserves.

The remainder of our paper is organized as follows. Section II briefly introduces the cointegration methodology and estimates an empirical model of lapse rate. Section III performs the Monte Carlo simulation followed by risk analyses and various robustness checks. Section IV summaries the results and discusses the directions of future research.

II. An Empirical Model of Lapse Rate

Few empirical studies look into the relationship between lapse rate and interest rate. Using two different measures of lapse rate[3], Outreville (1990) finds merely weak connection between lapse rate and interest rate in the United States and Canada. Stronger connections

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[2] We use "credit rate" to represent the discount rate that is used to calculate the actuarially fair premium, conventional (or book value) policy reserves, and cash values. It is a discount rate to the insurance company, but also represents the implicit return credited to policyholders in accumulating cash values. Since we assume that cash values are fixed at policy inception, the insurance company indeed offers a credit rate guarantee or a minimum return guarantee to its policyholders.

[3] Both the Life Insurance Marketing and Research Association and the American Council of Life Insurance define and report lapse rates.

are found from more recent actuarial studies[4] in the *Transactions of Society of Actuaries Reports*. These studies document that surrender increases with the spread between the policy's credit rate and the market interest rate. The inconsistent results among previous studies are probably due to the difference in sampling periods and in methodologies. The sample in Outreville (1990) covers up to 1979 only and misses the wide swing of interest rate during 1980s and 1990s, while the sample periods of the actuarial studies basically include the late 1980s and 1990s. Besides, Outreville (1990) performs OLS analysis with Cochrane-Orcutt adjustment for first-order serial correction of residuals whereas most analyses in actuarial reports are accomplished with univariate analysis without controlling variables. Since the results so far are relatively scarce and inconclusive, we aim to empirically construct a model that captures the connection between lapse rate and interest rate in this section.

1. Data

We obtain lapse rate data from the *Life Insurance Fact Book*, an annual statistical report of the American Council of Life Insurance. The data in the *Fact Book* represent information about life insurance companies authorized to sell insurance policies in the U.S. market. Our sample contains annual voluntary termination rates of all ordinary life

[4] See, for instance, Cox, Laporte, Linney, and Lombardi (1992) and the Annuity Persistency Study in the 1995-96 reports (p. 559-638).

insurance policies[5] in force from 1959 to 1995. Compared with the sample of previous papers, ours has more sample points, spans a longer period, and covers the era of highly volatile interest rates in the 1980s and early 1990s. We collect interest rate data from the U.S. Financial Database maintained by the Ministry of Education in Taiwan[6].

2. Estimation of the Cointegrated Vector Autoregression Model

We employ the augmented Dickey-Fuller (ADF) tests to test whether there is a unit root in lapse rate and interest rate. The ADF tests are done on both the levels and the first-order differences of lapse rate and interest rate. Figure 1 shows the levels and the first-order differences of these two series and Table 1 reports the results of the unit root test on lapse rate, interest rate, and their first-order differences. All of the ADF statistics for the levels of both series are not significant at 5% level, implying that the null hypothesis of a unit root cannot be rejected for lapse rate and interest rate. In addition, the corresponding statistics for their first-order differences are significant at 1% level and thus suggest the rejection of the null hypothesis. Based on these results, we conclude that lapse rate and interest rate follow nonstationary $I(1)$ processes individually.

[Insert Figure 1 Here]

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[5] Nearly all individual insurances are classified as "ordinary insurance" in the *Fact Book*. Roughly 0.2 percent of individual life insurance in force is classified as industrial insurance.

[6] Since the one-year Treasury bill rates are recorded monthly, we transform monthly interest rate to annual rate by the following compounding method:

$$\text{annual interest rate} = (1+\frac{m_1}{12})(1+\frac{m_2}{12})...(1+\frac{m_{12}}{12})-1$$

where m_i denotes the interest rate in month i, $i = 1, 2, \ldots, 12$.

[Insert Table 1 Here]

After the unit root test, the order of VAR model needs to be decided. The Akaike information criterion (AIC) is used for this purpose. The optimal model selected by AIC is VAR(3). Basing on this VAR(3) model, we use two maximum likelihood multivariate cointegration tests, the maximal eigenvalue test and the trace test, to conduct the cointegration test to determine the number of cointegration vectors. The results are reported in Table 2. Both tests reveal the existence of a cointegrating vector between lapse rate and interest rate.

[Insert Table 2 Here]

We then use the maximum likelihood methodology to estimate an error-correction model with one cointegrating vector of lapse rate and interest rate as the following:

$$\begin{bmatrix} \Delta L_t \\ \Delta I_t \end{bmatrix} = \begin{bmatrix} \underset{(-5.193)}{-0.243^{***}} \\ \underset{(-0.890)}{-0.199} \end{bmatrix} \begin{bmatrix} 1 & \underset{(-9.819)}{-1.053^{***}} & \underset{(-1.148)}{-0.008} \end{bmatrix} \begin{bmatrix} L_{t-1} \\ I_{t-1} \\ 1 \end{bmatrix} + \begin{bmatrix} \underset{(1.650)}{0.240} & \underset{(-0.881)}{-0.046} \\ \underset{(-0.210)}{-0.146} & \underset{(0.597)}{0.149} \end{bmatrix} \begin{bmatrix} \Delta L_{t-1} \\ \Delta I_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} \underset{(-0.094)}{-0.012} & \underset{(-2.934)}{-0.151^{***}} \\ \underset{(-1.037)}{-0.642} & \underset{(-2.085)}{-0.514^{*}} \end{bmatrix} \begin{bmatrix} \Delta L_{t-2} \\ \Delta I_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^L \\ \varepsilon_t^I \end{bmatrix} \tag{1}$$

where $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_t^L & \varepsilon_t^I \end{bmatrix}' \sim N(\mathbf{0}, \hat{\boldsymbol{\Sigma}})$ and $\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} 1.67\times10^{-4} & 8.09\times10^{-6} \\ 8.09\times10^{-6} & 7.28\times10^{-6} \end{bmatrix}$.

The specifications for individual variables in the error-correction VAR model and the results of their misspecification tests are in Table 3. Table 3 shows that the estimated models of lapse rate and interest rate are generally well specified, especially the equation for lapse rate.

[Insert Table 3 Here]

III. Monte Carlo Simulation

After specifying the relationship between lapse rate and interest rate, we turn to generate the distribution of policy reserves through Monte Carlo simulation. The simulation procedure consists of three risk layers, one on top of another. In the first layer we consider the mortality risk resulting from random survivorship. We adopt the probabilistic interpretation of the life table to estimate the risk of random survivorship. The second layer considers the interest rate risk due to the randomness of interest rate. We employ the cointegration model estimated in the previous section to resemble the interest rate risk. The lapse rate risk is then added to the simulation on top of random mortality and interest rate in the final layer. The lapse rate risk should be analyzed after the interest rate risk since it originates from the dependence of lapse rate on interest rate. Such layer-adding analysis permits us to retrieve the distribution of policy reserves with all risks being considered as well as to evaluate the marginal effects of various risk sources.

1. Simulation Setting

Consider a group of *N life-aged-a* policyholders. Assume that these policyholders have two causes of decrement: death and early surrender. For each of these policyholders, the probability of decrement during the age interval of x and $x+1$ due to death and early surrender is specified by $q_x^{(m)}$ and $q_x^{(l)}$, respectively[7], where x is a positive integer and $x \geqq a$. In addition, let $L^{(\tau)}(x)$ denote the cohort's number of survivors at age x out of the original N lives and let $D_x^{(i)}$ denote the random variable equal to the number of lives who will leave the group between ages x and $x+1$ for cause i, where $i = m, l$, or τ [8].

Let the T-year endowment policies issued to these policyholders have face amount of \$ F dollars payable at the end of the death year or at the end of the Tth year and net level annual premium \$ P dollars payable at the beginning of each surviving year. If policyholders surrender their policies during the age interval of x and $x+1$, they receive the amount of S_x at the end of the year. We assume that

$$S_x = \left(0.8 + 0.2 \times \frac{x-a+1}{T}\right) \times {}_{x-a+1}V_a\,, \qquad (2)$$ [9]

where ${}_{x-a+1}V_a$ is the policy reserves calculated with random future lifetime and deterministic interest rate as in Bowers et al. (1986; 1997) and $x < a+T$. Let L be the random variable denoting the present value of the cash flows generated by this portfolio. Then

$$L = \sum_{x=a}^{a+T-1} [(F \times D_x^{(m)} + S_x \times D_x^{(l)}) \times v_{x-a+1}] + F \times L^{(\tau)}(a+T) \times v_T$$
$$- [\sum_{x=a}^{a+T-1} P \times L^{(\tau)}(x) \times v_{x-a}]\,, \qquad (3)$$

where v_{x-a} is the discount factor for the cash flows at policy year x-a[10].

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[7] The superscript m indicates the cause of mortality and l the cause of policy lapse.

[8] The superscript τ refers to all causes. Notice that $L^{(\tau)}(a) \equiv N$.

[9] Although this formula comes from Model Provisions of Life Insurance Policy in Taiwan, it possesses the general property of surrender charges: surrender charges are usually high at the beginning and decrease as policies mature.

The random variable L represents the present value of insurers' liabilities associated with a pool of policies. The statistical properties of L are critical to the risk management of life insurance companies and are of great concerns to actuaries, insurance regulators, and various stakeholders of the companies. Our goal is to estimate the distribution of L.

In the following simulations, we specify that $N = 100{,}000$, $T = 20$, $F = 1{,}000$, $a = 30$, the interest rate used in calculating P and ${}_{x-a+1}V_a$ is 6%[11], and $q_x^{(m)}$ is distributed as in the 1980 CSO male mortality table. P is \$27.133 obtained by the equivalence principle. Note that we do not consider dividends, expenses, loadings, taxes, or new business in the simulation.

2. Mortality Risk

Our focus in this subsection is the risk arising from random survivorship exclusively. In other words, we assume that the market interest rate is fixed at 6% and there are no early surrenders. To assess the risk resulting from random survivorship, we assume that $D_x^{(m)}$ has a binomial distribution with parameters ($L^{(2)}(x)$, $q_x^{(m)}$). This assumption is justifiable if deaths among policyholders are mutually independent. We simulate 10,000 observations of $D_x^{(m)}$ for $30 \le x < 50$ and obtain the distribution of L. The results are as shown in Figure 2.

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[10] $\neg_{x-a} = \begin{cases} 1 & \text{if } x = a, \\ \dfrac{1}{(1+r_1)(1+r_2)\cdots(1+r_{x-a})} & \text{if } a < x < a+T, \end{cases}$ where r_{x-a} is the interest rate prevailing during the policy year x-a for $a < x < a+T$.

[11] This rate is also called the credit rate in the following text.

[Insert Figure 2 Here]

Our simulation shows that the mortality risk is trivial. The expected value of the distribution is close to zero and the standard deviation is about one percent of annual premiums only. Even the 95th percentile of the distribution is less than two percent of annual premiums[12]. In other words, the insurer could keep premium-surplus ratio as high as fifty for the insolvency probability of five percent. The insignificance of the mortality risk is mainly due to the assumption of independence among policyholders' deaths and the large size of the pool. We experiment with the size of the pool and confirm that a pool with n times of policies (and thus n times of premium income) has a standard deviation about $\sqrt{n}$ times. Since the premium income increases faster than the standard deviation as the pool size increases, the risk measured by the ratio of the standard deviation to premium income decreases with the pool size. When the pool is sufficiently large, the risk of the pool relative to premiums diminishes. The result of insignificant mortality risk is consistent with the findings of previous studies.

3. Interest Rate Risk

In this second layer of simulation we include an additional risk factor: interest rate. The fundamental problem with the randomness of interest rate lies in the fact that, as opposed to the mortality risk, it is not possible to diversify the interest rate risk away by selling a large

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[12] The 95th percentile is about 1.64 times of standard deviations away from the mean. The number 1.64 along with the 2.87 kurtosis and the skewness of 0.119 once leads us to suspect that the distribution is a normal distribution. However, the Jarque-Bera test rejects the hypothesis of normality.

number of policies because every policy is subject to the same or highly correlated interest rates. Thus, the interest rate risk is expected to be more imperative to the insurer than the mortality risk.

To capture the dynamics of interest rate, we adopt the cointegration model developed in section II to simulate 10,000 interest rate paths of monthly one-year T-bill rates for twenty years[13]. Combining the 10,000 interest rate paths with the 10,000 sets of $D_x^{(m)}$ simulated in section III.2, we obtain the distribution of L under the consideration of stochastic interest rate as well as random survivorship. The shape and summary statistics of this distribution are shown in Figure 3.

[Insert Figure 3 Here]

The interest rate risk is obviously momentous. The mean reserve that is supposed to be zero or close to zero with the presence of the mortality risk now increases to $1,397,287, about fifty percent of annual premiums. The positive sign of the mean arises from both the convexity of the present value function with respect to interest rate and the slightly upward trend in the mean of simulated interest rates. The mean of simulated interest rates increases from the initial value of 6% to about 6.3% in the 20th simulated year[14]. Since interest rates

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[13] In simulating interest rates, we modify our cointegration model by setting the coefficients associated with lapse rate in the interest rate equation to be zero since these coefficients are not significant and it might not be reasonable to allow lapse rate to affect interest rate. In other words, we treat interest rate as an exogenous variable. Furthermore, we assume that the market interest rates at time -1 and -2 are 6%, the interest rate used in calculating the fair premium.

[14] This trend is due to the constraint of non-negative interest rate. Since the initial interest rate is 6%, the downward movements of interest rates cannot exceed 6% while the upward movements have no upper bound. As simulations move on and the cumulative deviations of interest rates from 6% enlarge, the deviations might not symmetric on average any more due to the non-negative constraint and would be biased positively.

on average are higher than the 6% rate used to price the insurance, policy reserves increase from zero. Furthermore, even though mean interest rate could have remained constant, policy reserves would still be greater than zero due to the convexity of discounting. The decrease of policy reserves resulting from an increase in interest rate thus would be less than the increase of reserves with an equivalent decline in interest rate. Therefore, reserves that are supposed to be zero under a constant interest rate environment turn out to be greater than zero within a stochastic interest rate environment.

The enormous figure of the mean results from the fact that the insurance policy analyzed is a long-term contract. The long maturity of the insurance aggravates the effect of the convexity. Long term discounting makes the increase of reserves resulting from interest rate declines substantially larger than the decrease with equivalent rises in interest rates. The large mean indicates that the insurance will be severely under-priced if the stochastic interest rate is mistakenly treated as fixed. In other words, the insurance sold under a stochastic interest rate environment but priced under the assumption of a deterministic interest rate would result in serious under-estimation of the contract value.

In addition to the large mean, the standard error of $9,072,683, which is more than three times of the total annual premiums, also signifies the severity of the interest rate risk. Furthermore, the 95th percentile of the distribution is almost seven times of annual premiums

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Therefore, the mean interest rate increases slowly with time. If we had allowed negative interest rates, then the slim rising trend would have disappeared.

meaning that the insurer has to keep tremendous amount of surplus to maintain an acceptable solvency probability. In summary, the large mean, the large standard error, the large 95th percentile, and the long right tail of the distribution of policy reserves all suggest that the interest rate risk of policy reserves is substantial.

4. Lapse Rate Risk

The third risk factor that we consider in the simulation is lapse rate. Life insurance policyholders may surrender their contracts to take advantage of the higher-yield alternatives in the financial markets. Such behavior makes the cash flows of life insurance policies sensitive to market interest rate and may significantly change the risk characteristics of policy reserves. Taking early surrenders into consideration therefore is essential to risk management of policy reserves.

To evaluate the lapse rate risk, we simulate 10,000 sample paths of lapse rate based on our empirical cointegration model[15]. These lapse rates are then combined with mortality rates in the calculation of reserves so that the policyholders are subject to two decrements. We apply again the concept of random survivorship group to simulate 10,000 sets of $D_x^{(m)}$ and $D_x^{(l)}$ for $30 \le x < 50$ under the assumption that both $D_x^{(m)}$ and $D_x^{(l)}$ are binomially distributed with parameter sets ($\mathcal{L}^{(\tau)}(x)$, $q_x^{(m)}$) and ($\mathcal{L}^{(\tau)}(x) - D_x^{(m)}$, $q_x^{(l)}$), respectively[16].

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[15] To start the simulation, we assume that the lapse rates at time *-1* and *-2* are 8%, the average lapse rate of the sample period.

[16] The assumption that $D_x^{(l)}$ has a binomial distribution is equivalent to assuming that surrender decisions among policyholders are mutually independent. Although policyholders have the same propensity to lapse their policies and the propensity is dependent upon interest rate, their lapse decisions might not necessarily

Combining the resulting 10,000 cash flow paths with the 10,000 interest rate paths simulated in section III.3, we obtain the distribution of L under the consideration of random survivorship, stochastic interest rate, and interest-rate-sensitive lapse rate. The simulation results are presented in Figure 4.

[Insert Figure 4 Here]

The right to lapse affects the distribution of policy reserves dramatically. The surrender option surprisingly helps the insurance company: the expected value of reserves turns from positive to negative and the standard deviation as well as the 95th percentile decreases significantly. The mean of reserves is now -$70,719, different from $1,397,287 when there are no lapses. The negative figure implies that the insurance company is expected to make profit from this pool of policies. An obvious reason for the dramatic reduction in the mean is the surrender charge that is about 20% for the first year and gradually declines over time. Policyholders who surrender their policies hence are compensated with only associated policy reserves minus surrender charges[17]. If we assume that the surrender charge is zero, then the mean reserve bounces back to $960,068 as we can see from Figure 5.

[Insert Figure 5 Here]

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affect each other. The assumption therefore is reasonable. The reason why the distribution parameter of $D_x^{(1)}$ is $L^{(2)}(x) - D_x^{(m)}$ instead of $L^{(2)}(x)$ is because of the assumption that policyholders who can choose to lapse their policies during the age interval of x and $x+1$ are those who survive to age $x+1$.

[17] This does not necessarily mean that insurance companies treat policyholders unfairly. Surrender charges are used to cover policy expenses that have not been recouped from received premiums. Since we do not consider expenses, surrender charges look like benefiting the insurance company.

Notice that the mean of Figure 5 is still smaller than that of Figure 3. The drop of the mean results from the lapses that happened during periods of low interest rates. The policyholders who choose to surrender their policies when market interest rates are low indeed relinquish the valuable credit rate guarantee offered by the insurance company at policy inception and hence benefit insurers. Although lapses occurring during high interest rate periods impair insurers' profits, the convexity of policy reserves with respect to interest rate makes the losses smaller than the gains. Comparing Figure 5 with Figure 3, we can observe that the effect of shrinking the right tail through lapses happening during the low-interest-rate periods is greater than the effect of shrinking the left tail due to lapses happening during the high-interest-rate periods. The net effect of lapses thus results in a decrease of the mean.

Such decrease, or equivalently a negative aggregate value for a pool of surrender options, is not seen in previous studies like Albizzati and Geman (1994), Grosen and Jorgensen (1997), or Grosen and Jorgensen (2000). These studies typically assume that the policyholder's lapse behavior is fairly "rational."[18] When interest rate is low, policyholders are not supposed to exercise their surrender options because the surrender option is now out of money and the credit rate guarantee is in the money. When interest rate is high, most policyholders are supposed to lapse their policies because the surrender option is now deep in

[18] Although Albizzati and Geman do allow certain degree of "irrationality" in the lapse decision of policyholders, the assumed range of lapse rate (3% to 60%) is much larger than the observed one (5.0% to 11.6%).

the money while the credit rate guarantee still has some value. The history however demonstrates that some policyholders do lapse their policies even when interest rate is very low. For instance, during the early 1960s, one-year interest rate was very low but lapse rates were still over 5%. Even in Japan where recent interest rates were extremely low, more than one tenth of policies lapsed in 1997, 1998 and 1999, respectively[19]. On the other hand, the history also shows that only a certain portion of policyholders would lapse their policies even when interest rates are very high. For example, lapse rates never exceeded 12% during the extraordinary period of high interest rate in the early 1980s. Therefore, previous studies assuming that the decision to lapse is equivalent to the "rational" early exercise of options overestimate the (aggregate) values of surrender options.

In addition to reducing the expected value of policy reserves, lapse also acts to mitigate the risk of reserve. The standard deviation drops more than fifty percent to \$4,064,278 in Figure 4. The 95th percentile also decreases to a level of only forty-four percent of that in Figure 3. This beneficial effect of lapse to insurers sustains no matter whether the surrender charges exist or not as we can see from Figure 5[20]. This mitigation of risk is reasonable because lapses make the losses and the gains of insurers caused by the variability of interest rates smaller. With the emergence of lapses, insurers make smaller

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19 The number is estimated with the data from the Life Insurance Business in Japan (http://www.seiho.or.jp/english/index.html).

20 Since we assume that the surrender charges do not affect the likelihood of lapse, the charges would change only the location of the distribution but not the shape of the distribution.

profit with rising interest rate and lose less when interest rate declines. Lapses therefore moderate the interest rate risk of policy reserves, a result that is in line with the findings of decreased effective duration of policy reserves in Babbel (1995), Briys and de Varenne (1997), and Santomero and Babbel (1997).

To sum up, lapses that are sensitive to interest rate can be advantageous to insurance companies in terms of the mean and the risk of policy reserves. The surrender charges and the lapses happening during low interest rate era serve to offset the adverse impact of stochastic interest rate on the expected value of policy reserves. Lapses also reduce the risk of policy reserves in the sense that they narrow the range of losses and gains resulting from the variability of interest rate. The bad aspect of lapses is causing the liquidity problem that the U.S. life insurers have experienced painfully. Aside from the liquidity concern, granting policyholders the right to surrender policies early might be beneficial to the insurance company.

IV. Conclusions

Risk management of policy reserves is vital to the solvency of life insurers. To portray the risk profile of policy reserves, one needs to model relevant cash flows and discount rates stochastically. Scholars have developed reserving techniques in a stochastic mortality and interest rate environment. The literature to date, however, does not consider the behavior and the consequences of early surrender. Early surrender of policyholders could make cash flows of life insurance policies sensitive to the interest rate. The characteristics of interest-rate-sensitive cash flows are material to the value and the risk of policy reserves. The main contribution of this article is to incorporate the impact of lapses into the estimation of the distribution of policy reserves.

To estimate such distribution, we first establish an empirical model for lapse rate and interest rate. The cointegration technique leads us to find a long run as well as a short run relationship between interest rate and lapse rate. Therefore, we conclude that lapse rate depends on interest rate and the cash flows of life insurance policies are sensitive to interest rate. We then perform Monte Carlo simulations featured with three risk layers: the mortality risk from random survivorship, the interest rate risk from stochastic interest rates, and the early surrender risk from interest-rate-dependent lapse rates. A pool of level-premium endowment insurance with cash-value schedules fixed at policy inception is used as an illustration. Different from previous findings, we find that the surrender option may indeed benefit the insurance company in that the emergence of early surrender helps to reduce the value as well as the risk of policy reserves. The empirical lapse behavior during low interest rate era that is not identified in the literature turns the aggregate value of surrender options to be negative.

Our findings have several implications for life insurance companies and regulators. First, traditional pricing or reserving methods that assume a deterministic interest rate would incur large estimation errors when applied in a volatile interest rate environment. Actuaries therefore should refer to modern term structure theories of finance when pricing and reserving for life insurance contracts. Second, actuaries should pay serious attention to the early surrender behavior. Ignoring or arbitrarily specifying early surrender behaviors of policyholders would cause large estimation errors in pricing and reserving when lapse rate is linked to interest rate. Ignorance or arbitrary assumption about lapse rate would probably lead to the mispricing of insurance policies that may undermine the competitiveness of the policies and, therefore, result in unpleasant outcome for the issuers. Third, early surrender does not necessarily go against the insurers. Although early surrender might cause liquidity problem and reduce the profitability of the insurers because of the uneven expense pattern of insurance contracting, early surrender might actually benefit the insurers through surrender charges, "irrational" lapse behaviors, and mitigation of the impact of stochastic interest rate on reserves. It implies that minimizing lapse rate might not be the optimal strategy for insurers. Fourth, the interest-rate-sensitive lapse rate could reduce the impact of the time lag associated with the prior approvals of new policies. Regulators in some jurisdictions require life insurance companies to obtain approvals before launching their new products. Such requirements, however, increase the risk of insurers since the market interest rate might be significantly different from the pre-specified credit rate of the policy when the policy is eventually launched. Since early surrender acts to mitigate the impact of stochastic interest rate on reserves, it could reduce the uncertainty introduced by the prior approval system.

Our paper can be extended in the following directions. First, similar analyses can be performed on other life insurance products. Analyses on a pool consisting of various insurance products will be particularly interesting for its resemblance to the real cases and for the sake of the diversification concern among insurance products. Second, this article can be extended from a static analysis (closed-block business) to a dynamic analysis (going-on business) since the policies issued with different guaranteed credit rates at different market interest rate levels are affected by lapses differently. Third, the propensity to lapse is different across policy ages, that is, younger policies tend to have higher probability to lapse than older ones. In conjunction with the consideration of initial expenses – the cost of procuring, underwriting, and issuing new business, lapses might display different impact on the insurance company. This is an interesting issue to explore in our future work. Finally, the importance of lapse rate calls for more thoughts of researchers about the seemingly irrational lapse behaviors of policyholders. Why there always exist some policyholders surrendering their policies at any levels of interest rate? Is there a "natural" lapse rate due to certain frictions in insurance transactions? What deters policyholders from surrendering their policies at high interest rate level? Research issues related to lapse rate seem to be abundant out there.

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Table 1: Unit root test of lapse rate (L_t) and interest rate (I_t)

	Augmented Dickey-Fuller Test[a]		
Variable	H_{20}:	H_{30}:	H_{40}:
L_t	---	-1.708	-2.804
I_t	---	-2.481	-2.386
ΔL_t	-3.008**[b]	---	---
ΔI_t	-5.116**	---	---

a. The regressions of the ADF test used to test the null hypothesis of a unit root include neither an intercept nor a time trend in H_{20}, include an intercept in H_{30}, and include both an intercept and a time trend in H_{40}, respectively. The optimal lags of the regressions are decided based on AIC in order to include enough lags of lagged variable to eliminate autocorrelations of residuals.

b. The critical values of ADF tests are based on MacKinnon (1991).

** Significant at 1% level.

Table 2: The maximal eigenvalue test and the trace test of Johansen (1991)[a]

		The Maximal Eigenvalue Test		
Null	Alternative	$\hat{a}_{\max}$	95% Critical Value	90% Critical Value
$r=0$	$r=1$	22.42*	15.87	13.81
$r\le 1$	$r=2$	3.81	9.16	7.53
		The Trace Test		
Null	Alternative	$\hat{a}_{trace}$	95% Critical Value	90% Critical Value
$r=0$	$r\ge 1$	26.23*	20.18	17.88
$r\le 1$	$r\ge 2$	3.81	9.16	7.53

a. The tests are performed based on $\Delta \mathbf{y}_t = \mathbf{C}_1 \Delta \mathbf{y}_{t-1} + ¥\sim_{t-1} + « + £_t$, where $\mathbf{C}_1$ is a 2×2 matrix polynomial in the lag operator, **a** is the first-order difference operator, « is an intercept vector, ¥ is a 2×2 constant matrix, and $£_t$ is a white noise error term vector. We use Microfit 4.0 to obtain relevant statistics.

Table 3: Error-correction models for lapse rate and interest rate[a]

Variable	$\mathbf{a}L_t$			$\mathbf{a}I_t$		
	Coefficient	t-value	p-vlaue	Coefficient	t-value	p-value
ΔL_{t-1}	0.240	1.650	0.110	-0.146	-0.210	0.836
ΔI_{t-1}	-0.046	-0.881	0.385	0.149	0.597	0.555
$\mathbf{a}L_{t-2}$	-0.012	-0.094	0.926	-0.642	-1.037	0.308
$\mathbf{a}I_{t-2}$	-0.151	-2.934	0.006	-0.514	-2.085	0.046
ECM_{t-1}	-0.275	-5.808	0.000	-0.199	-0.890	0.381
R^2	0.799			0.252		
DW[b]	1.577			1.985		
$SCorr$[c]	$\chi^2(1)=1.736[0.188]$			$\chi^2(1)=0.013[0.909]$		
$Hetero$[d]	$\chi^2(1)=0.005[0.944]$			$\chi^2(1)=0.701[0.402]$		
$Normal$[e]	$\chi^2(2)=0.980[0.612]$			$\chi^2(2)=0.225[0.893]$		
FF[f]	$\chi^2(1)=0.764[0.382]$			$\chi^2(1)=0.117[0.732]$		

$$ECM_{t-1} = L_{t-1} - 1.053 I_{t-1} - 0.008$$

a. These error-correction models are estimated by Microfit 4.0.

b. Durbin-Watson test.

c. Lagrange multiplier test of residual serial correlation.

d. Based on the regression of squared residuals on squared fitted values.

e. Based on a test of skewness and kurtosis of residuals.

f. Ramsey's RESET test using the square of fitted values.

Figure 1: Time Series of Lapse Rate, Interest Rate, and Their First-Order Differences

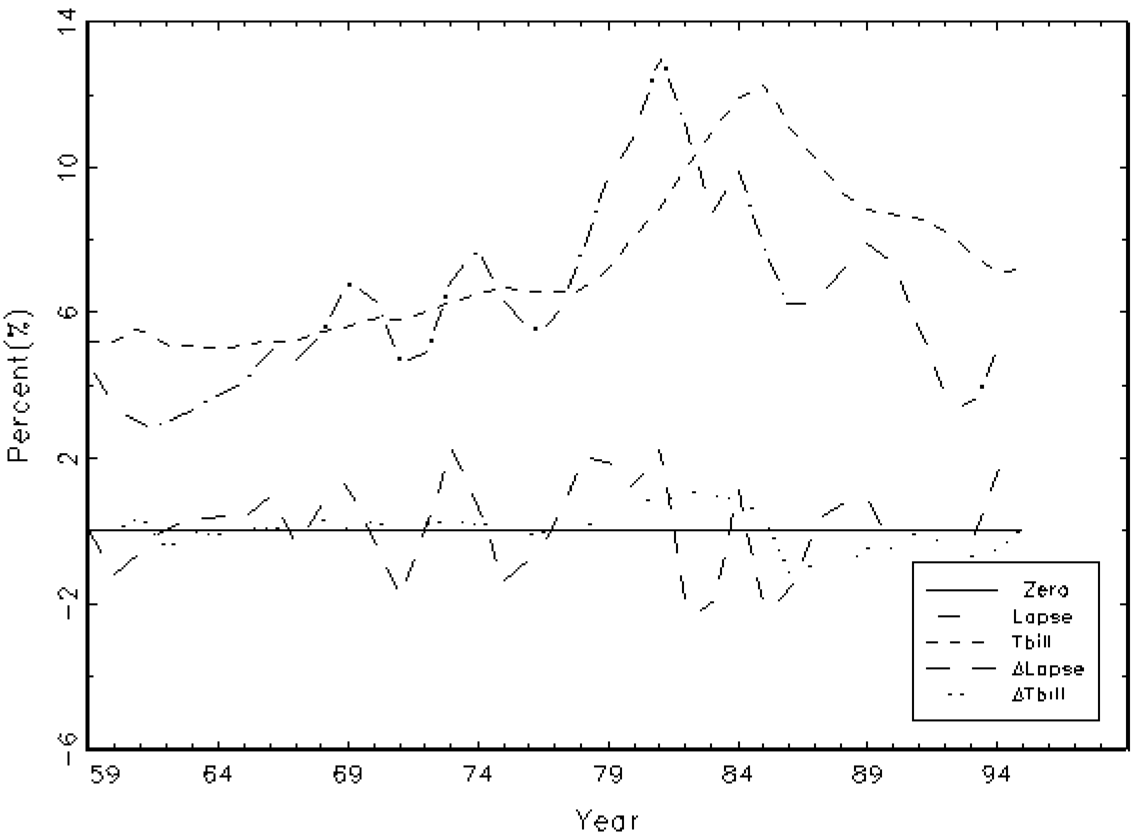
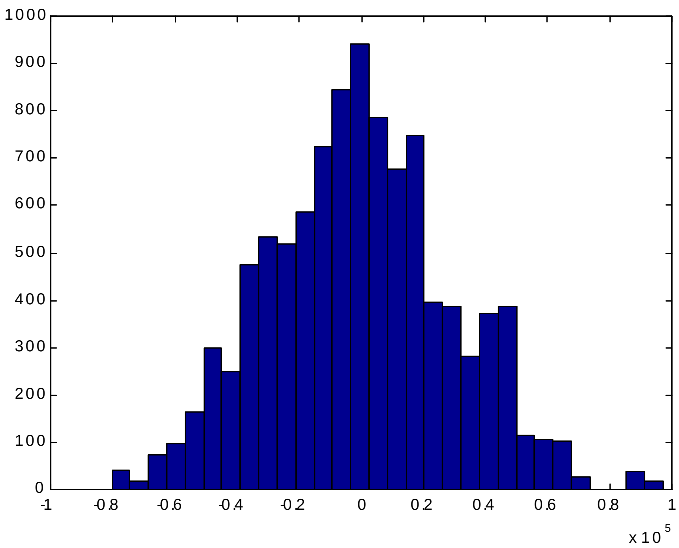
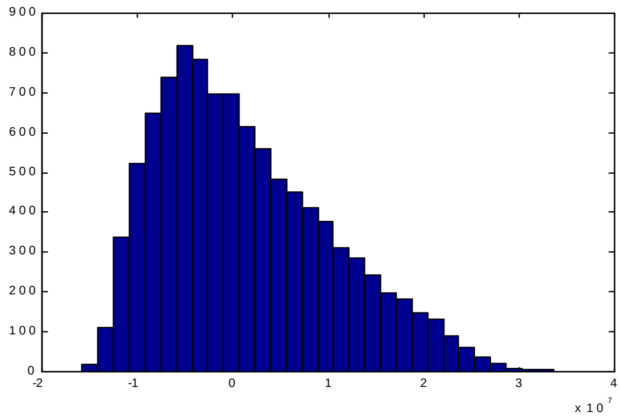


Figure 2: One Decrement (Mortality) with Constant Interest Rate



•Mean: -5.11e+002
•Median: -1.39e+003
•Standard Deviation (Std): 2.95e+004
•Skewness: 1.19e-001
•Kurtosis: 2.87e+000
•VaR (95%): 4.81e+004
•(VaR-Mean)/Std: 1.64e+000

Figure 3: One Decrement (Mortality) with Stochastic Interest Rate



Mean = 1,397,287

Median = -269,721

Std = 9,072,683

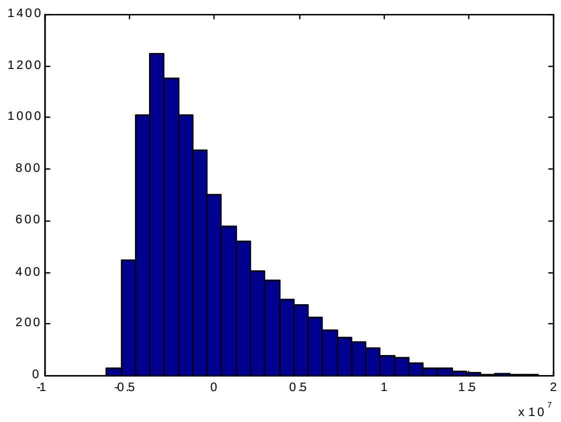
Skewness = 0.6536

Kurtosis = 2.7911

VaR (95%) = 18,819,865

(VaR-Mean)/Std =1.9203

Figure 4: Two Decrements (Mortality & Lapse) with Stochastic Interest Rate



Mean = -70,719

Median = -1,182,151

Std = 4,064,278

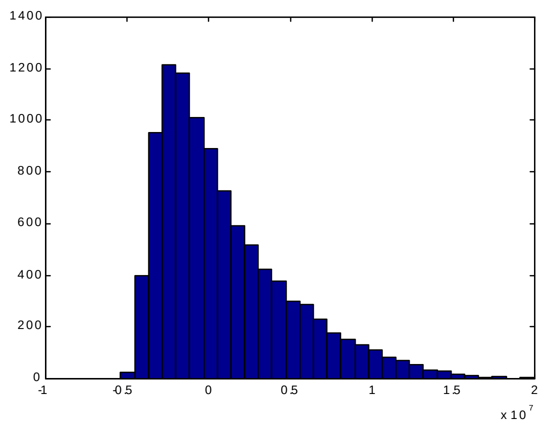
Skewness =1.1909

Kurtosis = 4.1843

VaR (95%) = 8,312,521

(VaR-Mean)/Std =2.0627

Figure 5: Two Decrements (Mortality & Lapse) with Stochastic Interest Rate and Zero Surrender Charge



Mean = 960,068

Median = -128,392

Std = 4,035,297

Skewness = 1.1748

Kurtosis =4 .1401

VaR (95%) = 9,246,962

(VaR-Mean)/Std = 2.0536