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1. Introduction

The put-call parity（hereafter, PCP）formalized by Stoll (1969) uses the no-arbitrage principle to price put（call）options relative to call（put）options. It has been the subject of numerous empirical studies, but these studies typically focuse on “direct” tests of whether arbitrage strategies earn ex post profits. They target on testing the implication of whether the absolute deviation from fair value is less than the cost of arbitrage. Earlier empirical studies testing the PCP include Gould and Galai (1974), Klemkosky and Resnick (1979,1980), Evnine and Rudd (1985), Chance (1987), and Ronn and Ronn (1989), among others. Their conclusions are best summarized by noting that while PCP holds, on average, there are frequent, substantial violations of PCP involving both overpricing and underpricing of calls or puts. However, all previous studies that test PCP use American options. As shown by Merton (1973), the PCP need not hold for American options, because the possibility of an early put exercise cannot be completely ruled out when the portfolio is established. Therefore, it is not possible to conclude from these studies whether, or to what extent, observed PCP violations are due to market inefficiency or due to the value of early exercise.

Kamara and Miller (1995) avoid the early exercise problem by testing European options on the S＆P500 stock index traded on the Chicago Board of Options Exchange (CBOT). Using daily and intradaily prices, they find violations of PCP that are much less frequent and smaller than those reported in studies using American options. Furthermore, these violations reflect the premia for immediacy risk.

Some problems regarding the “direct” test of PCP have been addressed by the recent disagreement regarding the efficiency of the market for exploiting arbitrage opportunities. While many studies support evidence of inefficiency, Harris (1989), Kleidon (1992) and Miller, Muthuswamy and Whaley (1997) caution that non-synchronous trading may create the illusion of apparent arbitrage opportunities. Kamara and Miller (1995) find that these violations of PCP suggest that the trading strategies underlying PCP are subject to significant liquidity (immediacy) risk. Variations in the deviations from PCP bounds are systematically positively related to proxies for liquidity risk in the stock and option markets. Their empirical studies provide evidence that liquidity (immediacy) risk is a substantial impediment to the role of arbitrage in pricing assets and is likely to produce deviations from predictions of arbitrage-based asset pricing models.

Previous arbitrage models have ignored the impact of short-sale restrictions, early liquidation before maturity, the opportunity cost of funds for index arbitrage, and the magnitude of transaction costs. Neal (1996) provides a detailed analysis of actual S＆P500 index futures arbitrage trades and directly relates these trades to the predictions of index arbitrage models. He shows that (1) short-sales restrictions are unlikely to have a large effect on mispricing. About half the arbitrage trades are executed for institutions. Since institutions are typically net long in stocks, they can avoid short-sale restrictions by selling the stock directly; (2) an estimate of the implied opportunity cost of arbitrage funds is 88 basis points higher than the Treasury bill rate; (3) the average price discrepancy captured by arbitrage trades is small, which is consistent with an efficient market for exploiting arbitrage opportunities; (4) early liquidation is the rule, not the exception, which is consistent with the finding of Sofianos (1993). He concludes that " the ability of these models to explain arbitrage trades, however, is surprisingly low."

In contrast with previous studies, this paper presents a model of the option price adjusting (mean-reverting) to a functional form of put-call parity. The adoption of mean reversion is appealing in several aspects. First, the threshold of mean reversion should be interpreted more broadly than as simply reflecting proportional transaction costs, but also as resulting from the tendency of traders to wait for sufficiently large arbitrage opportunities to open up before entering the market and trading （Neal (1996), Sofianos (1993)）. Second, the assumption of an instantaneous trade can be replaced with the presumption that it takes some time to observe an arbitrage opportunity and then execute transactions and that trading is infrequent (Neal (1996)). Third, in a market with heterogeneous agents who face different levels of transactions costs, margin requirements, or position limits, agents essentially face no-arbitrage bands of different sizes. Fourth, in a market with heterogeneous participants, asset prices may reflect irrational bubbles on "fads" resulting in the persistence of PCP deviations.

The first contribution of this paper is to employ the variance ratio (VR) statistic to test for the mean reversion and make an appropriate allowance for heteroskedasticity when basing inference on the VR statistic by using the Gibbs sampling approach in the context of a three-state Markov-switching model. The framework provides a rationale for the behavior of option prices, since it allows us to understand the question of whether option prices do adjust to put-call parity, and if so, how fast do they adjust as market frictions are present. As suggested by Kim, Nelson, and Startz (1998), the sampling distribution of the VR is substantially affected by the particular pattern of heteroskedasticity during the sample period. Simulation methods that assume heteroskedasticity or allow for persistence in heteroskedasticity, but not conditioned on the particular pattern of the historical period, produce a biased test, leading us to reject the null hypothesis of no mean reversion too often.

The second contribution of this paper is to adopt a resampling strategy, as suggested by Kim, Nelson, and Startz (1998), that standardizes historical returns, using the Gibbs sampling approach to allow for uncertainty in parameters and states while conditioning on the information in the data. Gibbs sampling is a Markov chain Monte Carlo simulation method for approximating joint and marginal distributions by sampling from conditional distributions[1]. Since dividend payments and liquidity premiums exhibit seasonal patterns (Harvey and Whaley (1994) ; Eleswarapu and Reinganum (1993)), Gibbs sampling makes appropriate use of the information in the historical data. At the end of each iteration of the Gibbs sampling, we compare the estimate of the VR from the standardized historical data with the corresponding VR from "randomized" data. We then estimate a p-value by counting how often the former falls below the latter.

It is finally found that dividend uncertainty and/or the trading system might play key roles in explaining equity index arbitrage behavior. The empirical result shows that PCP deviations from electronic screen-traded DAX index option, which is calculated as if the dividends are reinvested in the index, displays mean reversion at long horizons. On the other hand, those deviations from the floor-traded S&P 500 index option, which is not corrected for dividend payments, vary randomly.

Section 2 of this paper first defines the deviation from put-call parity. Section 3 describes the data and introduces two comparable index options. Section 4 presents a VR statistic for testing mean reversion in a framework of a three-state Markov-switching variance model. Section 5 introduces an extended version of the Bayesian Gibbs sampling approach and data augmentation. Section 6 describes the VR tests based on historical and standardized deviations of data. Section 7 presents empirical results. Finally, Section 8 concludes the paper.

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[1] Useful references include Casella and George (1993), Gelfand and Smith (1990), and Chib and Greenberg (1996).

2. Deviations from Put-call Parity

Put-call parity (PCP) is a well-known relation that exists, in a perfect capital market, between the prices of European call and put options with similar terms on the same underlying stock. In a frictionless market, the following relation must hold：

$$C_t - P_t = I_t - D_t - Xe^{-r(T-t)/365} \quad \text{(1a)}$$

$$D_t = \sum_{\tau=t}^{T} d_\tau e^{-r(\tau-t)/365} \quad \text{(1b)}$$

where r_t is the known annualized continuous risk-free interest rate

C_t is the market price of the European call at time t

P_t is the market price of the European put at time t

T is the expiration date of the option

X is the exercise (or strike) price of the option

I_t is the market value of the index at time t

D_t is the present value of the sum of the τ -time known non-stochastic dividends (d_τ) to be paid during the option period.

If the parity condition described in equation（1）is valid and the financial markets are efficient, then the riskless interest rate for the options' maturity can be inferred from equation（1）. As mentioned above, when market frictions are present, the deviations from PCP can fluctuate within a bounded interval without giving rise to any arbitrage profit. In other words, the implied interest rate derived from equation（1）may not be riskless.

The implied interest rate as suggested by Brenner and Galai (1986) from equation（1）of the European PCP is

$$r_t^{'} = -\frac{365}{(T-t)} \times Ln\left(\frac{I_t - D_t - C_t + P_t}{X}\right) \quad \text{(2)}$$

Thus, the deviations （e_t） from PCP can be written as

$$r_t^{'} = r_t + e_t \qquad (3)$$

where r_t denotes the riskless rate of return.

As described in Neal (1996), the “implied” interest rate, $r_t^{'}$, can be treated as the opportunity cost of arbitrage funds. Since arbitrageurs face uncertainty about execution prices, the future value of dividends, the problem of immediacy and early liqudation, and the magnitude of tracking error, the index arbitrage does not necessarily provide a risk-free return. It is reasonable to assume that the opportunity cost of arbitrage funds will exceed the risk-free rate and the deviation can be regarded as the “risk premium” or reflects the cost of transaction[2]. Neal (1996) further found that the implied opportunity cost of arbitrage funds exceeds the Treasury bill rate and the arbitrage decision is sensitive to the opportunity cost of funds.

Neal (1996) estimates the implied opportunity cost of arbitrage funds and shows that the cost is 88 basis points higher than the Treasury bill rate. An opportunity cost exceeding the risk-free rate is consistent with the risk of index arbitrage as noted in Kawaller (1987).

The presence of the term e_t allows for short-run deviations from PCP. If the sequence $\{e_t\}$ exhibits mean reversion, then the deviations from equilibrium must be temporary. Thus, PCP is said to hold. On the other hand, if the sequence $\{e_t\}$ is serially random, then the deviations from equilibrium are permanent in nature. Thus, we can reject the theory of PCP. Our “indirect” test, as shown in the following sections, is to determine whether $\{e_t\}$ exhibits mean reversion.

[2] These transaction costs include bid-ask spread, commission fees, differential interest rates, and execution costs, etc. in both stock and option markets.

3. Data and Two Comparable Equity Index Option: S&P500 and DAX

Neal (1996) and previous studies ignore the effect of dividend uncertainty on equity index arbitrage[3]. To illustrate the importance of dividend uncertainty on arbitrage, we consider two European index options, S&P500 and DAX, for comparison purposes. The daily closing prices of two equity indices and options are used. These data, provided by DATASTREAM, cover the period of August 23, 1999 through March 20, 2003. Because options on the S&P500 index are not corrected for dividend payments, the put-call parity may contain an additive term of dividend payments as shown in (1a) and (1b), but with D_t unknown. On the other hand, the DAX index is an example of a performance index (Grünbichler and Callahan (1994)). On ex-dividend days, the DAX is calculated as if the dividends are reinvested in the index so that the put-call parity for DAX index options can be expressed as

$$C_t - P_t = I_t - Xe^{-r(T-t)/365}$$

In the above equation, the dividend payments do not appear on the functional form of PCP. In other words, the PCP deviations (e_t) from the implied interest rates in equation (3) are absent of dividend risk.

Like Neal (1996), we implement the realized dividends, as reported in the S&P500 and DAX bulletins, as proxies for the future dividends on the PCP model. The uncertainty of dividend payments can produce large errors of option pricing as indicated by earlier studies (Harvey and Whaley (1992)).

These are two other specific features which lead to a relatively low level of risk associated with ex ante arbitrage strategies in DAX as compared with S&P500[4]. First,

[3] As suggested by Harvey and Whaley (1992), Neal (1996) uses the realized dividends, as reported in the S&P500 bulletin, as proxies for future dividends on the cost-of-carry model.
[4] For details, refer to Grünbichler, Longstaff, and Schwartz (1994) and Bühler and Kempf (1995).

the DAX index is narrow, consisting of only 30 blue chips of the German equity market which represent about 60% of the market capitalization and 85% of the trade volume. Arbitragers are able to trade a perfect matching basket at reasonable costs and in a reasonable span of time. Thus, tracking error risk can be avoided and the execution risk is relatively low. Second, there is little execution risk in the derivatives (futures and options) market as the German Futures and Options Exchange (DTB) is an electronic screen-trading market[5]. It is believed that price discovery is faster in a screen-based trading system, because it is less costly to operate and may therefore offer lower bid-ask spreads. The possibility of remote access may increase the number of traders and thereby also lead to an increase in liquidity.

Based on these specific features of Germany's markets, one would expect that arbitrage opportunities can be exploited very quickly and that ex ante arbitrage strategies are nearly risk free. Consequently, the PCP would not allow for large and long-lasting arbitrage opportunities.

The deviations e_t from equation (3) for S&P500 and DAX are drawn in Figures 1 (a) and (b), respectively. A preliminary look shows that shocks to the deviations for both indices may be temporary in nature with a tendency to revert to some mean level. However, the mean for S&P500 is subject to more occasional level shifts over time. The actual patterns for S&P500 and DAX need more careful investigation.

[5] After September 1998, the DTB merged with SOFFEX, the Swiss Options and Futures Exchanges, to create Eurex, a cross-border exchange.

4. Variance Ratio Statistics in a Markov-Switching Variance Model

The indirect test in our model employs variance ratio (VR) statistic, which is due to Cochrane (1998) and Lo and MacKinlay (1988). This test allow heteroscedasticity in the data and, more importantly, does not require the assumption of normality. Under the null of a heteroscedastic random walk process, Lo and MacKinlay（1989）show that the variance ratio test is more powerful than the Box-Pierce Q test and the Dickey-Fuller unit root test against several alternative hypotheses. In addition, it offers a very straight-forward interpretation of how rapidly a series reverts back to, or diverges from, its mean value.

If one-period deviations are serially random, then we have

$$e_t \sim iid\left(\mu, \sigma^2\right) \qquad (4)$$

and since the k-period deviation is the accumulation of k successive e_t,

$$Var\left(e_t^k\right) = k\,\sigma^2 \qquad (5)$$

The VR statistic is defined in the deviation context as

$$VR(k) = \frac{Var\left(e_t^k\right)}{Var\left(e_t\right)}\frac{1}{k} \qquad (6)$$

which is unity under the serially random hypothesis. On the other hand, if the series exhibits mean reversion, so that changes in either direction tend to be offset over time by moving back toward the starting point, then $Var\left(e_t^k\right)$ will be less than k times as large as $Var\left(e_t\right)$, such that VR will be less than unity. We therefore take values for VR of one or above as evidence against the PCP model.[6]

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[6] The sample variance ratio, $VR(k)$, can be expressed as one plus a positively-weighted sum of the first k-1 sample autocorrelations. As shown by Cochrane (1988), the approximated value of the sample variance ratio is：

$$VR(k) = 1 + 2\sum_{j=1}^{k-1}\frac{k-j}{k}\rho(j)$$

where $\rho(j)$ is the jth-order sample autocorrelation of one-period deviations e_t in this study. If the variance ratio is greater than one, less than one, or equal to one, then autocorrelations between

One explanation for mean reversion is the presence of a transitory component in asset prices. To judge whether a sample VR is significantly below unity, one needs to know the sampling distribution of the VR under the null hypothesis. Poterba and Summers (1988) and Lo and MacKinlay (1989) use the VR to test for mean reversion in stock prices and conclude that a transitory component accounted for a substantial fraction of the variance in stock returns over horizons of several years. The inference was based on a Monte Carlo simulation of the sampling distribution of the VR under the null hypothesis of serially random returns.

Kim, Nelson, and Startz (1991) estimate the sampling distribution of the VR by randomizing actual returns and also suggest a "stratified randomization" that preserves the historical pattern of high and low volatility periods. The fact that the latter reveals substantially weaker evidence of mean reversion than the former suggests that the specific pattern of heteroskedasticity in the sample period may play an important role in inference. However, their approach assumes that the econometrician has certain knowledge of the pattern, yet does not exploit any information from the pattern of heteroskedasticity in the estimation of the VR. Furthermore, a resampling of returns is limited by segregation into subperiods according to volatility.

Kim, Nelson, and Startz (KNS, hereafter)(1998) find that the sampling distribution of the VR is affected by the particular pattern of heteroskedasticity, such as the Great Depression, during the sample period. They use a model with a three-state Markov-Switching process estimated by an extension of the Bayesian Gibbs sampling approach of Albert and Chib (1993) in which the parameters as well as the unobserved states are viewed as random variables for which we obtain a

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deviations are positive, negative, or zero and the series of deviation exhibit "mean aversion", "mean reversion", or "random walk", respectively.

conditional distribution given the data. It suggests two changes in the way that that interpret the VR in the presence of heteroskedasticity. First, they fully utilize the information in the data about the pattern of heteroskedasticity in simulating the sampling distribution of the VR without pretending to have prior knowledge of the pattern. Second, they modify the VR statistic to make more efficient use of the information in the data about mean reversion by weighting observations appropriately based on information in the data about the timing and magnitude of volatility changes.

Following KNS, we consider the following three-state Markov-switching model of stock index returns：

$$e_t \sim N\left(0, \sigma_t^2\right), \tag{7}$$

$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} + \sigma_3^2 S_{3t} \tag{8}$$

$$S_{Kt} = 1 \text{ if } S_t = k \text{, and } S_{Kt} = 0, \quad \text{otherwise; } k = 1, 2, 3 \tag{9}$$

$$P_{i1} + P_{i2} + P_{i3} = 1 \text{, where } \Pr\left[S_t = j \middle| S_{t-1} = i\right] = p_{ij}, \quad i, j = 1, 2, 3 \tag{10}$$

$$\sigma_1^2 < \sigma_2^2 < \sigma_3^2 \tag{11}$$

where e_t is demeaned deviation, and S_t is an unobserved state variable which evolves according to a first-order Markov process with transition probabilities in (10).

The adoption of a three-state Markov switching variance model is suggested by KNS for several reasons. Porterba and Summers (1988) use VR statistics to investigate whether stock prices are mean-reverting, taking data that consists of monthly total returns on all NYSE stocks for both value-weighted and equal-weighted portfolios from 1926 through 1985. In measuring the statistical significance of the VR statistic, they implement an estimate of the standard error based on Monte Carlo simulations assuming independently and normally-distributed returns. Although stock returns are actually unconditionally non-normal and heteroskedastic with high persistence, Poterba and Summers (1988) show that the empirical distribution of the

VR statistic with heteroskedasticity is no different from that with homoskedasticity. In other words, it suggests that the degree of persistence in heteroskedasticity does not affect the distribution of the VR statistic very much.

Instead of using Monte Carlo simulations, which require a distributional assumption, Kim, Nelson, and Startz (1991) employ "randomization" methods to estimate the unknown distribution of the VR for the same sample period[7]. To estimate the distribution of the VR statistic under the null, they first shuffle the date to destroy any time dependence, and then recalculate the test statistic for each reshuffling. They presente results for a "stratified randomization" that preserves the historical pattern of heteroskedasticity. Their results suggest that significance levels are much lower than previously reported. Even though their stratified randomization provides a way to retain information in historical heteroskedasticity in returns, their division of the sample into low- and high-variance states is arbitrary and limited.

KNS criticize the above two papers for (1) Poterba and Summers (1988) reported a Monte Carlo experiment that mimics the actual persistence of volatility, but does not preserve the historical pattern. This may be valid when the particular historical pattern of heteroskedasticity, such as the Great Depression, does not affect the sampling distribution of the VR statistic. (2) In the presence of persistence in heteroskedasticity, the randomization destroys any time dependence in variance. Thus, the usual randomization method may fail, because errors are not interchangeable.

Employing a three-state Markov-switching variance model, KNS find that the empirical distribution of the VR is much different from that in the homoskedastic case,

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[7] Randomization focuses on the null hypothesis that one variable is distributed independently of another. Randomization shuffles the data to destroy any time dependence and then recalculates the test statistic for each reshuffling to estimate its distribution under the null. Repeating the experiment, we count how many times the calculated variance ratio after randomization falls below the value of the actual historical statistic in order to estimate the significance level. The advantage of this approach is that the null hypothesis is very simple and no assumptions are made concerning the distribution of stock prices.

when the pattern of heteroskedasticity is the historical one. The distribution has wider variance and is more skewed than in the homoskedastic case. This suggests that the VR tests of Poterba and Summers (1988) based on Monte Carlo experiments and those of Kim, Nelson, and Startz (1991) based on the usual randomization method have the wrong size, rejecting the null of random returns in favor of mean reversion too often.

5. Bayesian Gibbs Sampling and Data Augmentation

The Markov-switching variance model from （7） to （11） could traditionally be estimated using the maximum likelihood estimation method of Hamilton (1989) and Hamilton and Susmel (1994). However, the MLE approach, which is based on asymptotic normality, may not be valid when the sample size is not large enough. In addition, it is difficult to determine how large the sample size should be in order for asymptotic normality to hold. In this paper we employ an extended version of Albert and Chib's（1993）Bayesian Gibbs sampling approach to estimate the model[8].

The Gibbs sampler is a path-breaking technique for generating random samples from a multivariate distribution by using conditional distributions without having to compute the full joint density. In regime-switching models, the full joint density is extremely difficult to calculate, but the conditional distributions are easy to evaluate. In addition, the Gibbs sampling approach can provide us with a way to deal with uncertainty associated with underlying parameters and unknown states of the model. It is an iterative Monte Carlo technique that generates a simulated sample from the joint distribution of a set of random variables by generating successive samples from their conditional distribution[9].

In the Gibbs sampling approach, all the parameters of the model are treated as random variables with an appropriate but unknown prior distribution. Thus, random variables to be drawn in the above model from （7） to （11） are given by

$$\tilde{S}_t = \{S_t, t = 1,2,...T\}, \quad \sigma_1^2, \sigma_2^2, \sigma_3^2 \text{ and } \tilde{P} = \{P_{11}, P_{12}, P_{21}, P_{22}, P_{31}, P_{32}\}$$

Starting from arbitrary initial values of the parameters, Gibbs sampling proceeds by taking：

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[8] Advantages of the Gibbs sampling approach over the maximum likelihood method are discussed in detail in Albert and Chib(1993).

[9] For more details of the Gibbs sampling approach to a three-state Markov-switching variance model given below, readers are referred to Kim and Nelson (1999).

Step 1 : a drawing from the conditional distribution of $\tilde{S}$ given the data, σ_1^2, σ_2^2, σ_3^2 and $\tilde{P}$; then

Step 2 : a drawing from the conditional distribution of σ_1^2 given the data, $\tilde{S}$, σ_2^2, σ_3^2 and $\tilde{P}$; then

Step 3 : a drawing from the conditional distribution of σ_2^2 given the data, $\tilde{S}$, σ_1^2, σ_3^2 and $\tilde{P}$; then

Step 4 : a drawing from the conditional distribution of σ_3^2 given the data, $\tilde{S}$, σ_1^2, σ_2^2 and $\tilde{P}$; then

Step 5 : a drawing from the conditional distribution of $\tilde{P}$ given the data, $\tilde{S}$, σ_1^2, σ_2^2 and σ_3^2.

By successive iteration from step 1 to step 5, the procedure simulates a drawing from the joint distribution of all the state variables and parameters in the model. It is straightforward then to summarize the marginal distribution of any of these, given the data[10].

KNS find that the sampling distribution of the VR is affected by the particular pattern of heteroskedasticity, and that this effect is also substantially important in the case of daily deviations of PCP. Simulation methods that assume heteroskedasticity or allow for persistence in heteroskedasticity, but are not conditioned on the particular pattern of the historical period, produce a biased test, leading the investigator to reject the null hypothesis of no mean reversion too often. Since dividend payments and a liquidity premium exhibit seasonal patterns (Harvey and Whaley (1994), Eleswarapu and Reinganum (1993)) in the data, we follow KNS and consider two new tests of mean reversion that are conditioned on the information that the data contain the historical pattern of heteroskedasticity. We employ a resampling strategy for

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[10] For details of the Gibbs sampling approach to a three-state Markov-switching variance model given above, readers are referred to Kim and Nelson (1999) P219-224.

estimating the sampling distribution of the VR that standardizes historical returns using estimated variances. Instead of conditioning on the estimates of these variances and the dates of regime switches, the Gibbs sampling approach is used so as to allow for uncertainty in these parameters and states while being conditioned on the information in the data.

6. Tests Based on the Variance Ratios of Historical and Standardized Deviations

This section suggests a modification of the VR statistic to make more efficient use of the information in the data about mean reversion by weighting observations appropriately based on the information in the data about the timing and magnitude of volatility changes.

Assume that

$$e_t \sim \left(0, \sigma_t^2(\theta)\right)$$

$$\theta = \left\{\sigma_1^2, \sigma_2^2, \sigma_3^2, P_{11}, P_{12}, P_{21}, P_{22}, P_{31}, P_{32}\right\}$$

where e_t is the demeaned deviation which shows heteroskedasticity with variance σ_t^2

θ is a vector of parameters that describes the dynamics of σ_t^2

By standardizing historical returns before calculating the VR test statistic, appropriate weights can be assigned to observations depending on their volatility. An additional complication of this approach is that unlike the test based on original returns, the test statistic itself is subject to sampling variation due to uncertainty in the parameters that describe the dynamics of heteroskedasticity. Thus, we compare two distributions：the distribution (due to parameter uncertainty) of the VR test statistic for standardized historical returns and the distribution of the VR test statistic under the null hypothesis estimated from randomizing the standardized returns.

A natural way to randomize returns without losing time dependence in historical returns would be the following:

Step 1：Standardize e_t to obtain $\left\{e_t^* = \dfrac{e_t}{\sigma_t}, t = 1,2,...T\right\}$

Step 2：Randomize the standardized deviations e_t^* to obtain $\left\{e_t^{r^*}, t = 1,2,...T\right\}$

Step 3：Destandardize $e_t^{r^*}$ to obtain $\left\{ e_t^r = e_t^{r^*} \times \sigma_t, t = 1,2,...T \right\}$

From the above procedure, we obtain the following four data series：

1. original series e_t
2. standardized series e_t^*
3. randomized standardized series $e_t^{r^*}$
4. randomized de-standardized series e_t^r

Step 4：Calculate VRs for each series. They are respectively $VR(k), VR^*(k), VR^{r^*}(k)$ and $VR^r(k)$, $k = 1,2,...K$.

Step 5：Repeat the Gibbs sampling and steps 1 to 4.

These steps are repeated, say, M times to get the posterior distribution of the VR for standardized historical returns, $VR^*(k)$, and the empirical distribution of the VR under the null of no mean reversion, $VR^{r^*}(k)$. To estimate the significance level for the test of mean reversion, we count how many times the variance ratio for the standardized and randomized returns $\left(VR^{r^*}(k)\right)$ from Gibbs-sampling-augmented randomization falls below the variance ratio for standardized historical returns $\left(VR^*(k)\right)$ from Gibbs sampling. For comparison purposes, we also conduct the same analysis for the original returns. Thus, the p-value, which is defined as

$$p = \frac{No.\,of\left[VR(k) > VR^r(k)\right]}{M} \quad \text{for original returns}$$

and

$$p^* = \frac{No.\,of\,[VR^*(k) > VR^{r^*}(k)]}{M} \quad \text{for standardized returns.}$$

It is important to calculate the above p-values exactly rather than using a standard deviation under an assumption of normality for VR, because its sampling distribution is skewed.

If the variance of deviations for each time point σ_t^2 or the parameters θ that govern the evolution of σ_t^2 are known, then the above procedure would be straightforward. In practice, σ_t^2 or the parameter θ associated with σ_t^2 has to be estimated using historical data. Because θ is subject to parameter uncertainty, the standardized deviations are also subject to sampling variation.

To incorporate the effect of uncertainty in the parameters associated with the variance of deviations, we augment the Gibbs-sampling approach introduced in Section 5 with the standardizing step of the above procedure. As in Section 5, each run of Gibbs sampling based on historical returns provides us with particular realizations of the set, $\{S_t, t = 1,2,...T\}$ and $\{\sigma_1^2, \sigma_2^2, \sigma_3^2\}$, which are used to calculate σ_t^2 according to Equation（8）. Using σ_t^2, t =1,2,…T, simulated in this way, we can proceed with the above Steps 1 through 3.

If the above procedure is repeated, say, 10,000 times, with each iteration augmented by simulations of σ_t^2 from each run of Gibbs sampling, then we have 10,000 sets of randomized returns. These artificial histories are conditioned on the information about the pattern of heteroskedasticity contained in the historical returns, incorporate parameter uncertainty, and are consistent with the null of mean reversion due to randomization. For each of these 10,000 sets of artificial histories, the variance ratio statistic is calculated, which can be used to estimate the empirical distribution of the variance ratio statistic. To estimate the significance level, we calculate the p-values to know how many times the variance ratios for the artificial histories fall below the variance ratios for original historical returns.

7. Empirical Results

The procedures for Gibbs-sampling described in the previous sections are applied here to the PCP deviations of S&P 500 and the DAX. Gibbs-sampling is run such that the first 2,000 draws are discarded and the next 10,000 are recorded. We employ almost non-informative priors for all the models' parameters. Table 1 presents the marginal posterior distributions of the parameters that result from Gibbs-sampling for the PCP deviations of S&P 500 and the DAX, respectively. At the end of each run of Gibbs-sampling, we have a simulated set of $\{S_t,\ t=1, 2...T\}$ and thus, of $\{S_{jt},\ t=1, 2,...T,\ j=1, 2, 3\}$, $\sigma_j^2,\ j=1, 2, 3$ and $\tilde{P}$. Figures 2(a), 2(b), 2(c) and Figures 3(a), 3(b), and 3(c) depict probabilities of low-, medium-, and high-variance states for the PCP deviations of S&P 500 and the DAX, respectively, that result from the Gibbs-sampling simulation.

Using the particular realizations of the states and the parameters for each run of Gibbs-sampling, we can calculate σ_t^2 for $t=1, 2,\cdots T$ using equation (8). Thus, when all the iterations are over, we have 10,000 sets of realized variances $\tilde{\sigma}_T^2=\{\sigma_t^2,\ t=1, 2,\cdots T\}$ of PCP deviations. Figures 2(d) and 3(d) plot the average of 10,000 sets of $\tilde{\sigma}_T^2$, which are our estimates of the variance of the S&P 500's and the DAX's PCP deviations. Tables 2 (a) and (b) present variance ratios for original daily deviations from PCP for S&P 500 and DAX, respectively. Only the DAX displays mean reversions at long horizons. The smallest p-value is 0.028 at 45 days lag.

Table 3 (a) and (b), in which variance ratios for standardized daily deviations from PCP for S&P 500 and DAX are presented, respectively. The DAX also displays mean reversion at long horizons and its smallest p-value is 0.028 at a lag of 40 days. The evidence is weak that the standardized returns approach to estimating the VRs suggests that mean reversion, if it is present, occurs at shorter lags.

8. Summary

Previous studies indicate that when market frictions are taken into account, the deviations from PCP can fluctuate within a bounded interval without giving rise to any arbitrage profit. This study presents a model of the option price mean reverting to a function form of PCP. The variance ratio test is employed to examine whether the deviations of PCP exhibit mean reversion. We make appropriate allowance for heteroskedasticity when basing inference on the VR statistic by using the Gibbs-sampling approach in the context of a three-state Markov switching model.

The empirical result shows that PCP deviations from the electronic screen-traded DAX index options, which are calculated as if the dividends are reinvested in the index, display mean reversion at long horizons. On the other hand, those deviations from floor-traded S&P 500 index options, which do not correct for dividend payments, vary randomly.

Table 1：The estimated parameters from the Bayesian Gibbs-sampling approach to a three-state Markov-switching model of heteroskedasticity for S&P 500's and DAX's daily PCP deviations

	Posterior distribution			
	S&P 500		DAX	
Parameter	Mean	Std.	Mean	Std.
P_{11}	0.8912	0.0344	0.9244	0.0168
P_{12}	0.0561	0.0402	0.0713	0.0165
P_{21}	0.0197	0.0230	0.0677	0.0511
P_{22}	0.8926	0.0489	0.6355	0.0975
P_{31}	0.2575	0.0689	0.6257	0.1234
P_{32}	0.0882	0.0519	0.0390	0.0379
σ_1^2	0.0216	0.0064	0.0280	0.0026
σ_2^2	0.0559	0.0174	0.0843	0.0274
σ_3^2	1.1807	0.2525	2.6624	0.7211

Table 2(a)：Variance ratios for original daily deviations from PCP in S&P500

Lag(days) K	VR(k)	VR sampling distribution $(VR^r(k))$ Mean	Std	Median	p-value
2	1.4616	1.0343	0.0621	1.0340	1.0000
5	2.1525	1.0796	0.1266	1.0738	1.0000
10	1.1489	0.9957	0.0578	0.9950	0.9950
15	1.1816	0.9699	0.0922	0.9671	0.9854
20	1.2202	0.9536	0.1177	0.9491	0.9824
25	1.2907	0.9561	0.1385	0.9490	0.9864
30	1.3059	0.9564	0.1602	0.9467	0.9790
35	1.2527	0.9491	0.1812	0.9376	0.9440
40	1.1663	0.9425	0.2004	0.9288	0.8667
45	1.1181	0.9464	0.2172	0.9284	0.7957
50	1.0792	0.9496	0.2341	0.9290	0.7294

Note：1. The sampling distribution is based on Gibbs-sampling-augmented randomization.

2. The p-value is the frequency with which the simulated VR is smaller than the historical sample value, which is observed in the Gibbs-sampling-augmented randomization under the null hypothesis.

Table 2(b)：Variance ratios for original daily deviations from PCP in DAX

Lag(days)		VR sampling distribution $\left(VR^{r}\left(k\right)\right)$			
K	VR(k)	Mean	Std	Median	p-value
2	1.2283	1.0245	0.0648	1.0245	0.9991
5	1.5501	1.0438	0.1184	1.0401	0.9999
10	1.0528	0.9789	0.0471	0.9794	0.9456
15	1.0141	0.9520	0.0808	0.9529	0.7782
20	0.9131	0.9313	0.1043	0.9315	0.4293
25	0.8369	0.9294	0.1237	0.9257	0.2279
30	0.7666	0.9291	0.1450	0.9230	0.1305
35	0.6626	0.9216	0.1671	0.9131	0.0513
40	0.5893	0.9153	0.1859	0.9049	0.0282
45	0.5635	0.9174	0.2007	0.9050	0.0237
50	0.5484	0.9189	0.2158	0.9029	0.0254

Note：1. The sampling distribution is based on Gibbs-sampling-augmented randomization.

2. The p-value is the frequency with which the simulated VR is smaller than the historical sample value, which is observed in the Gibbs-sampling-augmented randomization under the null hypothesis.

Table 3(a)：Variance ratios for standardized daily deviations from PCP in S&P500

Lag(days)	VR posterior distribution $(VR^{*}(k))$			VR sampling distribution $(VR^{r^{*}}(k))$			
k	Mean	Std	Median	Mean	Std	Median	p-value
2	1.5519	0.0209	1.5503	1.0001	0.0346	0.9998	1.0000
5	2.6193	0.0887	2.6043	0.9998	0.0758	0.9981	1.0000
10	1.3209	0.0387	1.3120	0.9982	0.0645	0.9984	1.0000
15	1.4444	0.0788	1.4244	0.9971	0.1047	0.9938	0.9996
20	1.5463	0.0975	1.5243	0.9963	0.1358	0.9911	0.9997
25	1.6176	0.0995	1.6000	0.9959	0.1619	0.9890	0.9995
30	1.6019	0.0992	1.5897	0.9958	0.1850	0.9862	0.9959
35	1.5043	0.0997	1.4955	0.9961	0.2063	0.9827	0.9811
40	1.3640	0.1008	1.3590	0.9966	0.2263	0.9803	0.9228
45	1.2431	0.1001	1.2410	0.9971	0.2449	0.9760	0.8256
50	1.1392	0.1031	1.1396	0.9975	0.2622	0.9726	0.7141

Note：1. The sampling distribution is based on Gibbs-sampling-augmented randomization

2. The p-value is the frequency with which the realizations of the Gibbs sampling of the posterior distribution are smaller than the corresponding realization under the null hypothesis.

Table 3(b)：Variance ratios for standardized daily deviations from PCP in DAX

Lag(days)	VR posterior distribution $(VR^{*}(k))$			VR sampling distribution $(VR^{r^{*}}(k))$			
k	Mean	Std	Median	Mean	Std	Median	p-value
2	1.3804	0.0299	1.3852	1.0001	0.0343	1.0002	1.0000
5	2.0731	0.0830	2.0853	1.0001	0.0752	0.9979	1.0000
10	1.1008	0.0201	1.1017	0.9970	0.0642	0.9966	0.9388
15	0.9341	0.0389	0.9332	0.9963	0.1050	0.9931	0.2978
20	0.8265	0.0461	0.8252	0.9960	0.1367	0.9906	0.1166
25	0.8670	0.0449	0.8664	0.9963	0.1632	0.9876	0.2313
30	0.8349	0.0455	0.8347	0.9970	0.1868	0.9845	0.2062
35	0.7005	0.0483	0.7003	0.9976	0.2081	0.9801	0.0641
40	0.6078	0.0504	0.6068	0.9983	0.2281	0.9769	0.0277
45	0.6136	0.0526	0.6121	0.9991	0.2467	0.9728	0.0411
50	0.6213	0.0542	0.6196	1.0000	0.2645	0.9683	0.0580

Note：1. The sampling distribution is based on Gibbs-sampling-augmented randomization.

2. The p-value is the frequency with which the realizations of the Gibbs sampling of the posterior distribution are smaller than the corresponding realization under the null hypothesis.

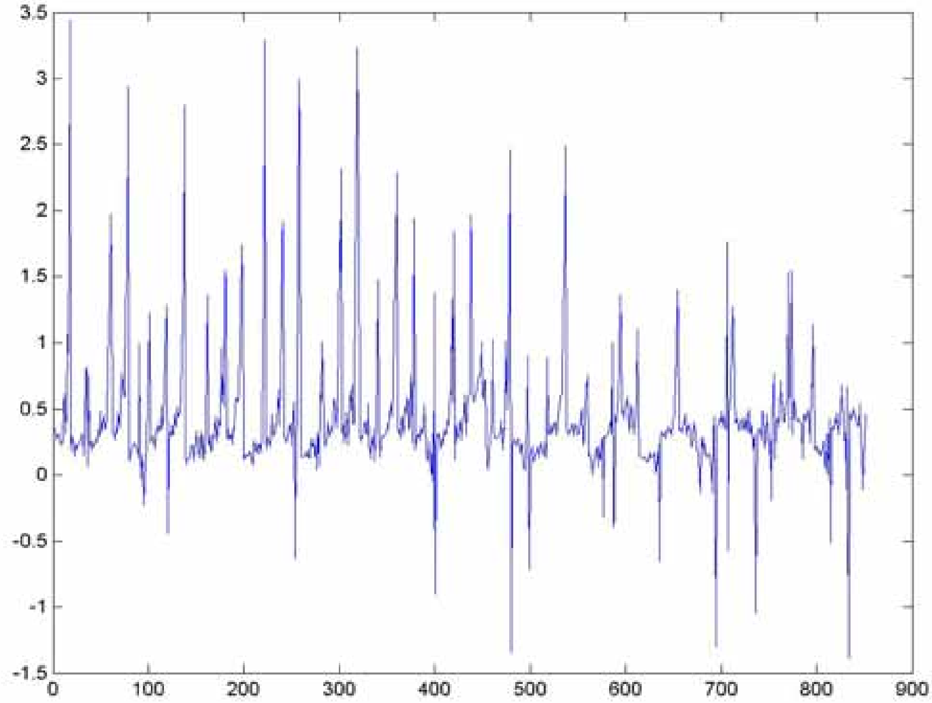

Figure 1(a)：Daily PCP Deviations from S&P500

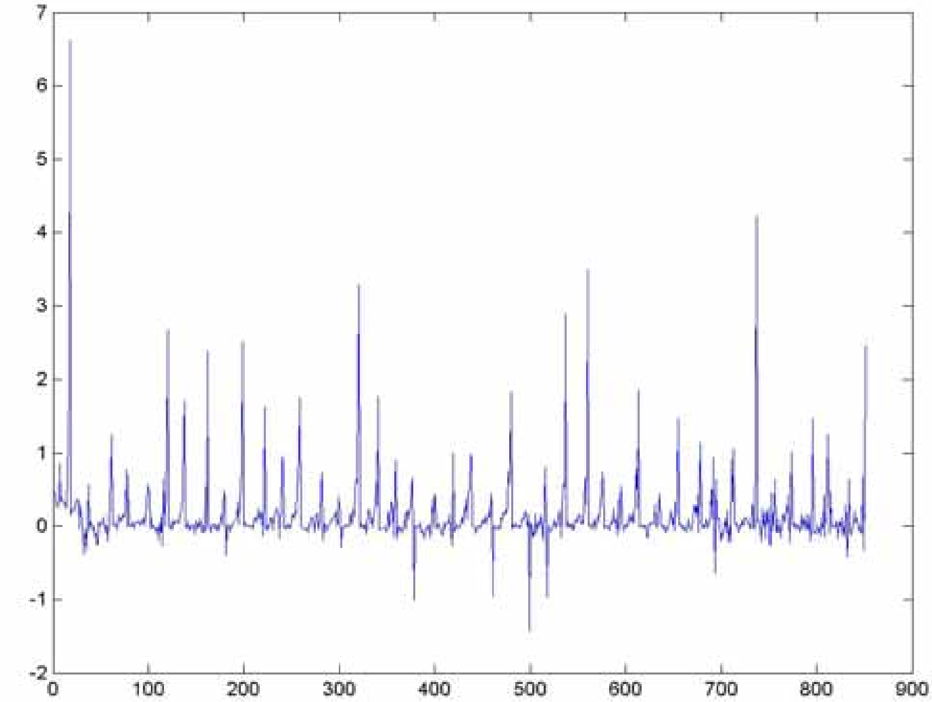

Figure 1(b)：Daily PCP Deviations from DAX

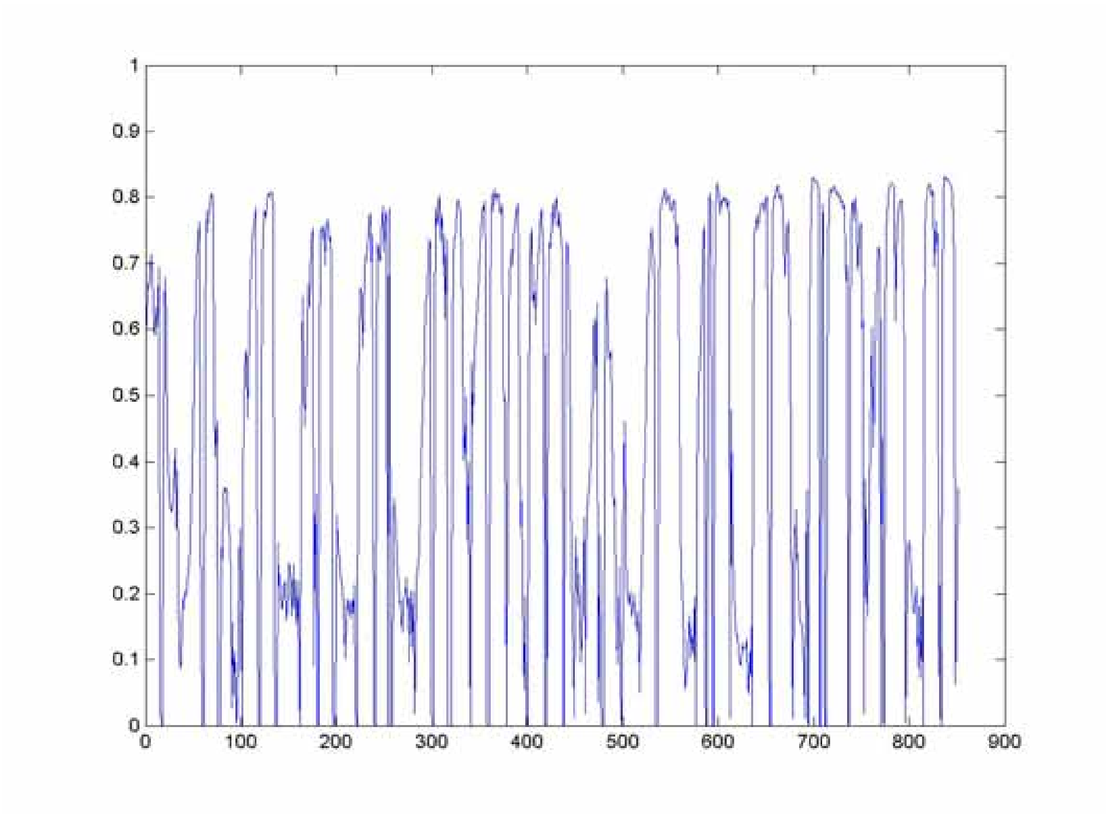

Figure 2(a)：Probability of a Low-variance State for S&P500's PCP Deviations (Gibbs Sampling)

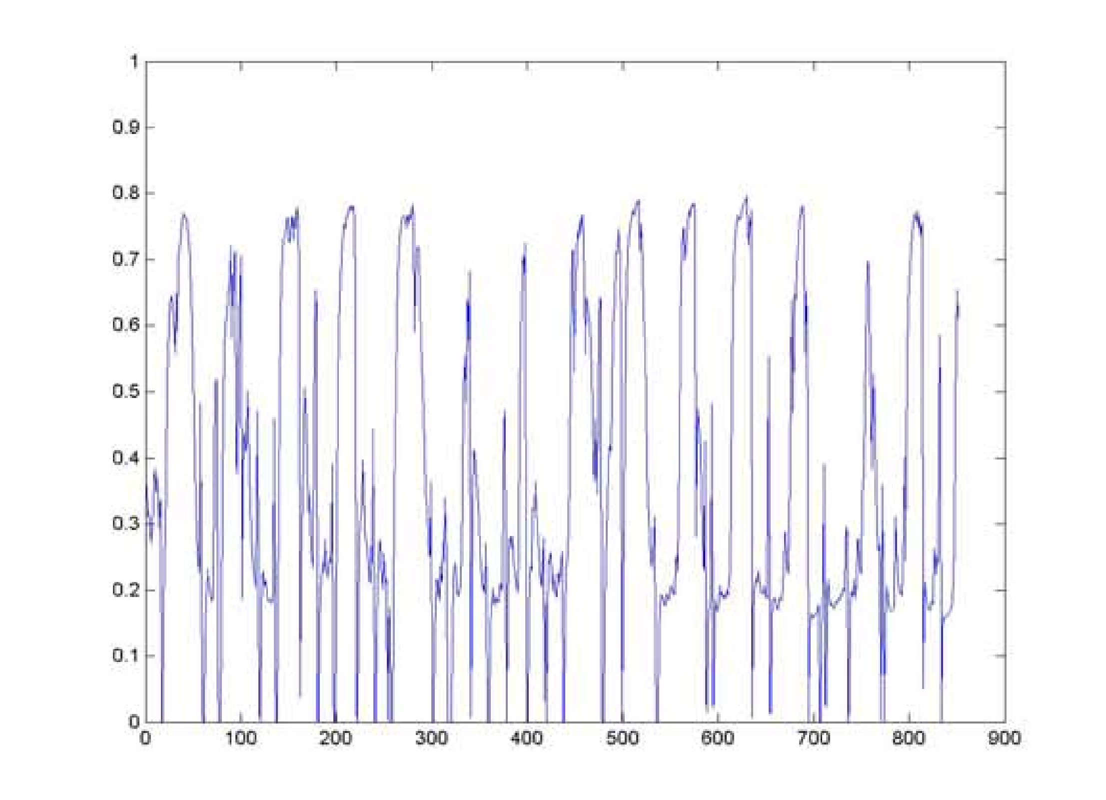

Figure 2(b)：Probability of a Medium-variance State for S&P500's PCP Deviations (Gibbs Sampling)

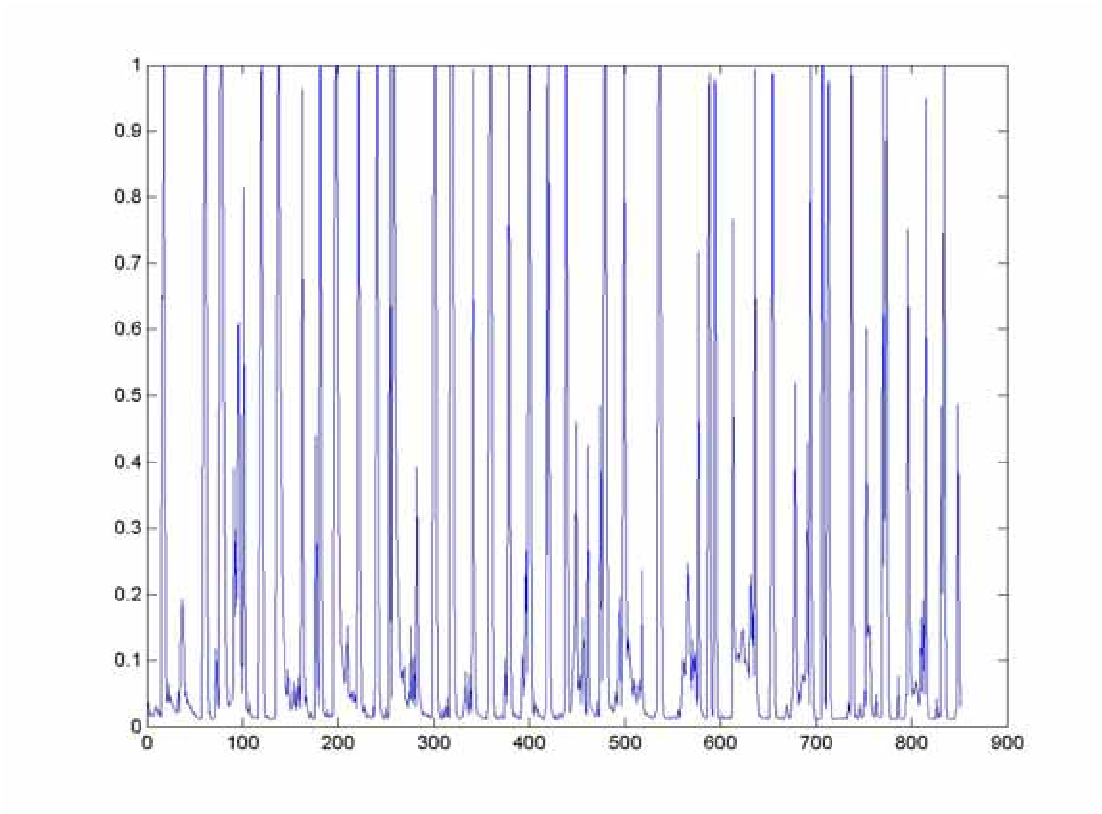

Figure 2(c)：Probability of High-variance State for S&P500's PCP Deviations (Gibbs Sampling)

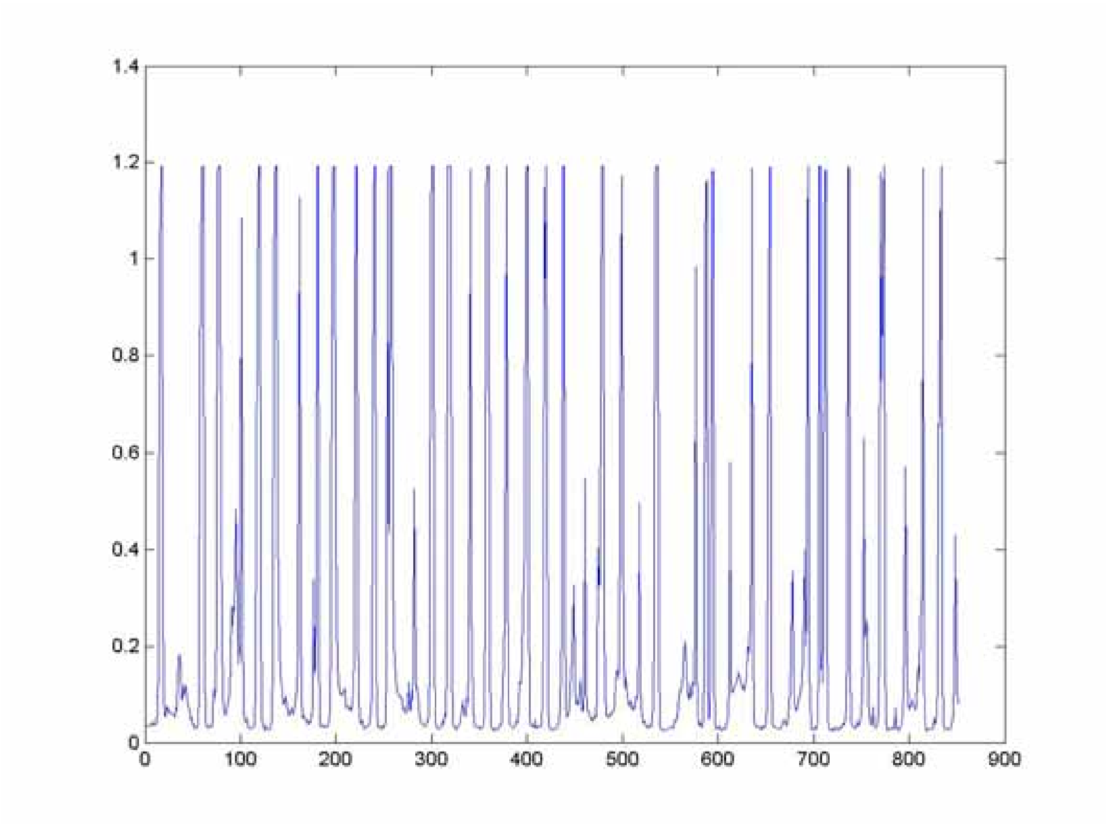

Figure 2(d)：Estimated Variance of S&P500's PCP Deviations (Gibbs Sampling)

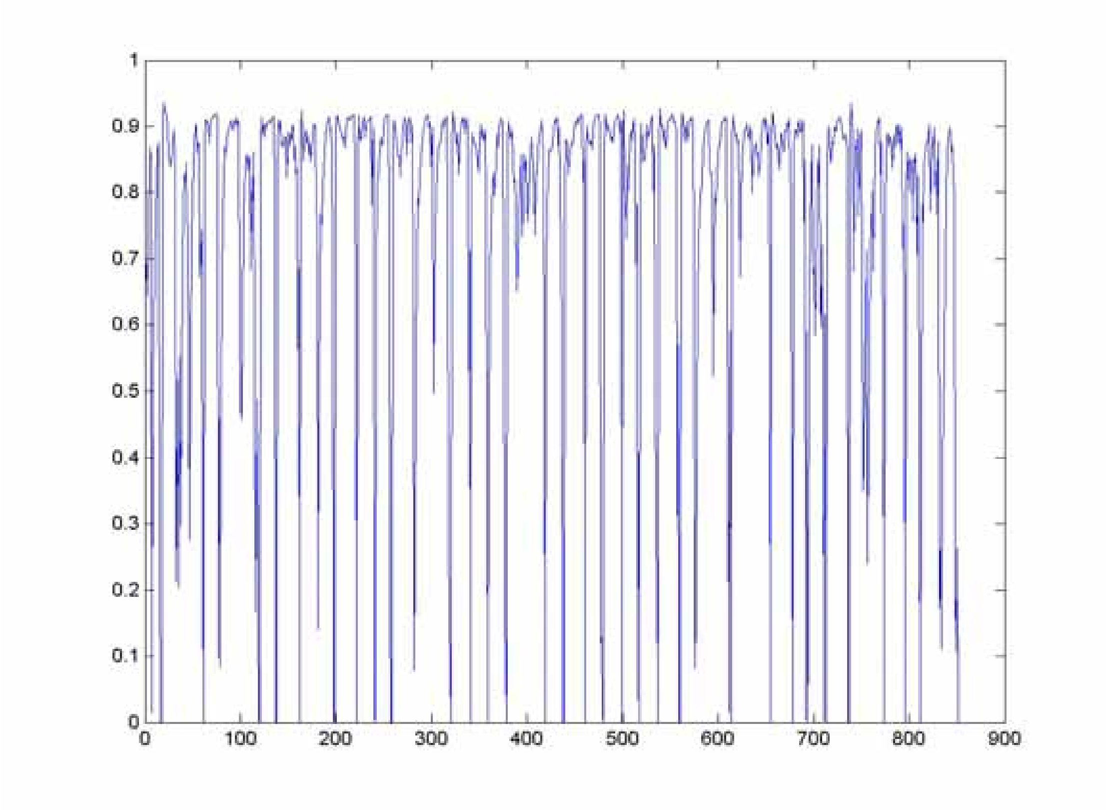

Figure 3(a)：Probability of a Low-variance State for DAX’s PCP Deviations (Gibbs Sampling)

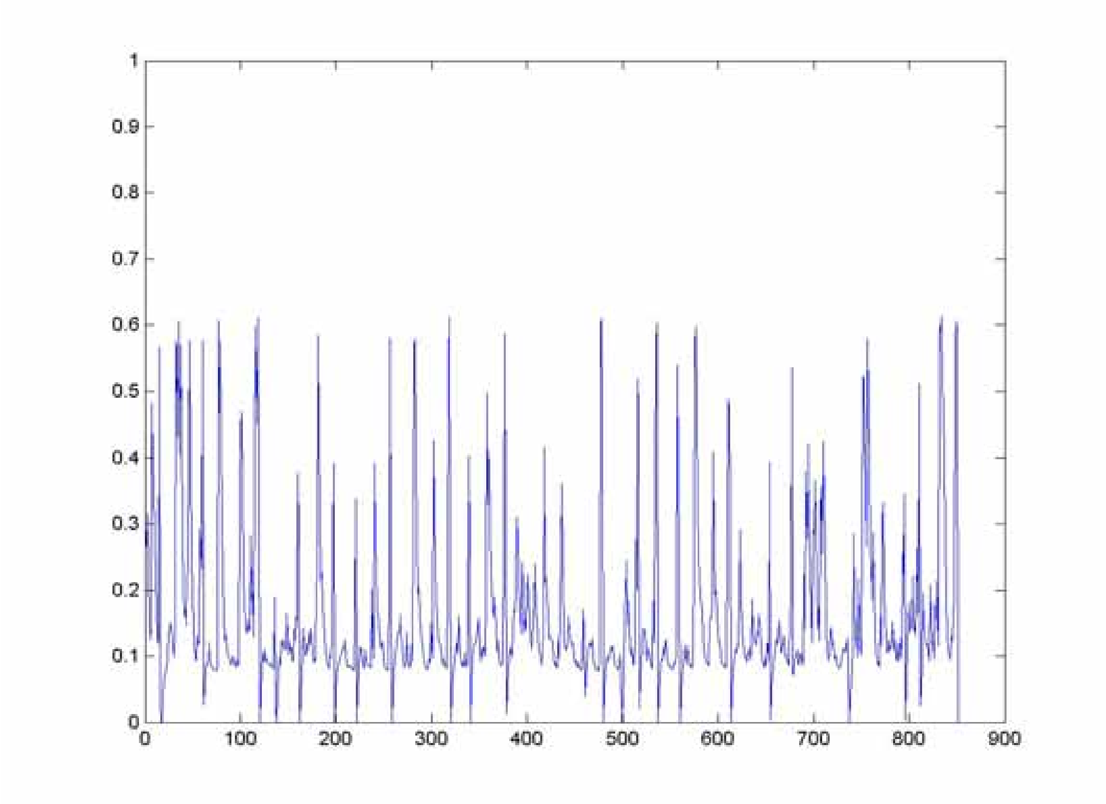

Figure 3(b)：Probability of a Medium-variance State for DAX’s PCP Deviations (Gibbs Sampling)

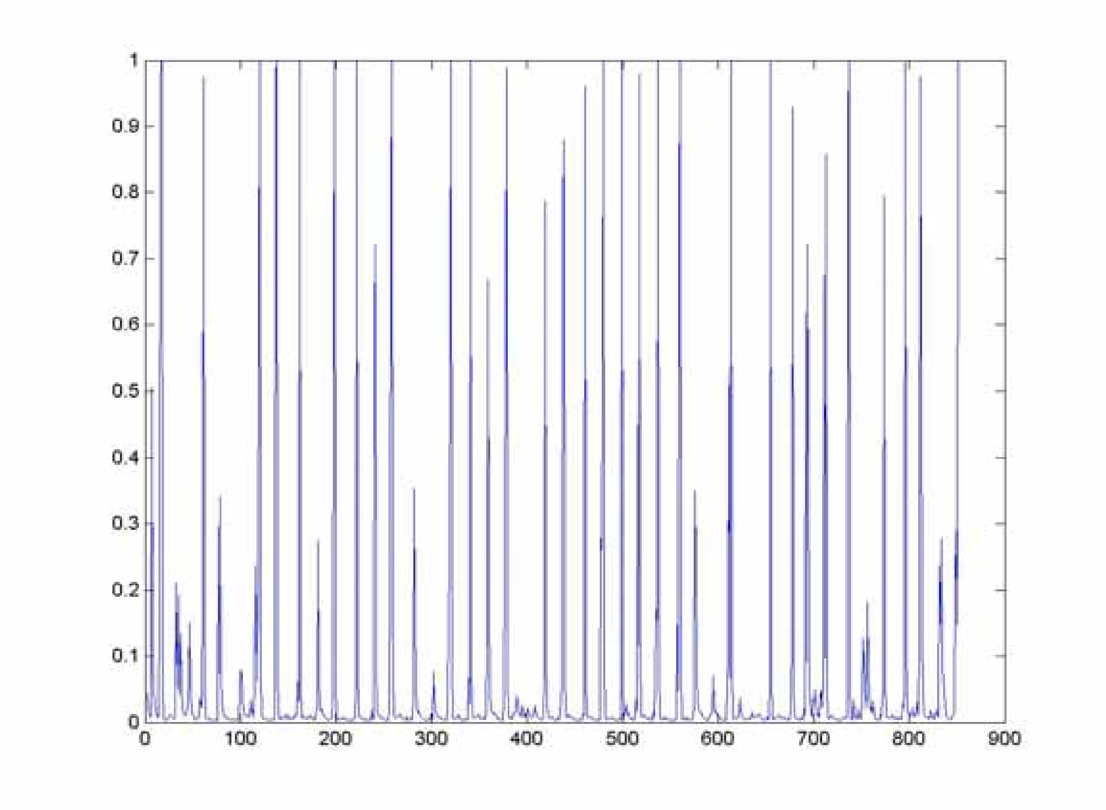

Figure 3(c)：Probability of a High-variance State for DAX's PCP Deviations (Gibbs Sampling)

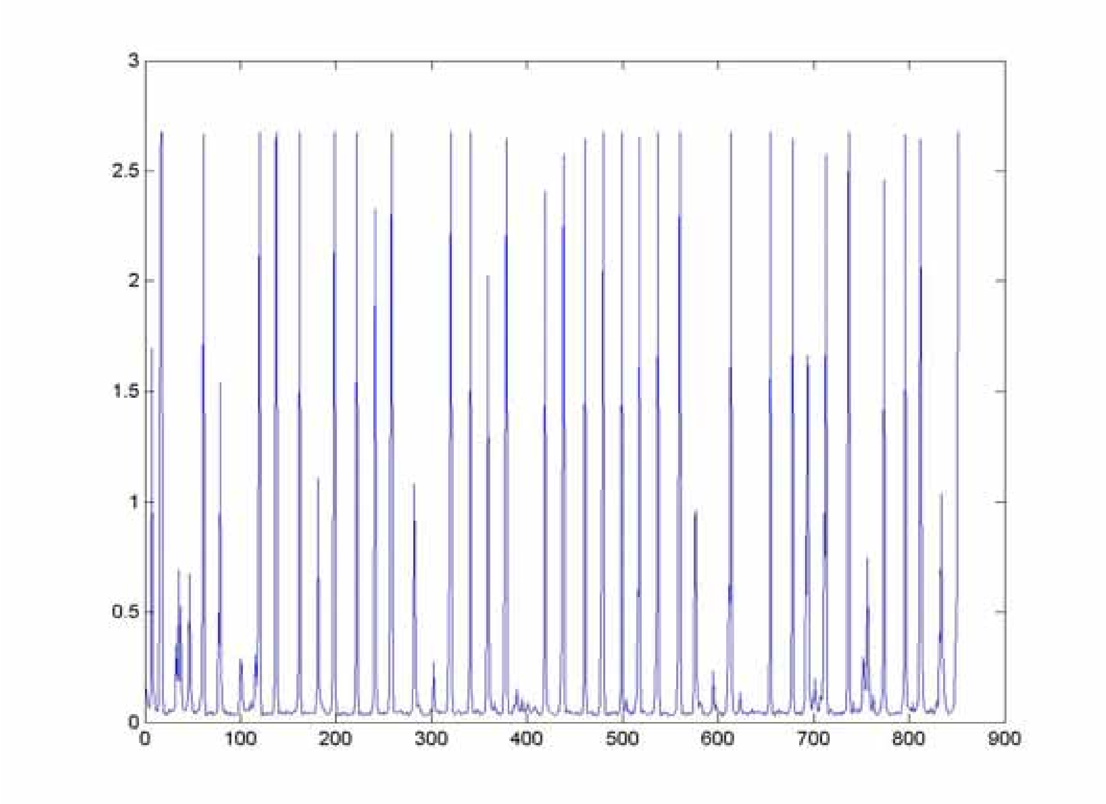

Figure 3(d)：Estimated Variance of DAX's PCP Deviations (Gibbs Sampling)

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