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Valuation of Quanto Interest-Rate Exchange Options in a Cross-Currency LIBOR Market Model

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Valuation of Quanto Interest-Rate Exchange Options in a Cross-Currency LIBOR Market Model

Abstract

This paper extends the single-currency LIBOR Market Model (LMM) to the cross-currency LMM (CLMM) based on the Amin and Jarrow (1991) framework, and the resulting model is utilized to price quanto interest-rate exchange options (QIREOs). Four different types of quanto interest-rate exchange options are priced and discussed in this article. Hedging strategies and calibration procedures are also examined in detail for practical implementation. Furthermore, Monte-Carlo simulation is provided to evaluate the accuracy of the theoretical models.

Key words: Quanto Interest-Rate Exchange Options, Cross-Currency LIBOR Market Model.

中文摘要

本研究將對匯率連動利率交換選擇權（Quanto Interest-Rate Exchange Options；QIREOs）進行評價與分析。跨通貨 LIBOR 市場模型(the Cross-Currency LMM) 由單一國家經濟環境延伸至兩國經濟環境，適用於評價匯率連動之利率衍生性商品。因此文中將採用跨通貨 LIBOR 市場模型，對 QIREOs 進行評價推導理論封閉解(Closed-Form Solution)，並作避險策略與實務應用的探討。為能供實務運用，並將探討如何進行參數校準(Calibration)；文中亦將進行蒙地卡羅模擬（Monte Carlo Simulation）以分析模型理論解的準確性。

中文關鍵詞：匯率連動利率交換選擇權，LIBOR 市場模型

報告內容

1. Introduction

Quanto interest-rate exchange options (QIREOs), also known as interest-rate difference options, are options written on the difference between two interest rates that are available in different currencies or between two interest rates in one currency, with the final payments made in domestic currency. Interest rate volatility during the past decade has magnified the risk due to an unfavorable shift in the term structure of interest rates, thereby leading to a dramatic increase in the number and types of contingent claims that incorporate options on change in the level of interest rates. These products have been developed to enhance the ability of asset/liability managers to alter their interest-rate exposure. As a result, QIREOs are evolved to exploit interest-rate differentials without directly incurring exchange-rate risk.

In contrast with QIREOs, yield-spread options (YSOs) are written on the underlying difference between two interest rates in a single-currency economy and denominated in the same currency. When a cross-currency economy degenerates to a single-currency one, QIREOs will become YSOs. Consequently, YSOs are special cases of QIREOs.

The applications of QIREOs are quite extensive and similar to those of differential swaps. However, QIREOs provide more flexibility in certain applications. First, QIREOs provide a mechanism for achieving a payoff based on the differential of interest rates available in two different currencies, which is not directly affected by movements in exchanges rates. Second, as compared with differential swaps, the major advantage of QIREOs is that they can be used to fit a very specific strategy since they can be tailored to provide payoffs that depend on whether the spread of two interest rates is above or below a specified level, or within or outside a specified range on a specific date in the future. Third, QIREOs can provide added precision to a strategy involving differential swaps. For example, a portfolio manager might use a differential swap to capitalize on anticipated yield-curve movements while also purchasing an QIREO on the spread in order to limit his downside risk. Moreover, money market investors may use QIREOs to take advantage of a high-yield currency; asset managers may adopt QIREOs to enhance their portfolio return; liability managers and other borrowers can employ QIREOs to reduce their effective borrowing rates. More details regarding the applications of QIREOs can be seen in Schwartz and Smith (1993).

Despite the wide applications of QIREOs, the academic literature has paid little attention to how to price such options. Only few articles were written on the YSOs. Longstaff (1990), Fu (1996) and Miyazaki and Yoshida (1998) derived the pricing formulas for YSOs. However, their formulas were all conducted in a single-currency economy. Their model setting and formulas are not appropriate for pricing the QIREOs since the "quanto-effect" in a cross-currency economy is not taken into account.

Besides, Longstaff (1990) derived the pricing formula for an YSO under the Cox, Ingersoll and Ross (CIR) model. His model has several problems in practical application. First, the instantaneous short rate modeled in the CIR model is abstract and market-unobservable, thereby leading to difficulty in the parameter calibration. In addition, the compounding period of the underlying rate is infinitesimal, which contradicts with the market convention of being discretely compounded on the basis of the LIBOR rates. Hence, it is complex to recover model parameters from market-observed data.

Fu (1996) and Miyazaki and Yoshida (1998) derived the pricing formulas of YSOs based on the Heath-Jarrow-Morton (HJM) model. Under the HJM framework the instantaneous forward rates are not observable in the market, so the recovery of the model parameters from the market-observed data is a difficult and complicated task. Moreover, the pricing formulas of widely traded interest rate derivatives, such as caps, floors, swaptions, etc., based on the Gaussian HJM model are not consistent with market practice. This results in some difficulties in the calibration procedure. Furthermore, as examined in Rogers (1996), the Gaussian HJM forward rates can become negative with a positive probability, which may cause some pricing errors.

The primary purpose of this article is to extend the (single-currency) LIBOR market model (LMM) to a cross-currency LMM (CLMM) based on Amin and Jarrow (1991, AJ) and then utilize the results to derive pricing formulas of QIREOs. The pricing model of QIREOs is more general and suitable for pricing Quanto interest-rate derivatives, and if the model setting degenerates to the single-currency case, the pricing model of QIREOs will become the pricing model of YSOs in the LMM framework. In addition, pricing QIREOs under CLMM can avoid the problems as mentioned earlier and is more tractable for practice.

The LMM has been developed by Musiela and Rutkowski (1997), Miltersen, Sandmann and Sondermann (1997), and Brace, Gatarek and Musiela (1997, BGM). The reasons for adopting the CLMM rather than the instantaneous short rate models, such as the Vasicek model and the CIR model, etc., or the HJM model are attributed to the following advantages. First, the interest rates modeled in the CLMM are the LIBOR rates, which are market-observable and consistent with the market convention of being discretely compounded. Hence, the model is more suitable for practical implementation. Second, the cap and floor pricing formulae in the LMM framework follow the Black's formula, which is consistent with market practice and makes the calibration procedure easier. Moreover, BGM have shown that under the forward measures forward LIBOR rates have a lognormal volatility structure that prevents the forward LIBOR rates from becoming negative with a positive probability. As a result, pricing errors arising from negative rates are avoided. Thus, pricing QIREOs based on the CLMM is more straightforward and convenient than pricing based on the instantaneous short rate models or the HJM model for practical application.

The remainder of this article is organized as follows. Section 2 briefly describes the results in AJ (1991) and adopts them to derive the CLMM. Section 3 outlines each type of QIREO and derives the pricing formulae for the four different types of QIREOs based on the CLMM. The hedging strategy of each option is also presented. Section 4 provides the calibration procedure for practical implementation and examines the accuracy of the pricing formulae via Monte-Carlo simulation. Section 5 concludes the paper with a brief summary.

2. The Arbitrage-Free Cross-Currency LIBOR Market Model

In this section, we briefly specify the results of the cross-currency LMM.

Assume that trading takes place continuously in time over an interval $\left[0,\tau\right], 0<\tau<\infty$. The uncertainty is described by the filtered spot martingale probability space $\left(\Omega, F, Q, \{F_t\}_{t\in[0,\tau]}\right)$ where the filtration is generated by independent standard Brownian motions $W(t)=\left(W_1(t), W_2(t), ..., W_m(t)\right)$. Q denotes the domestic spot martingale probability measure. The filtration $\{F_t\}_{t\in[0,\tau]}$ which satisfies the usual hypotheses represents the flow of information accruing to all the agents in the economy.[1] The notations are given below with *d* for domestic and *f* for foreign:

$f_k(t,T)$ = the k^{th} country's forward interest rate contracted at time *t* for instantaneous borrowing and lending at time *T* with $0\le t\le T\le\tau$, where $k\in\{d, f\}$.

$P_k(t,T)$ = the time *t* price of the k^{th} country's zero coupon bond (ZCB) paying one dollar at time *T*.

$r_k(t)$ = the k^{th} country's risk-free short rate at time *t*.

$\beta_k(t)$ = $\exp\left[\int_0^t r_k(u)\,du\right]$, the k^{th} country's money market account at time *t* with an initial value $\beta_k(0)=1$.

$X(t)$ = the spot exchange rate at $t\in[0,\tau]$ for one unit of foreign currency expressed in terms of domestic currency.

For some $\delta>0$, $T\in[0,\tau]$ and $k\in\{d,f\}$, define the forward LIBOR rate process $\{L_k(t,T);0\le t\le T\}$ as given by

$$1+\delta L_k(t,T)=\frac{P_k(t,T)}{P_k(t,T+\delta)}=\exp\left(\int_T^{T+\delta} f_k(t,u)\,du\right)$$

We present the results in the following Proposition.

Proposition 2.1 THE CLMM UNDER THE MARTINGALE MEASURE

Under the domestic spot martingale measure, the processes of the forward LIBOR rates and the exchange rate are expressed as follows:

$$\frac{dL_d(t,T)}{L_d(t,T)}=\gamma_{Ld}(t,T)\cdot\sigma_{Pd}(t,T+\delta)dt+\gamma_{Ld}(t,T)\cdot dW(t)$$

$$\frac{dL_f(t,T)}{L_f(t,T)}=\gamma_{Lf}(t,T)\cdot\left(\sigma_{Pf}(t,T+\delta)-\sigma_X(t)\right)dt+\gamma_{Lf}(t,T)\cdot dW(t)$$

$$\frac{dX(t)}{X(t)}=\left(r_d(t)-r_f(t)\right)dt+\sigma_X(t)\cdot dW(t)$$

where $t\in[0,T]$, $T\in[0,\tau]$.

The cross-currency LIBOR market model is very general. It is useful for pricing many kinds of quanto interest-rate derivatives. In Section 3, four variants of the cross-currency interest-rate exchange options are priced based on the CLMM.

3. Valuation of Quanto Interest-Rate Exchange Options

In this section, we derive the pricing formulae of four different types of quanto interest-rate exchange options (QIREOs) based on the cross-currency LIBOR market model. Introductions and analyses of each option are presented sequentially as follows.

3.1 Valuation of First-Type QIREOs

Definition 3.1 *A contingent claim with the payoff specified in (3.1.1) is called a First-Type QIREO* (Q_1IREO)

$$C_1(T)=N_d\omega\left[L_d^{\delta}(T,T)-L_f^{\eta}(T,T)\right]^+, \tag{3.1.1}$$

where

$L_d^{\delta}(T,T)$ = *the domestic T-matured LIBOR rates with a compounding period* δ

$L_f^{\eta}(T,T)$ = *the foreign T-matured LIBOR rates with a compounding period* η, $\eta \neq \delta$

N_d = *notional principal of the option, in units of domestic currency*

T = *the maturity date of the option*

$(x)^+$ = $Max(x,0)$

ω = *a binary operator (1 for a call option and -1 for a put option).*

Q_1IREO pricing is expressed in the following theorem, and the proof is provided in Appendix.

Theorem 3.1 *The pricing formula of Q_1IREOs with the final payoff as specified in (3.1.1) is expressed as follows:*

$$C_1(t) = \omega N_d P_d(t,T)\left[L_d^{\delta}(t,T)e^{\int_t^T \overline{\mu}_{1d}^{\delta}(u,T,T+\delta)du} N(\omega d_{11}) - L_f^{\eta}(t,T)e^{\int_t^T \overline{\mu}_{1f}^{\eta}(u,T,T+\eta)du} N(\omega d_{12})\right] \qquad (3.1.2)$$

where

$$d_{11} = \frac{\ln\left(\dfrac{L_d^{\delta}(t,T)}{L_f^{\eta}(t,T)}\right) + \int_t^T \left[\overline{\mu}_{1d}^{\delta}(u,T,T+\delta) - \overline{\mu}_{1f}^{\eta}(u,T,T+\eta)\right]du + \frac{1}{2}V_1^2}{V_1}$$

$$d_{12} = d_{11} - V_1$$

$$V_1^2 = \int_t^T \lVert \gamma_{Ld}^{\delta}(u,T) - \gamma_{Lf}^{\eta}(u,T) \rVert^2 \, du$$

$$\overline{\mu}_{1d}^{\delta}(t,T,T+\delta) = \gamma_{Ld}^{\delta}(t,T)\cdot\left[\overline{\sigma}_{P_d}^{s}(t,T+\delta) - \overline{\sigma}_{P_d}^{s}(t,T)\right]$$

$$\overline{\mu}_{1f}^{\eta}(t,T,T+\eta) = \gamma_{Lf}^{\eta}(t,T)\cdot\left[\overline{\sigma}_{P_f}^{s}(t,T+\eta) - \overline{\sigma}_{P_d}^{s}(t,T) - \sigma_x(t)\right]$$

$\omega = 1$ (a call) or -1 (a put).

Theorem 3.1 not only provides the pricing formula for the Q_1IREOs but also reveals a clue to the construction of a hedging (replicating) portfolio for the Q_1IREOs.

For hedging, we rewrite equation (3.1.2) as equation (3.1.3) (the proof is provided in Appendix) as follows

$$C_1(t) = \Delta_{1t}^{(1)}\left[P_d(t,T) - P_d(t,T+\delta)\right] - \Delta_{2t}^{(1)}\left[P_f(t,T) - P_f(t,T+\eta)\right], \qquad (3.1.3)$$

where

$$\Delta_{1t}^{(1)} = \omega N_d \left(1 + \delta L_d^{\delta}(t,T)\right)\frac{1}{\delta} N(\omega d_{11}) e^{\int_t^T \overline{\mu}_{1d}^{\delta}(u,T,T+\delta)du}$$

$$\Delta_{2t}^{(1)} = \omega N_d \left(1+\eta L_d^{\eta}(t,T)\right)\frac{1}{\eta} N\left(\omega d_{12}\right) QA_1\left(t,T+\eta\right)$$

$$QA_1\left(t,T+\eta\right) = \frac{P_d\left(t,T+\eta\right)}{P_f\left(t,T+\eta\right)}\rho_1\left(t,T\right)$$

$$\rho_1\left(t,T\right) = e^{\int_t^T \overline{\mu}_{1f}^{\eta}\left(u,T,T+\eta\right)du}.$$

Equation (3.1.3) serves as a guide to the formation of a hedging portfolio $H_t^{(1)}$ for an Q_1IREO. $H_t^{(1)}$ can be completed by a linear combination of four types of assets: holding long $\Delta_{1t}^{(1)}$ units of $P_d\left(t,T\right)$ and $\Delta_{2t}^{(1)}$ units of $P_f\left(t,T+\eta\right)$ and selling short $\Delta_{1t}^{(1)}$ units of $P_d\left(t,T+\delta\right)$ and $\Delta_{2t}^{(1)}$ units of $P_f\left(t,T\right)$.

3.2 Valuation of Second-Type QIREOs

Definition 3.2 *A contingent claim with the payoff as specified in (3.2.1) is called a Second-Type QIREO* (Q_2IREO)

$$C_2\left(T\right) = \overline{X} N_f \left[L_f^{\delta}\left(T,T\right) - L_f^{\eta}\left(T,T\right)\right]^{+}, \tag{3.2.1}$$

where

N_f = *notional principal of the option, in units of foreign currency*

$\overline{X}$ = *the fixed exchange rate expressed as the domestic currency value of one unit of foreign currency*.

The pricing formula of Q_2IREOs is expressed in Theorem 3.2 below and the proof is provided in Appendix.

Theorem 3.2 *The pricing formula of Q_2IREOs with the final payoff as specified in (3.2.1) is presented as follows:*

$$C_2\left(t\right) = \overline{X} N_f P_d\left(t,T\right)\left[L_f^{\delta}\left(t,T\right) e^{\int_t^T \overline{\mu}_{2f}^{\delta}\left(u,T,T+\delta\right)du} N\left(d_{21}\right) - L_f^{\eta}\left(t,T\right) e^{\int_t^T \overline{\mu}_{2f}^{\eta}\left(u,T,T+\eta\right)du} N\left(d_{22}\right)\right] \tag{3.2.2}$$

where

$$d_{21} = \frac{\ln\left(\dfrac{L_f^{\delta}\left(t,T\right)}{L_f^{\eta}\left(t,T\right)}\right) + \int_t^T \left[\overline{\mu}_{2f}^{\delta}\left(u,T,T+\delta\right) - \overline{\mu}_{2f}^{\eta}\left(u,T,T+\eta\right)\right]du + \frac{1}{2}V_2^2}{V_2}$$

$$d_{22} = d_{21} - V_2$$

$$V_2^2 = \int_t^T \left\|\gamma_{Lf}^{\delta}\left(u,T\right) - \gamma_{Lf}^{\eta}\left(u,T\right)\right\|^2 du$$

$$\overline{\mu}_{2f}^{*}\left(t,T,T+*\right) = \gamma_{Lf}^{*}\left(t,T\right)\cdot\left[\overline{\sigma}_{P_f}^{s}\left(t,T+*\right) - \overline{\sigma}_{P_d}^{s}\left(t,T\right) - \sigma_x\left(t\right)\right],\quad * \in \left\{\delta,\eta\right\}.$$

Once again, equation (3.2.2) can be written in terms of (3.2.3), and the proof is presented in Appendix.

$$C_2(t)=\Delta_{1t}^{(2)}\left[P_f(t,T)-P_f(t,T+\delta)\right]-\Delta_{2t}^{(2)}\left[P_f(t,T)-P_f(t,T+\eta)\right], \tag{3.2.3}$$

where

$$\Delta_{1t}^{(2)}=\overline{X}N_f\left(1+\delta L_d^{\delta}(t,T)\right)\frac{1}{\delta}N(d_{21})QA_2(t,T+\delta)$$

$$\Delta_{2t}^{(2)}=\overline{X}N_f\left(1+\eta L_d^{\eta}(t,T)\right)\frac{1}{\eta}N(d_{22})QA_2(t,T+\eta)$$

$$QA_2(t,T+*)=\frac{P_d(t,T+*)}{P_f(t,T+*)}\rho_2^*(t,T),\quad *\in\{\delta,\eta\}$$

$$\rho_2^*(t,T)=e^{\int_t^T\overline{\mu}_{2\,f}^{*}(u,T,T+*)du},\quad *\in\{\delta,\eta\}.$$

Equation (3.2.3) shows the composition of a hedging portfolio $H_t^{(2)}$ for an Q_2IREO: it holds long $\Delta_{1t}^{(2)}$ units of $P_f(t,T)$ and $\Delta_{2t}^{(2)}$ units of $P_f(t,T+\eta)$ and sells short $\Delta_{1t}^{(2)}$ units of $P_f(t,T+\delta)$ and $\Delta_{2t}^{(2)}$ units of $P_f(t,T)$.

3.3 Valuation of Third-Type QIREOs

Definition 3.3 *A contingent claim with the payoff as specified in (3.3.1) is called a Third-Type QIREO* (Q_3IREO)

$$C_3(T)=X(T)N_f\left[L_f^{\delta}(T,T)-L_f^{\eta}(T,T)\right]^+, \tag{3.3.1}$$

where

$X(T)$ = *the floating exchange rate expressed as the domestic currency value of one unit of foreign currency at time T.*

Theorem 3.3 *The pricing formula of Q_3IROs with the final payoff as expressed in (3.3.1) is presented as follows:*

$$C_3(t)=X(t)N_fP_f(t,T)\left[L_f^{\delta}(t,T)e^{\int_t^T\left[\overline{\mu}_{3\,f}^{\delta}(u,T,T+\delta)\right]du}N(d_{31})-L_f^{\eta}(t,T)e^{\int_t^T\left[\overline{\mu}_{3\,f}^{\eta}(u,T,T+\eta)\right]du}N(d_{32})\right] \tag{3.3.2}$$

where

$$d_{31}=\frac{\ln\left(\dfrac{L_f^{\delta}(t,T)}{L_f^{\eta}(t,T)}\right)+\int_t^T\left[\overline{\mu}_{3\,f}^{\delta}(u,T,T+\delta)-\overline{\mu}_{3\,f}^{\eta}(u,T,T+\eta)\right]du+\frac{1}{2}V_3^2}{V_3}$$

$$d_{32}=d_{31}-V_3$$

$$V_3^2=\int_t^T\left\Vert\gamma_{Lf}^{\delta}(u,T)-\gamma_{Lf}^{\eta}(u,T)\right\Vert^2du$$

$$\overline{\mu}_{3\,f}^{*}(t,T,T+*)=\gamma_{Lf}^{*}(t,T)\cdot\left[\overline{\sigma}_{P_f}^{s}(t,T+*)-\overline{\sigma}_{P_f}^{s}(t,T)\right],\quad *\in\{\delta,\eta\}.$$

Similarly, we rewrite (3.3.2) to obtain (3.3.3) as follows

$$C_3(t)=\Delta_{1t}^{(3)}\left[P_f(t,T)-P_f(t,T+\delta)\right]-\Delta_{2t}^{(3)}\left[P_f(t,T)-P_f(t,T+\eta)\right], \quad (3.3.3)$$

where

$$\Delta_{1t}^{(3)}=X(t)N_f\left(1+\delta L_f^{\delta}(t,T)\right)\frac{1}{\delta}e^{\int_t^{T-\delta}\overline{\mu}_{3f}^{\delta}(u,T,T+\delta)du}N(d_{31})$$

$$\Delta_{2t}^{(3)}=X(t)N_f\left(1+\eta L_f^{\eta}(t,T)\right)\frac{1}{\eta}e^{\int_t^{T-\eta}\overline{\mu}_{3f}^{\eta}(u,T,T+\eta)du}N(d_{32}).$$

Equation (3.3.3) also implies a composition for a hedging portfolio $H_t^{(3)}$ similar to that given in the previous theorems.

3.4 Valuation of Fourth-Type QIREOs

Definition 3.4 *A contingent claim with the payoff as specified in (3.4.1) is called a Fourth-Type QIREO* (Q_4IREO)

$$C_4(T)=\omega\left[X(T)N_fL_f^{\delta}(T,T)-N_dL_d^{\eta}(T,T)\right]^+. \quad (3.4.1)$$

ω = *a binary operator (1 for a call option and -1 for a put option).*

Theorem 3.4 below presents the pricing formula of an Q_4IREO. Its proof follows in a similar way as the previous options.

Theorem 3.4 *The pricing formula of Q_4IREOs with the final payoff as expressed in (3.4.1) is presented as follows:*

$$\begin{aligned}C_4(t)&=\omega X(t)N_fP_f(t,T)L_f^{\delta}(t,T)e^{\int_t^T\left[\overline{\mu}_{4g}^{\delta}(u,T,T+\delta)\right]du}N(\omega d_{41})\\&\quad-\omega N_dP_d(t,T)L_d^{\eta}(t,T)e^{\int_t^T\left[\overline{\mu}_{4d}^{\eta}(u,T,T+\eta)\right]du}N(\omega d_{42})\end{aligned} \quad (3.4.2)$$

where

$$d_{41}=\frac{\ln\left(\dfrac{X(t)N_fP_f(t,T)L_f^{\delta}(t,T)}{N_dP_d(t,T)L_d^{\eta}(t,T)}\right)+\int_t^T\left[\overline{\mu}_{4g}^{\delta}(u,T,T+\delta)-\overline{\mu}_{4d}^{\eta}(u,T,T+\eta)\right]du+\frac{1}{2}V_4^2}{V_4}$$

$$d_{42}=d_{41}-V_4$$

$$V_4^2=\int_t^T\|\gamma_g^{\delta}(u,T)-\gamma_{Ld}^{\eta}(u,T)\|^2\,du$$

$$\overline{\mu}_{4g}^{\delta}(t,T,T+\delta)=\gamma_{Lf}^{\delta}(t,T)\cdot\left[\overline{\sigma}_{P_f}^{s}(t,T+\delta)-\overline{\sigma}_{P_f}^{s}(t,T)\right]$$

$$\overline{\mu}_{4d}^{\eta}(t,T,T+\eta)=\gamma_{Ld}^{\eta}(t,T)\cdot\left[\overline{\sigma}_{P_d}^{s}(t,T+\eta)-\overline{\sigma}_{P_d}^{s}(t,T)\right].$$

In order to obtain a hedging portfolio, equation (3.4.2) is rewritten as equation (3.4.3).

$$C_4(t)=\Delta_{1t}^{(4)}\left[P_f(t,T)-P_f(t,T+\delta)\right]-\Delta_{2t}^{(4)}\left[P_d(t,T)-P_d(t,T+\eta)\right], \quad (3.4.3)$$

where

$$\Delta_{1t}^{(4)} = \omega X(t) N_f \left(1+\delta L_f^{\delta}(t,T)\right)\frac{1}{\delta} e^{\int_t^T \overline{\mu_{4g}^{\delta}}(u,T,T+\delta)du} N(\omega d_{41})$$

$$\Delta_{2t}^{(4)} = \omega N_d \left(1+\eta L_d^{\eta}(t,T)\right)\frac{1}{\eta} e^{\int_t^T \overline{\mu_{4d}^{\eta}}(u,T,T+\eta)du} N(\omega d_{42}).$$

Equation (3.4.3) shows the composition of a hedging portfolio $H_t^{(4)}$ for an Q_4IREO: holding long $\Delta_{1t}^{(4)}$ units of $P_f(t,T)$ and $\Delta_{2t}^{(4)}$ units of $P_d(t,T+\eta)$ and selling short $\Delta_{1t}^{(4)}$ units of $P_f(t,T+\delta)$ and $\Delta_{2t}^{(4)}$ units of $P_d(t,T)$.

In Section 4, we provide a calibration procedure and numerical examples showing the accuracy of the pricing formulae.

4. Calibration Procedure and Numerical Examples

In this section, we first provide a calibration procedure and then examine the accuracy of the pricing formula via a comparison with Monte Carlo simulation.

This subsection offers some practical examples that examine the accuracy of the pricing formulas as derived in the previous section and compare the results with Monte Carlo simulation. Based on actual market data, the 1-year and 3-year Q_2IREOs with $\delta=\eta=\frac{1}{2}$ year and $\omega=-1$ in Theorem 1 are priced at different semiannually dates, and the results are listed in Exhibits 3 and 4. The flat volatility of the exchange rate is assumed to be 20%. The notional value is assumed to be \$1. The simulation is based on 50,000 sample paths. Note that in the examples the domestic country is the U.S. and the foreign country is the U.K. By comparison to Monte Carlo simulation, the pricing formulas have been shown to be accurate and robust for the recent market data. The empirical examples associated with the other three theorems have also shown satisfactory accuracy.

Exhibit 1: The 1-Year Q_1IREO

Date	2006/1/2	2006/7/3	2007/1/1	2007/7/2
Thm 1	1.2683×10^{-3}	1.2802×10^{-3}	5.0714×10^{-3}	9.5540×10^{-3}
M.C.	1.2682×10^{-3}	1.2811×10^{-3}	5.0779×10^{-3}	9.5594×10^{-3}
s.e.	1.2812×10^{-5}	1.2560×10^{-5}	2.3516×10^{-5}	2.8212×10^{-5}

The prices of a 1-year Q_1IREO with semiannual accrual periods are presented in this exhibit. The abbreviations M.C. and s.e. represent the results of Monte Carlo simulations and their standard errors, respectively.

Exhibit 2: The 3-Year Q_1IREO

Date	2006/1/2	2006/7/3	2007/1/1	2007/7/2
Thm 1	4.0575×10^{-3}	3.1143×10^{-3}	5.6667×10^{-3}	7.3662×10^{-3}
M.C.	4.0546×10^{-3}	3.1229×10^{-3}	5.6588×10^{-3}	7.3560×10^{-3}
s.e.	7.4232×10^{-5}	6.3378×10^{-5}	8.5226×10^{-5}	9.9884×10^{-5}

The prices of a 3-year Q_1IREO with semiannual accrual periods are presented in this exhibit.

5. Conclusions

We have extended the single-currency LMM to the cross-currency LMM based on the Amin and Jarrow (1991) structure and then utilized the resulting model to price four different types of QIREOs with four theorems. The derived pricing formulae represent the general formulae of Margrabe (1978) in the framework of the cross-currency LMM and are familiar to practitioners for easy practical implementation. These pricing formulae have been shown to be very accurate as compared with Monte-Carlo simulation.

Moreover, we have provided the hedging strategies for the QIREOs via the pricing formulae and discussed the calibration procedure in detail. Since the LIBOR rate is market observable and its related derivatives, such as caps and swaptions, are actively traded in the markets, it is easier to calibrate these model parameters than with the instantaneous interest-rate models. Thus, the QIREO-pricing formulae derived under the cross-currency LIBOR market model are more tractable and feasible for practical implementation.

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計畫成果自評

We have completed the research program as the plan:

1. We derive the pricing formulae of QIREOs by adopting the martingale method and the cross-currency LIBOR market model.
2. The calibration of parameters and hedging strategies are also discussed in detail for practical implementation.
3. Monte Carlo simulation is provided to examine the accuracy of the QIREOs pricing formulae.
4. The research has been written in English and submitted to the journal listed in 『國科會管理一學門財務領域國際期刊分級排序專案計畫』as being a working paper.