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Pricing and Hedging European Energy Derivatives: A Case Study of WTI Oil Options

Chih-Chen Hsu**
Department of Finance, National Chung Cheng University

Shih-Kuei Lin
Department of Money and Banking, National Chengchi University

Ting-Fu Chen
Department of Money and Banking, National Chengchi University

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Abstract

This study extends the mean-reversion dynamic framework of (Pilipovic, *Energy risk: Valuing and managing energy derivatives*, 1997) and (Schwartz, The stochastic behavior of commodity prices: Implications for pricing and hedging, *Journal of Finance* **52**, 1997, 923) and focuses on developing a variety of continuous-time commodity-pricing and hedging models by analyzing the pricing and hedging errors found in an empirical investigation of options contracts on light sweet crude oil traded on the New York Mercantile Exchange. Thus, this study contributes to furthering the applicability of the models developed. The inclusion of the benchmark Black-Scholes pricing model generates systematic biases that are consistent with (Bakshi, Cao and Chen, *Handbook of Quantitative Finance and Risk Management*, 2010). The mean-reversion jump-diffusion and seasonality option-pricing model best describes the extreme price volatility experienced during a financial collapse, but the mean-reversion and seasonality option-pricing model offers the best pricing and hedging capability for other periods. The performances of hedging models are generally consistent with pricing errors.

Keywords Mean-reversion; Jump-diffusion; Seasonality; Systematic biases

JEL Classification: C52, G13

1. Introduction

With the rise and fall of international crude oil prices and the increasing need for hedging in the market, crude oil derivatives were rapidly developed to allow traders to avoid the volatility risks associated with energy prices. Most participants in

*Corresponding author: Chih-Chen Hsu, Department of Finance, College of Management, National Chung Cheng University, No.168, Sec. 1, University Rd., Min-Hsiung, Chai-yi County 621, Taiwan. Tel: +886-5-2720-411 ext. 34212, Fax: +886-5-2720-818, email: finccm@ccu.edu.tw.

the energy-commodity markets use futures and options to hedge against crude oil-related risks.

Understanding the price dynamics of crude oil is essential to developing proper pricing formulae and to executing an optimal hedging strategy. The pricing of the oil market is complex. The price of crude oil is affected by seasonal and geopolitical factors (such as war), in addition to changes in its demand and supply. Because crude oil prices are exposed to – and can be affected by – drastic structural changes caused by external shocks, the highest crude oil price from the last five years is five times higher than the lowest crude oil price during that same period.[1] Price volatility increases the risk to participants in the crude oil markets, which directly or indirectly destabilizes industry production and profit generation and, in turn, can lead to increased levels of anxiety throughout the entire global economy. The present study develops a variety of theoretical futures- and option-pricing formulae by considering these stylized factors affecting the price of crude oil. With the goal of constructing the best-fitting futures- and option-pricing models for crude oil market participants, this empirical investigation is conducted using actual market data.[2]

The option-pricing models proposed by Black and Scholes (1973) and Merton (1973) (BS models) were derived by relying on the assumption that the price dynamics of underlying assets are consistent with changes in the Brownian motion process. However, the characteristics of energy derivatives are not identical to financial market derivatives. Consequently, the option-pricing model proposed by Black and Scholes (1973) and Merton (1973) may not provide the most accurate pricing for crude oil derivatives.

Schwartz (1997) argues that the long-run dynamics of spot prices on the commodities futures market will tend to return to their long-run averages (i.e., the Mean-Reversion (MR) process), and applied a mean-reversion model in the exploration of futures and option pricings of commodities such as oil, copper, and gold. The results indicated that the price dynamics of commodities possess significant MR characteristics. Since then, many studies – including Miltersen and Schwartz (1998), Bjørk and Landen (2000), Jaillet *et al.* (2004), and Koekebakker and Lien (2004) – also employed the MR model in their studies in developing evaluation models for their research assets. Pilipovic (1997) analyzed the indexed options of the S&P 500 and energy prices (using price or log-price distributions) and found that the S&P 500 index option lognormal model corrects to the benchmark models, whereas various other energy commodities – such as West Texas Intermediate (WTI) crude oil (also known as light sweet crude oil on international markets), heating oil, and natural gas – are better treated by the MR pricing model.

[1]Data source: Futures Industry Association website.

[2]The dataset consists of the daily futures and option prices of West Texas Intermediate (WTI) crude oil that is also known as light sweet crude oil on the NYMEX from July 1, 2007 to April 30, 2012.

After observing the effects of seasonality in the energy market, Pilipovic (1997) found that energy commodity prices are related to the seasonal demand for energy. For example, the demand for heating oil is higher in winter than in summer; thus, the price of heating oil is comparatively higher in winter than in summer. Pilipovic (1997) added this characteristic to the MR model using crude oil and electricity prices as research data – assuming that seasonality is a trigonometric model – to explain the trend in the long-run volatility of the price. Weron (2006) and Mayer *et al.* (2011) believe that the time trends in long-run price changes should be included because adding the time trend as a seasonal factor allows seasonality to be more consistent with changes in reality.

Deng (1999) further combined both the commodity price characters of MR and jump diffusion into a mean-reversion jump-diffusion (MRJD) model to describe the price dynamics of energy – with electricity as an example – and applied a variety of different jump models to describe the unique spike phenomenon in electricity prices. In fact, as with changes in other energy prices, it is not difficult to show that dramatic changes in crude oil prices are frequently affected by external factors, including an increase or decrease in crude oil production, accidents in oil fields, and political wrangling or war involving oil-producing countries. These often result in drastic short-run fluctuations in crude oil prices. Cartea and Figueroa (2005) also applied the MRJD model to the energy market to analyze the dynamic process of spot price returns in the electricity market in the United Kingdom. The analysis of the distribution of the rate of return offered an explanation for the existence of jump phenomena in the market price for electricity. Thus, embedding jump risks into the MR model transforms it into a MRJD model for deriving the closed-form solution of the futures price.

From a financial perspective, crude oil is typically thought to be part of the commodities market. However, crude oil spot prices have certain characteristics that distinguish crude oil from most other financial products. In this study, we use an assortment of pricing models to explore the pricing and hedging performance of crude oil options contracts trading on the New York Mercantile Exchange (NYMEX);[3] these models are themselves based on a variety of continuous-time models of spot-price dynamics that have been successfully employed in other commodity markets, including the BS, MR, MRJD, MRS (mean-reversion and seasonality), and MRJDS (mean-reversion jump-diffusion and seasonality) pricing models. Moreover, we analyze the pricing errors between the theoretical and actual prices of WTI crude oil options in these five models with a moneyness-maturity categorization. A continuous-time framework has the substantial advantage of allowing closed-form valuation for options contracts. Furthermore, in exploring the performance of options in hedging, this study employs a dynamic delta-hedging portfolio that replicates options prices regardless of transaction costs and analyzes the effect of each evaluation model and of the number of days per delta adjustment on the performance of a particular hedging strategy.

[3]NYMEX merged into the CME group in 2008.

Our empirical investigation sampled the Bloomberg dataset of the WTI light sweet crude oil price, futures, and options that traded on the NYMEX between 1 July 2007 and 30 April 2012. To examine the fitness of the option-pricing formulae that are developed, the sample data are divided into the financial collapse period and the non-financial collapse period. The empirical results show that the MRJDS model best describes the extreme price dynamics that characterize a financial collapse. However, during other periods, the MRS model is more accurate because it generates a smaller pricing error. The performance of the MR series model was actually better than that of the Black-Scholes model, which indicates that the WTI crude oil price dynamic has MR characteristics. This conclusion is consistent with the findings by Schwartz (1997) that commodity price dynamics are more suited for the theory of the mean-reversion model. In an analysis of hedging performance, the pros and cons of the model are generally consistent with the pricing error. Regardless of which period was considered, the hedging error associated with adjusting delta for each 5-day period is approximately twice as large as that of the daily delta adjustment. Furthermore, the study also compared various models to show that a model to obtain a better price evaluation and hedging performance is best chosen based on spot characteristics at different periods; this is contrary to the position that a more complex model is better.

The contents of this paper are organized as follows. Section 2 introduces the assumptions of the theoretical models. Section 3 introduces the development of the model and hedging ratios. Section 4 conducts an empirical investigation. Section 5 draws conclusions.

2. Models and Hypotheses

This section reviews a variety of price dynamics and the model assumption for the further development of European option-pricing formulae and their hedging ratios.

2.1. Black-Scholes Model

The option-pricing model proposed by Black and Scholes (1973) hypothesizes a perfect market in which interest rate volatility remains constant and trades occur continuously. Under such a framework, the dynamic process of the spot price under a risk-neutral measure Q may be obtained through measurement conversion as:

$$\frac{\mathrm{d}S_t}{S_t} = r\mathrm{d}t + \sigma \mathrm{d}W_t^Q \tag{1}$$

where S_t is the spot price; σ is the instantaneous volatility of the spot price; W_t^Q is Geometric Brownian motion under a risk-neutral measure; and r is a risk-free rate, which indicates that the instantaneous rate of return of the spot price is equal to a risk-free rate under the risk-neutral Q measure.

The dynamic process of the spot price assumed by the Black-Scholes model can be employed to calculate the underlying asset price at a specific time in the future. However, observations of energy market characteristics show that energy prices typically revert to the long-run average price. This feature cannot be presented in the Black-Scholes model and the BS model may not be the best model for describing energy price dynamics.

2.2. Mean-Reversion (MR) Model

According to the single-factor model in Schwartz (1997), if commodity prices are assumed to possess MR characteristics, then, under the physical probability measure, P, the MR model for stock price dynamics is:

$$\mathrm{d}S_t = \alpha(m - \ln S_t)S_t\mathrm{d}t + \sigma S_t\mathrm{d}W_t^P \tag{2}$$

where S_t is the spot price; α represents the MR speed with which spot prices revert back to the long-run level, which is a constant that is always greater than 0; m is the long-run average parameter of the spot price, with long-run log-spot prices converging to this level; σ represents the volatility level of the instantaneous rate of return of the spot price; and W_t^P is Brownian motion under the physical probability measure.

To achieve the price dynamic under the risk-neutral measure, we follow the evaluation methods in Bjerksund and Ekern (1995) and Schwartz (1997) through the adjustment of the market price of risk. Subtracting the market price of risk from the spot price moving trend under the physical probability measure allows for the transition of the dynamic process of spot pricing to a risk-neutral measure:

$$\mathrm{d}S_t = \alpha(m - \eta - \ln S_t)S_t\mathrm{d}t + \sigma S_t\mathrm{d}W_t^Q \tag{3}$$

where η is the market price of risk. The log-spot price $Y_t = \ln S_t$ can be similarly expressed as follows:

$$\mathrm{d}Y_t = \alpha\left(m_{MR}^* - Y_t\right)\mathrm{d}t + \sigma\mathrm{d}W_t^Q \tag{4}$$

where $m_{MR}^* = m - \eta - \frac{\sigma^2}{2\alpha}$ is a long-run average parameter of the log-spot price under a risk-neutral measure.

Although Pilipovic (1997) notes that the MR model explains energy markets better than the Black-Scholes model, its explanatory ability is inadequate when the market incurs extreme price volatilities because of major events, such as policy changes in energy-producing countries or natural disasters that can cause prices to change substantially in the short-run. In such cases, the MR model does not provide good model fitness for the real price changes.

2.3. Mean-Reversion Jump-Diffusion (MRJD) Model

Merton (1976) divides stock price movements into two parts. First, there is the randomness of supply and demand that guides the market itself; second, there is the dramatic change in price that is caused by an important piece of market information that creates a jump phenomenon. If price-jump characteristics are embedded in the MR model, the result is a MRJD model.

Clewlow and Strickland (2000) hypothesized that the dynamic process of adding spot-price jumps to the MR model expressed under natural measures is as follows:

$$\mathrm{d}S_t = \alpha(m - \ln S_{t-})S_{t-}\mathrm{d}t + \sigma S_{t-}\mathrm{d}W_t^P + S_{t-}(e^J - 1)\mathrm{d}N_t \tag{5}$$

where S_t is the spot price at time t, and S_{t-} is the spot price at t - time. If no jump occurs in the underlying asset prices at time t, then S_t and S_{t-} are equal; J is the jump amplitude parameter – which follows a normal distribution $N(\theta, \delta^2)$ – that is used to describe changes in the price level during spot price jumps; and N_t is the Poisson process at jump intensity λ.

According to the MRJD hypothesis by Cartea and Figueroa (2005), price jumps are a part of dispersible non-systematic risk and are independent of Brownian motion for the underlying asset-price dynamics. Therefore, under a risk-neutral assumption, only the mean of the compound Poisson distribution, $\lambda\bar{J}$, must be directly deducted, which is composed of the jump intensity, λ, and the average jump magnitude, $\bar{J}, \bar{J} \equiv E^Q[e^J - 1] = (e^{\theta+\frac{1}{2}\delta^2} - 1)$; there is no additional risk premium. The deduction of the market price of risk from the long-run average allows for the transition of the dynamic process of the spot price from a physical-probability measure to a risk-neutral measure:

$$\mathrm{d}S_t = \alpha\left(m - \eta - \frac{\lambda\bar{J}}{\alpha} - \ln S_{t-}\right)S_{t-}\mathrm{d}t + \sigma S_{t-}\mathrm{d}W_t^Q + S_{t-}(e^J - 1)\mathrm{d}N_t \tag{6}$$

The log-spot price dynamic under the risk-neutral measure can be written as:

$$\mathrm{d}Y_t = \alpha\left(m^*_{\mathrm{MRJD}} - Y_t\right)\mathrm{d}t + \sigma\mathrm{d}W_t^Q + J\mathrm{d}N_t \tag{7}$$

where $m^*_{\mathrm{MRJD}} = m - \eta - \frac{1}{\alpha}\left(\frac{\sigma^2}{2} + \lambda\bar{J}\right)$.

This dynamic is used to characterize the process of price changes caused by the addition of jump risks into the MR model. Consistent with the jump-diffusion model proposed by Merton (1976), the jump events of the price are a non-systemic risk that is dispersible, and the distribution of the jump process is independent of the Brownian motion of the underlying assets. According to Dritschel and Protter (1999) and Jensen (1999), such a jump-diffusion model is constructed under the hypothesis of an incomplete market. According to the second fundamental theorem of financial economics, in a complete market there is one and only one risk-neutral measure. In an incomplete market, however, there may be more than one risk-neutral measure that can be determined. Different preferential conditions and hedging considerations may produce different risk-neutral equivalent measures. This study adheres to the logic of Bakshi *et al.* (1997), Hilliard and Reis (1999), and Koekebakker and Lien (2004), which estimate the market price of risk and the jump-diffusion parameters in obtaining the theoretical pricing of options implied under the risk-neutral measure.

2.4. Mean-Reversion and Seasonality (MRS) Model

An analysis of the energy and stock markets reveals that both markets are cyclical. For example, U.S. heating oil prices reach a relative peak each winter but remain relatively low during the spring and fall, and demand for different goods in different seasons will affect price changes, such as the cyclical nature of the stock market as it relates to taxes. Branch (1977), de Bondt and Thaler (1987), and Chen *et al.* (2007) conducted relevant studies on the cyclical nature of stock markets and collectively refer to these types of cyclical behaviors as seasonality. Pilipovic (1997) used a trigonometric function to express the impact of seasonality on markets as:

$$h_t = a_1 \cos(2\pi(t - a_3)) + a_2 \cos(4\pi(t - a_4)) \tag{8}$$

where h_t represents the seasonality factor; a_1 and a_2 are coefficients, representing the size of seasonality; a_3 is the annualized location parameter used to indicate seasonal peaks and troughs; and a_4 is the semi-annualized location parameter. Pilipovic used trigonometric functions to describe the cyclical nature of seasonality, but did not consider the time trend of the spot price. Weron (2006) and Mayer *et al.* (2011) further added a time trend to the seasonality model to obtain the mathematical expression as follows:

$$g_t = a_1 + a_2 t + a_3 \cos\left(\frac{2\pi(t - a_4)}{250}\right) \tag{9}$$

where a_1 is a constant; a_2 is the term of the time trend designed to capture the changing trend of energy prices over time; a_3 is the size of seasonality that describes the effect of seasonality on log-price; and a_4 represents the position of seasonality, which describes the peak and trough locations of energy prices during the year.

By adding seasonal factors to the MR model, the model becomes known as the mean-reversion model with seasonality. Assume that log-spot price Y_t is divided into the seasonality section g_t and a non-seasonality log-spot price section X_t under the MR model, the dynamic is as follows:

$$Y_t = g_t + X_t \tag{10}$$

$$\mathrm{d}X_t = \alpha\left(m - \frac{\sigma^2}{2\alpha} - X_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t^P \tag{11}$$

The spot price under a risk-neutral measure can be obtained by employing the technique of change measure:

$$\mathrm{d}Y_t = \mathrm{d}g_t + \mathrm{d}X_t = \alpha\left(m^*_{MRS} + \frac{1}{\alpha}\frac{\mathrm{d}g_t}{\mathrm{d}t} - X_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t^Q \tag{12}$$

Because $X_t = Y_t - g_t$, substituting this expression into equation (12) allows the dynamic-process MRS with seasonal effects in the MR model under the risk-neutral measure to be obtained:

$$\mathrm{d}Y_t = \alpha\left(m^*_{MRS} + g_t + \frac{1}{\alpha}\frac{\mathrm{d}g_t}{\mathrm{d}t} - Y_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t^Q \tag{13}$$

where $m^*_{MRS} = m - \eta - \frac{\sigma^2}{2\alpha}$ is the long-run parameter excluding seasonality. This process is comparable to adding seasonality to the MR model, as it relates to long-run equilibrium and time. In other words, the long-run equilibrium is no longer merely a constant term, but a variable that will change over time.

2.5. Mean-Reversion Jump-Diffusion and Seasonality (MRJDS) Model

If we assume that asset price movements obey the MR model while also taking into account the characteristics of price-jump behavior and seasonality, then the model becomes a mean-reversion jump-diffusion model with seasonality. The model assumptions are the same as for the MRS model. The difference is that the log-spot price X_t is an MR model that contains jump events. Therefore, the dynamic process of spot price in a risk-neutral measure can be expressed as:

$$\mathrm{d}Y_t = \alpha\left(m^*_{\mathrm{MRJDS}} + g_t + \frac{1}{\alpha}\frac{\mathrm{d}g_t}{\mathrm{d}t} - Y_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t^Q + J\mathrm{d}N_t \tag{14}$$

where $m^*_{\mathrm{MRJDS}} = m - \eta - \frac{1}{\alpha}\left(\frac{\sigma^2}{2} + \lambda\bar{J}\right)$; J is the jump amplitude parameter, which follows a normal distribution $N(\theta, \delta^2)$ and is used to describe the changing price situation during jumps in the spot price; and N_t is the Poisson process with jump intensity λ.

3. Futures- and Option-Pricing Formulae and Hedging Ratio

Beginning with the Black-Scholes model as a reference model, this section develops a variety of European energy option-pricing formulae and their delta hedges based on MR-related price dynamics that were introduced in the second section. The details of the derivation of each pricing and hedging formula may be found in the appendix.

3.1. Black-Scholes Model-Based Futures and Option-Pricing Formula

Following the Black-Scholes model, the futures price may be obtained by the expected future spot price under a risk-neutral measure, $F(t,T) = E^Q[S_T|\mathcal{F}_t]$, in which the $\mathcal{F}_t$ is a right continuous information at time t. Despite the revenues from storage cost, convenience yield, and asset-price changes, the rate of return of tradable assets should be a risk-free rate. Thus, the futures price is $F_{BS}(t, T) = S_t e^{r(T-t)}$; otherwise, there would be arbitrage opportunities.

The European call-option-pricing formula that can be obtained using Black-Scholes option-pricing process at time t is:

$$C_{BS}(t, T, K) = S_t N(d_1) - Ke^{-r(T-t)} N(d_2) \tag{15}$$

where

$$d_1 = \frac{\ln \frac{S_t}{K} + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \ d_2 = d_1 - \sigma\sqrt{T-t}$$

In the options market, issuers of options commonly use a delta-hedging ratio to calculate the value of a holding asset when the spot price changes. Mathematically, we write the delta hedge as follows:

$$\frac{\partial C_{BS}(t, T, K)}{\partial S_t} = N(d_1) \tag{16}$$

Equation (16) shows that the purchase of $N(d_1)$ units of an underlying asset can reduce the risk of a price change caused by one unit of option issued. In a subsequent empirical analysis, this study will employ replicating portfolio construction and dynamic hedging to monitor the performance of the delta hedging parameters of various models during the hedging of options.

3.2. Characteristic-Function and Futures-Pricing Formulae for Mean-Reversion-Related Models

The characteristic function for the log-spot price is given as follows:

$$f_{MR}(t, T, Y_t; \phi) \equiv E^Q[e^{i\phi Y_T} | Y_t = y] \tag{17}$$

where $t \le T$ and $i = \sqrt{-1}$. For the MR model and according to the price dynamic in equation (4), the characteristic function can be solved as equation (18) (refer to Appendix A for details):

$$f_{MR}(t, T, Y_t; \phi) = \exp\left\{ i\phi Y_t e^{-\alpha(T-t)} + m^*_{MR} i\phi\left(1 - e^{-\alpha(T-t)}\right) - \frac{\phi^2\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right)\right\} \tag{18}$$

The futures price of the underlying S_t follows the MR model with maturity T under the risk-neutral measure given by $f(t, T, Y_t; -i)$; that is:

$$F_{MR}(t, T) = S_t^{e^{-\alpha(T-t)}} \exp\left\{ m^*_{MR}\left(1 - e^{-\alpha(T-t)}\right) + \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right)\right\} \tag{19}$$

where $m^*_{MR} = m - \eta - \frac{\sigma^2}{2\alpha}$.

The characteristic function for the log-spot price-adopted MRJD model can be obtained by running equation (7) through the Ito-Doeblin formula, as in equation (20):

$$f_{\mathrm{MRJD}}(t,T,Y_t;\phi)=\exp\Big\{i\phi Y_t e^{-\alpha(T-t)}+m^*_{\mathrm{MRJD}}i\phi\Big(1-e^{-\alpha(T-t)}\Big)\\ -\frac{\phi^2\sigma^2}{4\alpha}\Big(1-e^{-2\alpha(T-t)}\Big)-\lambda\int_T^t\Big(e^{i\phi e^{-\alpha(T-s)}\theta+\frac{-1}{2}\phi^2e^{-2\alpha(T-s)}\delta^2}-1\Big)ds\Big\} \tag{20}$$

where $m^*_{\mathrm{MRJD}}=m-\eta-\frac{1}{\alpha}\left(\frac{\sigma^2}{2}+\lambda\bar{J}\right)$ is the long-run parameter of the spot price under a risk-neutral measure. The characteristic function of the MRJD model can be divided into two parts, with the first part generated by the MR character and the other part generated by the effect of the jump-diffusion factor.

The futures price is arrived at by substituting $\phi=-i$ in the characteristic function with $f_{MRJD}(t,\ T,\ Y_t;\ -i)$:

$$F_{\mathrm{MRJD}}(t,T)=S_t^{e^{-\alpha(T-t)}}\exp\Big\{m^*_{\mathrm{MRJD}}\Big(1-e^{-\alpha(T-t)}\Big)+\Big(\sigma^2\frac{1-e^{-2\alpha(T-t)}}{4\alpha}\Big)\\ +\lambda\int_t^T\Big(e^{e^{-\alpha(T-s)}\theta+\frac{\delta^2}{2}e^{-2\alpha(T-s)}}-1\Big)ds\Big\} \tag{21}$$

The log-spot price that follows the MRS model described in Section 2.4 can lead to the corresponding characteristic function by running equation (13) through the Ito-Doeblin formula, as in equation (22):

$$f_{MRS}(t,T,Y_t;\phi)=\\ \exp\Big\{i\phi\Big(g_T-g_te^{-\alpha(T-t)}\Big)+i\phi Y_te^{-\alpha(T-t)}+m^*_{MRS}i\phi\Big(1-e^{-\alpha(T-t)}\Big)-\frac{\phi^2\sigma^2}{4\alpha}\Big(1-e^{-2\alpha(T-t)}\Big)\Big\} \tag{22}$$

where Y_t is the log-spot price with the seasonality-styled fact; g_t is the seasonality factor; and $m^*_{MRS}=m-\eta-\frac{\sigma^2}{2\alpha}$ is the long-run average parameter of the spot price under a risk-neutral measure. Similarly, the futures price under the MRS model may be obtained by the characteristic function and represented as equation (23):

$$F_{MRS}(t,T)=\exp\Big\{g_T+(\ln S_t-g_t)e^{-\alpha(T-t)}+m^*_{MRS}\Big(1-e^{-\alpha(T-t)}\Big)+\frac{\sigma^2}{4\alpha}\Big(1-e^{-2\alpha(T-t)}\Big)\Big\} \tag{23}$$

The seasonality factor, g_t, is a non-random time-trend function. Thus, the MRS futures formula is similar to that based on the MR futures formula in equation (19). The only difference is whether the futures price is subject to an existing seasonal factor.

According to the MRJDS model described in Section 2.5, the characteristic function for the spot log-price dynamic following equation (14) can be derived as:

$$\begin{aligned} f_{\mathrm{MRJDS}}(t,T,Y_t;\phi) = \exp\Big\{ & i\phi\left(g_T - g_t e^{-\alpha(T-t)}\right) + i\varphi Y_t e^{-\alpha(T-t)} + m^*_{\mathrm{MRJD}} i\phi\left(1-e^{-\alpha(T-t)}\right) \\ & -\frac{\phi^2\sigma^2}{4\alpha}\left(1-e^{-2\alpha(T-t)}\right) - \lambda\int_T^t \left(e^{i\phi e^{-\alpha(T-s)}\theta + \frac{-1}{2}\phi^2 e^{-2\alpha(T-s)}\delta^2} - 1\right)ds \Big\} \end{aligned} \tag{24}$$

where $m^*_{\mathrm{MRJD}} = m - \eta - \frac{1}{\alpha}\left(\frac{\sigma^2}{2} + \lambda\bar{J}\right)$ is the long-run average parameter of the spot price under a risk-neutral measure. The corresponding MRJDS futures-price formula includes styled facts of seasonality and jump risks, and can be obtained as:

$$\begin{aligned} F_{\mathrm{MRJDS}}(t,T) = \exp\Big\{ & g_T + (\ln S_t - g_t)e^{-\alpha(T-t)} + m^*_{\mathrm{MRJDS}}\left(1-e^{-\alpha(T-t)}\right) \\ & + \frac{\sigma^2}{4\alpha}\left(1-e^{-2\alpha(T-t)}\right) + \lambda\int_t^T \left(e^{e^{-\alpha(T-s)}\theta + \frac{\delta^2}{2}e^{-2\alpha(T-s)}} - 1\right)ds \Big\} \end{aligned} \tag{25}$$

3.3. Mean-Reversion-Related Model-Based Option-Pricing Formula and Delta Hedge Ratio

To implement the option-pricing formula, we adopted the fast Fourier transform (FFT) to compute call options based on the characteristic function method. Following Carr and Madan (1999), let $s = \ln S$ be the log-spot price, $k = \ln K$ be the log-strike price, $q(t, s_t)$ denote the risk-neutral density of the log-spot price, and $C(t,T,k)$ represent the value of the call option. To ensure that the call option is an absolutely integrable function for the Fourier transform, Carr and Madan consider the modified call price $c(t,T,k)$ by multiplying the damping factor, $\exp(\omega k)$, with a proper real number ω. The modified call option is given by $c(t, T, k) = \exp(\omega k)\, C(t, T, k)$, where $C(t,T,k) = \int_k^\infty e^{-r(T-t)}(e^{s_T} - e^k)q(T,s_T)ds_T$. The damping factor allows the Fourier transform of the modified call option, $c(t,T,k)$, to exist:

$$\psi(t,T,\xi) = \int_{-\infty}^{\infty} e^{i\xi k} c(t,T,k)dk = \frac{e^{-r(T-t)} f(t,T,Y_t;\phi = \xi - (\omega+1)i)}{\omega^2 + \omega - \xi^2 + i\xi(2\omega+1)} \tag{26}$$

where $f(t,T,Y_t;\ \phi)$ is the characteristic function for the corresponding model in equations (18), (20), (22), and (24). Thus, the call option prices can be numerically obtained by the inverse transform:

$$C(t,T,k) = \frac{e^{-\omega k}}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi k}\psi(t,T,\xi)d\xi = \frac{e^{-\omega k}}{\pi}\mathrm{Re}\left(\int_0^\infty e^{-i\xi k}\psi(t,T,\xi)d\xi\right) \tag{27}$$

where $\mathrm{Re}(\cdot)$ denotes the real part of a complex number. Use the Trapezoid rule to integrate the call option price in equation (27), which is approximated as:

$$C(t,T,k) \approx \frac{e^{-\omega k}\eta}{2\pi}\operatorname{Re}\left(\psi(t,T,\xi_1) + e^{-i\xi_N k}\psi(t,T,\xi_N)\right) + \frac{e^{-\omega k}\eta}{\pi}\operatorname{Re}\left(\sum_{j=2}^{N-1} e^{-i\xi_j k}\psi(t,T,\xi_j)\right) \tag{28}$$

where η, the grid points, are the step size in equal length, $\xi_j = \eta\ (j-1)$; and N is the number of steps.

Given the option-pricing formula, the delta-hedging ratio may be obtained by applying partial differentiation to equation (28) with respect to the spot price S_t, as in (further details may be found in Appendix B):

$$\frac{\partial C(t,T,k)}{\partial S_t} = \frac{e^{-\omega k}\eta}{2\pi}\operatorname{Re}\left(\psi(t,T,\xi_1)\frac{i\varphi(\xi_1)e^{-\alpha(T-t)}}{S_t} + e^{-i\xi_N k}\psi(t,T,\xi_N)\frac{i\varphi(\xi_N)e^{-\alpha(T-t)}}{S_t}\right) + \frac{e^{-\omega k}\eta}{\pi}\operatorname{Re}\left(\sum_{j=2}^{N-1} e^{-i\xi_j k}\psi(t,T,\xi_j)\frac{i\varphi(\xi_j)e^{-\alpha(T-t)}}{S_t}\right) \tag{29}$$

where $\phi(\xi) = \xi_j - (\omega + 1)i$.

This section analytically derives a variety of formulae for European call options and delta-hedging ratios. The next section will empirically investigate the fitness of these developed models within different moneyness-maturity categories.

4. Empirical Analysis

4.1. Data Sampling

The NYMEX is currently the largest commodity futures exchange in which light sweet crude oil (WTI) futures and options are the world's most actively traded energy products. Based on FIA annual reports,[4] both WTI futures and WTI options led the transaction volume of energy futures and options contracts from 2007 to 2012. There are about 140 million WTI futures contracts and 32 million WTI options contracts (including American and European options) traded per year. Both contracts represent 1000 barrels, which is the standard quantity for physical crude oil delivery transactions: this provides an instantaneous price convergence between the physical and derivatives markets.

The WTI contract is the most liquid benchmark for the global price of crude oil, and these crude oil transactions are typically hedged by derivatives directly derived from the price risk of WTI contracts. This shows our investigating targets play an important role in managing risk and enable investors to make sound judgments in the energy sector worldwide.

The sampled data consist of daily WTI crude oil spot prices, futures and options prices from the Bloomberg dataset. The sampled period is from 1 July 2007 to 30

[4]Futures Industry Association (FIA) annual reports can be reached through the FIA official website: http://www.futuresindustry.org/volume-.asp

April 2012, which includes the financial collapse period from 1 July 2007 to 31 December 2008 and the non-financial collapse period from 1 January 2009 to 30 April 2012.

Trading in WTI options is fully automated. The exercise style of the WTI options is European; thus, contracts can be exercised only on their maturity dates. Moreover, liquidity is concentrated in the nearest expiration contract. A valuable feature of the Bloomberg dataset is that for each option price, C_n, we have a record of the contemporaneous spot WTI crude oil price, S_n. The combined time series $\{C_n, S_n\}$ is, therefore, synchronized.

To filter data, which is necessary for an empirical analysis, we refer to Kim (2009) and Rhee *et al.* (2012) to apply the following principles. We focus on the samples that are valid in one year. The last price of each options contract before 4:15 p.m. Central Time (CT) on each trading day are used in our empirical analysis. We exclude only the last transacted option in the sample if the same option was traded several times during any time period. Data are excluded when their time to maturity is less than 3 days and when their prices are less than $0.02.

The remaining live exercise data were collected each transaction day. Thus, we initiated effective investigative samples based on a range of strike prices from $20 to $160.[5] Second, based on the spot price on the maturity day of the contract, we added $3 and subtracted $3 sequentially so that the samples were effective for our investigation until the price reached an upper bound of $160 (through sequential addition) and a lower bound of $20 (through sequential subtraction). Finally, prices that did not meet the arbitrage restriction were excluded.

To explore an appropriate crude oil pricing model for use in different periods, the research period is divided into financial collapse and non-financial collapse periods, consistent with Campello *et al.* (2010). The sampled dataset from 1 July 2007 to 31 December 2008 is classified as the financial collapse period, and the sampled dataset between 1 January 2009 and 30 April 2012 is classified as the non-financial collapse period. During the financial collapse period, WTI crude oil prices followed the economic recession and recovery and thus experienced periods of huge volatility, as depicted by the spot prices in Figure 1. Additionally, the U.S. Treasury bill rates are selected as the risk-free rates (taken from the U.S. Treasury website).

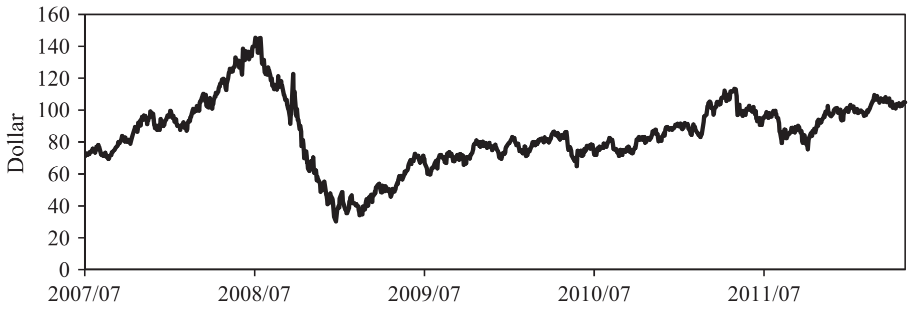
4.2. Statistics and Parameter Estimation Method

4.2.1. *Statistics*

This study further divides the options data into three categories based on the status of moneyness (S/K) consistent with the definition from Bakshi *et al.* (2010): when $0.95 < S/K \leq 1.05$, it is at the money (ATM) call options; when $S/K > 1.05$, it is in the money (ITM) call options; and when $S/K \leq 0.95$ it is out of the money (OTM) call options. In addition, the options are divided into the following five categories

[5]The historical spot WTI crude oil prices range from $20 to $160 over the period of our investigation.

Figure 1 WTI crude oil price.



The data of reported WTI crude oil price graph are sampled from the data set of Bloomberg. The sample period extends from 1 July 2007 to 30 April 2012.

based on different days to maturity: (1) less than 30 days left to maturity, extreme-short-run; (2) 30–60 days, short-run; (3) 60–120 days, near-term; (4) 120–180 days, middle-maturity; and (5) more than 180 days, long-run. Based on the above moneyness-maturity category, the entire sample is divided into 15 sub-categories.

Table 1 shows the statistics of European call-option prices on WTI options during the overall data period from 1 July 2007 to 30 April 2012, which describes the average call-option price and sample size based on the moneyness-maturity level category within the sample period. The total observations during this period are 246,162 call options. Among them, ITM and OTM call options accounted for 38% and 50%[6] of the total number of samples, respectively. Table 1 shows that the average price of OTM call options is 0.7830 in the extreme short-run; the average price of ITM call options is 27.8451 in the long-run.

4.2.2. Parameter Estimation

The parameter estimation employs the method of minimizing the sum of squared errors (minimize SSE) between actual and theoretical options and futures prices to estimate the parameters in an options evaluation study. The price information of futures for calibrating the model parameters are included because the WTI options are futures options. The spot dynamic models affect both the futures and options prices; therefore, they should be considered simultaneously for calibration. The parameters that are estimated in this study include volatility (σ), mean reverting speed (α), long-run average (m^*), jump amplitude parameters (θ and δ), jump intensive parameter (λ), and seasonality parameters (a_1, a_2, a_3, and a_4). The term Φ (σ, α, m^*, θ, δ, λ, a_1, a_2, a_3, a_4) is used to represent structure parameters that are estimated. This estimation method is one of option-implied volatility based on the model to minimize the sum of squared errors (minimized SSE) between the histori-

[6]These data are calculated by taking the sum of the observations in the moneyness (ITM, OTM) divided by the total observations individually.

Table 1 Sample properties of WTI options

The reported numbers are the average quoted bid-ask mid-point option price, and the total number of observation (in brackets), for each moneyness-maturity category. The sample period extends from 1 July 2007 to 30 April 2012 for a total of 246,162 calls. Daily information from the last quote (prior to 4:15 p.m. CT) of each option contract is used to obtain the summary statistics. S denotes the spot WTI price and K is the exercise price. OTM, ATM, and ITM denote out-of-the-money, at-the-money, and in-the-money options, respectively.

	Days to maturity (day)							
Moneyness		<30	30–60	60–90	90–120	120–180	180<	Sum of observations
OTM (S/K<0.95)	Aver. price	0.7830	1.2479	1.6595	1.9552	2.4505	3.4700	{122,215}
	Observations	{2700}	{7312}	{10,519}	{12,138}	{24,481}	{65,065}	
ATM (0.95<S/K<1.05)	Aver. price	3.1300	5.1214	6.5854	7.5593	8.9316	11.7288	{29,764}
	Observations	{2682}	{3019}	{3026}	{3057}	{5418}	{12,562}	
ITM (1.05<S/K)	Aver. price	24.1197	24.6479	24.8607	25.3658	25.7475	27.8451	{94,183}
	Observations	{10,195}	{10459}	{9985}	{9444}	{16,067}	{38,033}	
	Sum of observations	{15,577}	{20,790}	{23,530}	{24,639}	{45,966}	{115,660}	{246,162}

cal and theoretical prices of options and futures. The details of the parameter estimation process are as follows:

Step 1: We first screen out all the options on the same trading day, including options with different strike prices and different times to maturity. Let $\hat{C}(\Phi, t, T_i, K_j)$ represent the theoretical price of option and t denote the current time. T_i represents the ith option maturity date and K_j is the jth strike price; $C(t,T_i,K_j)$ represents the observed market price of option corresponding maturity, T_i, and the strike price, K_j. Similarly, define $F(t,T_i)$ as the market price of futures and $\hat{F}(\Phi, t, T_i)$ as the theoretical price of futures with ith maturity date. Let $\varepsilon_{i,j}^{C}(\Phi)$ and $\varepsilon_{i}^{F}(\Phi)$ be the difference between the theoretical price and the market price of options and futures, respectively, and they become the functions of the model parameters:

$$\varepsilon_{i,j}^{C}(\Phi) = C(t, T_i, K_j) - \hat{C}(\Phi, t, T_i, K_j) \tag{30}$$

$$\varepsilon_{i}^{F}(\Phi) = F(t, T_i) - \hat{F}(\Phi, t, T_i) \tag{31}$$

Step 2: We use the minimize SSE method to estimate the parameters of each model for all the options prices screened out on the same trading day. The mathematical expression for the minimize SSE estimation method is as follows:

$$\min_{\Phi} \sum_{i,j \in I} \left(\varepsilon_{i,j}^{C}(\Phi)^2 + \varepsilon_{i}^{F}(\Phi)^2 \right) \tag{32}$$

where I is the set of the total options and futures in the same selected trading day, i is the ith difference in maturity, and j is the jth difference in strike price. This step results in an estimate of the model parameters. We return to Step 1 until the two steps have been repeated for each day over the sample period; thus, we calculate the average value of various parameters within the observed periods.

To achieve a stable estimated result, both control mechanics are employed to stabilize the output of the parameter estimation in this study: one is the interaction numbers and the other is the termination tolerance. By employing MATLAB software to execute the calibration process (minimize SSE) through the built-in function "*lsqnonlin*", the parameters estimation process will be stopped when either the interactions reach setup times or the termination tolerance reaches the setup standard. In this study, we increase the iterations to 3000 times rather than the default value, which is 400 times. Furthermore, the termination tolerance on the objective function value is adapted to be 1e-10 rather than 1e-6, as the default to improve the stability of the estimation of the parameters.

The parameters of various models determined according to the estimation method described above are represented in Table 2 for the financial collapse period (1 July 2007 to 31 December 2008) and Table 3 for the non-financial collapse period (1 January 2009 to 30 April 2012). Table 2 shows the average daily model

Table 2 Implied parameters during financial collapse period

The structural parameters of a given model are estimated by minimizing the sum of squared pricing errors between the market price and the model-determined price for each option. The daily average of the estimated parameters is reported first, followed by its standard error of daily parameters in parentheses. BS, MR, MRJD, MRS, and MRJDS, respectively, stand for the Black-Scholes, the mean-reversion model, the mean-reversion with jump-diffusion model, the mean-reversion with seasonality model, and the mean-reversion with jump-diffusion and seasonality model.

	σ	α	m^*	λ	θ	δ	a_1	a_2	a_3	a_4
BS	0.2498	–	–	–	–	–	–	–	–	–
	(0.1800)									
MR	0.3542	0.5215	4.0077	–	–	–	–	–	–	–
	(0.1482)	(0.4197)	(1.4778)							
MRJD	0.2869	0.5870	3.5954	1.3776	0.3519	0.3119	–	–	–	–
	(0.1037)	(0.4909)	(1.8654)	(2.1794)	(0.4453)	(0.3302)				
MRS	0.3632	0.5957	1.8800	–	–	–	2.5913	0.0216	0.1838	83.9909
	(0.1472)	(0.3113)	(0.2932)				(0.2426)	(0.1414)	(0.2404)	(25.6482)
MRJDS	0.3243	0.7539	2.4719	0.7150	0.1103	0.5771	1.9393	0.0531	0.2588	87.9454
	(0.1212)	(0.4157)	(0.3802)	(1.5934)	(0.5572)	(0.3058)	(0.2235)	(0.0684)	(0.1183)	(28.5716)

Table 3 Implied parameters during non-financial collapse period

The structural parameters of a given model are estimated by minimizing the sum of squared pricing errors between the market price and the model-determined price for each option. The daily average of the estimated parameters is reported first, followed by its standard error of daily parameters in parentheses.

	σ	α	m^*	λ	θ	δ	a_1	a_2	a_3	a_4
BS	0.3810	–	–	–	–	–	–	–	–	–
	(0.1134)									
MR	0.3490	0.4842	4.4740	–	–	–	–	–	–	–
	(0.1085)	(0.3417)	(0.1985)							
MRJD	0.2821	0.4956	4.6586	0.5060	−0.2717	0.4093	–	–	–	–
	(0.1043)	(0.3675)	(0.2686)	(0.3862)	(0.3363)	(0.2200)				
MRS	0.3391	0.3998	2.4211	–	–	–	2.7366	−0.1267	1.0371	49.7879
	(0.1139)	(0.3613)	(1.0915)				(1.0072)	(0.2177)	(0.5532)	(53.9960)
MRJDS	0.2794	0.5775	1.1282	0.4890	−0.3524	0.4526	4.0946	0.0238	0.8367	112.6572
	(0.1057)	(0.3958)	(0.4158)	(0.2643)	(0.2406)	(0.1878)	(0.1731)	(0.1120)	(0.1195)	(32.2479)

parameters for each day estimated during the financial collapse period. The estimated BS model's average daily implied volatility is 24.98%; the MR model's average daily implied volatility is 35.42%; the MRJD model's average daily implied volatility is 28.69%; the MRS model's average daily implied volatility is 36.32%; and the MRJDS model's average daily implied volatility is 32.43%. The difference between the largest and the smallest estimated volatility is 11.34%. The estimated implied volatilities during the non-financial collapse period are shown in Table 3, and are as follows. The BS model's average daily implied volatility is 38.10%; the MR model's average daily implied volatility is 34.90%; the MRJD model's average daily implied volatility is 28.21%; the MRS model's average daily implied volatility is 33.91%; and the MRJDS model's implied estimated average daily volatility is 27.94%. The difference between the largest and the smallest estimated volatility is 10.16%.

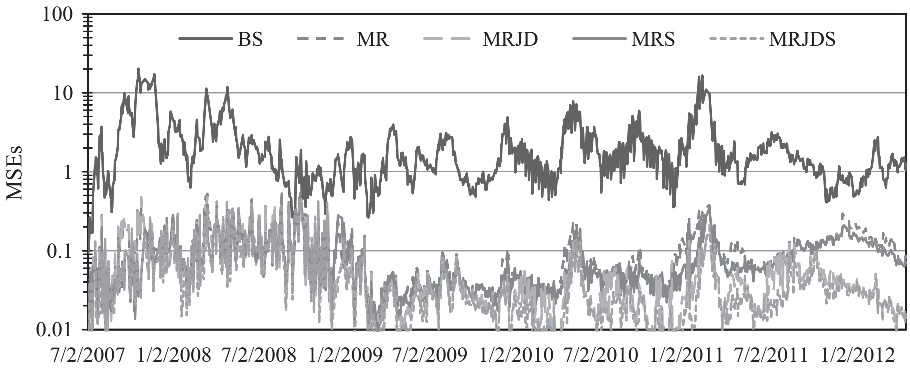
Analyzing the implied volatilities, the financial and non-financial collapse periods show that the volatilities estimated by the MRJD European energy-options model are smaller than those estimated by the MR European energy-options model. The likely reason for this difference is that the explanatory power of the model on volatility is distributed to the volatility of jump events once jump risks are considered, which is in the middle of the same extreme price fluctuation. Price jumps can be used to explain drastic changes in price instead of using volatility to explain all price changes. Similarly, comparison of the MRJDS European energy options estimated volatilities to the MRS European energy options estimated volatilities shows that the considered MRJDS volatility in the jump-diffusion model is smaller than that of the MRS model.

The decline of long-run average of MRS compared to MR is the result of the scattering of long-run average value by the intercept (a_1) caused by the effect of seasonality. The size of seasonality (a_3) is relatively higher in the non-financial collapse period than in the financial collapse period, which shows that seasonality is much more obvious after the financial collapse.

The estimated volatility parameter in the BS model is much higher during the period of non-financial collapse than during the period of financial collapse; this result is not intuitive. The explanation is that there is a steady growth trend of the sampled WTI crude oil spot price from July 2007 to July 2008, but the trend is reversed after that date. Although the price trend is reversed after July 2008, the financial collapse period ends on December 2008. Based on the defined financial collapse period (1 July 2007 to 31 December 2008), the volatility of sample data is diminished. This result is also consistent with Salisu and Fasanya (2012), who demonstrate that the standard derivation of the rate of return of WTI crude oil significantly increases during the period *ex post* financial collapse.

Figure 2 shows the daily in-sample mean square errors (MSEs) of the developed models adopted in this paper. From Figure 2, the MSEs of BS model are higher than MR-related models over the sample period. The observed majority of MSEs over MR-related models is less than 1 during the financial collapse period.

Figure 2 In-sample MSEs graph.



The tangled lines are daily in-sample mean square errors of BS, MR, MRJD, MRS, and MRJDS models from 1 July 2007 to 30 April 2012.

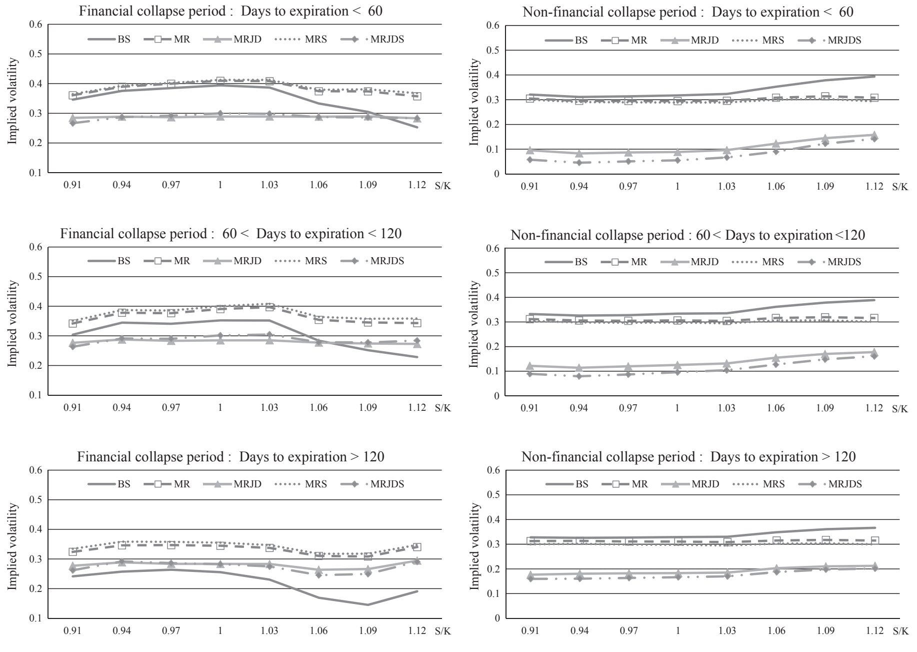
Moreover, it is obvious that the estimated result in the non-financial collapse period is stable and the observed majority of MSEs is less than 0.5. The results above demonstrate that the developed models and parameters are reliable and good to be employed to conduct further out-of-sample pricing and hedging performance analyses.

4.3. Implied Volatility Graphs

The spot implied volatility generates a number of diagnostic pieces of information that are useful in assessing the ability of the various dynamics to fit the market data. Following Rubinstein (1985) and Bakshi *et al.* (1997), we use the subsample data of moneyness (*S/K*), which ranges from 0.91 to 1.12 over the financial collapse period and non-financial collapse period, respectively, to compare the spot implied volatility patterns of each model across both moneyness and maturity categories. The basic procedure for backing out each model's implied-volatility series is as follows. First, we substitute the spot price and the interest rate of date t and the model parameters calibrated form the previous date $t-1$ except the spot volatility, such that the option price becomes the function of spot volatility. Thus, we can obtain the spot implied volatility by equating the market option price and the corresponding theoretical option price, which is the function of spot implied volatility.

Both subsamples, during the financial collapse period and during the non-financial collapse period, are tested, and the implied volatility graph is shown in Figure 3. The BS model is the most misspecified compared to the other models because implied volatility has the largest "smile" with significant skewness. In general, the patterns of MR and MRS are close to one another; in addition, the patterns of MRJD and MRJDS are close to one another in a moneyness-maturity category. The implied volatility of the MR and MRS models is relatively misspecified compared to that of the MRJD and the MRJDS models over the financial

Figure 3 Implied volatility graphs.



The left panels are implied volatility in the financial collapse period, from 1 July 2007 to 31 December 2008, and the right panels are the non-financial collapse period, from 1 January 2009 to 30 April 2012. The implied volatility graphs are respectively the BS, MR, MRJD, MRS, and MRJDS pricing formulae, for each moneyness-based category in different maturity category.

collapse period. Furthermore, the implied volatility of the MR and MRS models is persistently higher than the implied volatility of the MRJD and MRJDS models. These results show that the models that include jump diffusion diversify the spot implied-volatility in the Brownian term.

Regarding the diagnostic during the non-financial collapse period, the MR and the MRS models are less misspecified than the other models, which reveals that the model might have variant diagnostic information in different financial environments. Moreover, the implied volatility skewness of the BS model, the MRJD model, and the MRJDS model in short-run options is slightly more volatile than they are in the medium-run and long-run options. This result shows that the degree of implied volatility skewness is negative related with maturity.

4.4. Option-Pricing Error

The pricing error of options is the most commonly used index to examine the pricing performance of developed pricing formulae. The estimation processes of calculating pricing errors are shown as follows:

Estimate the first set of parameters from the sampled options data on the first day; these are used to calculate the theoretical option prices of the second day. Then, subtract the second day theoretical price from the second day's actual option price to obtain the value of the pricing error. Following the above process, the second day's actual data are used to estimate the second set of parameters, which is then used to calculate the third day's theoretical price. The third day's theoretical price and actual price are used to generate a second set of pricing errors. Mathematically, the aforementioned algorithm can be represented as follows:

$$\varepsilon(t, T_i, K_j) = C(t, T_i, K_j) - \hat{C}(\Phi_{t-1}, t, T_i, K_j) \tag{33}$$

where ε is the difference between the observed price and the theoretical price; $C(t, T_i, K_j)$ is the observed option price; and $\hat{C}(\Phi_{t-1}, t, T_i, K_j)$ is the theoretical option price calculated using the model parameters calibrated from the previous date.

Pricing error can be further computed as two indicators known as the relative error percentage and absolute error. Both indicators can be expressed as follows:

$$\text{Relative error percentage} = \frac{C(t, T_i, K_j) - \hat{C}(\Phi_{t-1}, t, T_i, K_j)}{C(t, T_i, K_j)} \tag{34}$$

$$\text{Absolute error} = \left| C(t, T_i, K_j) - \hat{C}(\Phi_{t-1}, t, T_i, K_j) \right| \tag{35}$$

To obtain the averaged and standard deviation of pricing errors in each interval, the daily pricing error is calculated first, then the number of days to maturity and the moneyness condition that one wishes to observe are screened and selected.

Based on the aforementioned algorithm, we are able to calculate the pricing errors of five developed pricing formulae with various moneyness statuses and times to maturity. The results are presented in Tables 4 and 5. For example, for OTM options with a maturity of 0–30 days over the financial collapse period, the absolute error values of the BS, MR, MRJD, MRS, and MRJDS models were \$0.5393, \$0.2584, \$0.2036, \$0.2151, and \$0.1813, respectively.

Table 4 reveals the complete analysis of absolute pricing errors during the financial collapse period. In the OTM scenario, The MRJDS pricing formula reveals the smallest absolute pricing error with 0–90 days time to maturity, but the MRJD pricing formula reveals the smallest absolute pricing error with more than 90 days time to maturity. In the ATM scenario, the MRJDS pricing formula reveals the smallest absolute pricing error with 0–90 days time to maturity, but the MRJD pricing formula reveals the smallest absolute pricing error with 90–120 days time to maturity, and with more than 180 days time to maturity.

In the ITM scenario, the MRJD pricing formula reveals the smallest absolute pricing error with 0–60 days time to maturity, but the MRJDS pricing formula reveals the smallest absolute pricing error with more than 60 days time to maturity. The analysis of absolute pricing error shows that, although the MRJDS pricing for-

Table 4 The mean and variance of the relative/absolute pricing errors during financial collapse period

For a given model we compute the price of each option using the previous day's implied parameters and implied stock volatility. The reported relative pricing error is the sample average of the difference between the market price and the model price for each call in a given moneyness-maturity category. The reported absolute pricing error is the sample average of the absolute difference between the market price and the model price for each call in a given moneyness-maturity category. The reported percent pricing error is the sample average of the market price minus the model price, divided by the market price. The results are obtained using the parameters implied by the previous day's options of a given moneyness (Out-, At-, or In-the-money; OTM, ATM, ITM) to price the current day's options of the same moneyness. The sample period is 1 July 2007–31 December 2008 call option prices. BS, MR, MRJD, MRS, and MRJDS, respectively, stand for the Black-Scholes, the mean-reversion, the mean-reversion with jump diffusion, the mean-reversion with seasonality, and the mean-reversion with jump diffusion and seasonality models.

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
Relative pricing error						
OTM						
BS	65.15% (0.2705)	62.14% (0.2789)	57.57% (0.2849)	50.43% (0.3207)	30.89% (0.3850)	−6.09% (0.6295)
MR	29.79% (0.3368)	23.92% (0.2594)	21.18% (0.2310)	13.91% (0.2085)	6.43% (0.1745)	3.85% (0.1468)
MRJD	16.00% (0.3263)	5.96% (0.2581)	5.09% (0.2252)	−0.65% (0.2048)	−1.65% (0.1480)	0.34% (0.1126)
MRS	24.85% (0.2945)	19.24% (0.2434)	16.44% (0.2255)	10.96% (0.2085)	5.94% (0.1694)	5.04% (0.1447)
MRJDS	10.51% (0.3226)	1.09% (0.2304)	−0.91% (0.2166)	−4.18% (0.2091)	−3.29% (0.1632)	1.16% (0.1184)
ATM						
BS	33.98% (0.2671)	32.15% (0.2114)	27.46% (0.1812)	21.86% (0.1667)	13.13% (0.1348)	−3.95% (0.1577)
MR	2.25% (0.1503)	2.37% (0.0806)	1.00% (0.0617)	−1.55% (0.0569)	−3.21% (0.0565)	−0.89% (0.0684)
MRJD	−1.72% (0.2121)	3.36% (0.0765)	1.76% (0.0611)	−1.03% (0.0582)	−2.68% (0.0574)	−0.59% (0.0668)
MRS	0.52% (0.1362)	2.07% (0.0683)	0.50% (0.0565)	−1.56% (0.0534)	−2.85% (0.0548)	−0.56% (0.0635)
MRJDS	−3.15% (0.2058)	3.21% (0.0672)	1.39% (0.0591)	−0.81% (0.0562)	−2.11% (0.0552)	−0.11% (0.0624)
ITM						
BS	0.17% (0.0399)	−0.32% (0.0638)	−2.87% (0.0926)	−5.55% (0.1107)	−9.27% (0.1292)	−15.45% (0.1821)
MR	0.49% (0.0269)	0.68% (0.0229)	0.21% (0.0279)	−0.44% (0.0321)	−0.84% (0.0368)	0.60% (0.0430)
MRJD	0.79% (0.0301)	0.79% (0.0254)	0.33% (0.0296)	−0.23% (0.0331)	−0.54% (0.0376)	0.89% (0.0436)
MRS	0.61% (0.0259)	0.82% (0.0217)	0.59% (0.0253)	0.04% (0.0286)	−0.33% (0.0353)	0.39% (0.0394)

Table 4 (*Continued*)

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
MRJDS	0.85% (0.0286)	0.89% (0.0242)	0.66% (0.0274)	0.25% (0.0300)	0.00% (0.0360)	0.68% (0.0418)
Absolute pricing error						
OTM						
BS	0.5393 (0.3980)	0.8088 (0.5752)	0.9585 (0.6482)	0.9202 (0.6879)	0.7677 (0.6024)	0.8948 (0.6895)
MR	0.2584 (0.2591)	0.2641 (0.2207)	0.2725 (0.2118)	0.2447 (0.2124)	0.2362 (0.2282)	0.3085 (0.3130)
MRJD	0.2036 (0.2077)	0.2033 (0.1916)	0.1980 (0.1949)	0.1800 (0.1986)	0.2027 (0.2109)	0.2776 (0.2967)
MRS	0.2151 (0.2373)	0.2149 (0.1934)	0.2314 (0.2016)	0.2292 (0.2241)	0.2292 (0.2361)	0.3209 (0.3234)
MRJDS	0.1813 (0.1962)	0.1668 (0.1806)	0.1734 (0.1883)	0.1843 (0.2148)	0.2081 (0.2343)	0.2985 (0.3308)
ATM						
BS	0.8789 (0.5036)	1.4540 (0.6219)	1.5466 (0.6594)	1.3118 (0.6762)	0.9043 (0.5827)	0.9829 (0.8769)
MR	0.2847 (0.2394)	0.3100 (0.2504)	0.2808 (0.2499)	0.2538 (0.2341)	0.3164 (0.2530)	0.4146 (0.3845)
MRJD	0.2831 (0.2542)	0.3081 (0.2521)	0.2873 (0.2584)	0.2513 (0.2388)	0.2994 (0.2555)	0.4079 (0.3866)
MRS	0.2441 (0.2161)	0.2670 (0.2461)	0.2672 (0.2696)	0.2618 (0.2569)	0.2952 (0.2770)	0.4183 (0.4266)
MRJDS	0.2367 (0.2081)	0.2524 (0.2257)	0.2465 (0.2460)	0.2545 (0.2423)	0.3125 (0.2677)	0.4114 (0.4006)
ITM						
BS	0.3217 (0.4957)	0.8171 (0.6510)	1.1891 (0.9114)	1.5139 (1.1774)	1.8895 (1.5665)	2.6305 (2.2403)
MR	0.3014 (0.2876)	0.3093 (0.2635)	0.3528 (0.3322)	0.3903 (0.3350)	0.4390 (0.3799)	0.6493 (0.6215)
MRJD	0.2590 (0.2770)	0.2950 (0.2503)	0.3357 (0.3241)	0.3870 (0.3233)	0.4379 (0.3655)	0.6428 (0.5972)
MRS	0.2964 (0.2749)	0.3073 (0.2687)	0.3485 (0.3395)	0.3914 (0.3540)	0.4501 (0.3980)	0.6642 (0.6334)
MRJDS	0.2621 (0.2603)	0.2993 (0.2557)	0.3296 (0.3208)	0.3674 (0.3275)	0.4301 (0.3693)	0.6232 (0.5969)

Table 5 The mean and variance of the relative/absolute pricing errors during non-financial collapse period

For a given model we compute the price of each option using the previous day's implied parameters and implied stock volatility. The reported relative pricing error is the sample average of the difference between the market price and the model price for each call in a given moneyness-maturity category. The reported absolute pricing error is the sample average of the absolute difference between the market price and the model price for each call in a given moneyness-maturity category. The reported percent pricing error is the sample average of the market price minus the model price, divided by the market price. The results are obtained using the parameters implied by the previous day's options of a given moneyness (Out-, At-, or In-the-money; OTM, ATM, ITM) to price the current day's options of the same moneyness. The sample period is 1 January 2009–30 April 2012 call option prices. BS, MR, MRJD, MRS, and MRJDS, respectively, stand for the Black-Scholes, the mean-reversion, the mean-reversion with jump diffusion, the mean-reversion model with seasonality, and the mean-reversion with jump diffusion and seasonality models.

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
Relative pricing error						
OTM						
BS	−31.15% (0.7581)	−30.30% (0.6779)	−30.80% (0.6187)	−32.58% (0.5681)	−40.06% (0.5793)	−59.04% (0.6718)
MR	−17.46% (0.4999)	−12.17% (0.3851)	−6.22% (0.3289)	−2.14% (0.2693)	1.04% (0.2322)	4.59% (0.1987)
MRJD	−13.39% (0.4850)	−12.63% (0.3561)	−7.99% (0.3067)	−3.54% (0.2486)	−0.36% (0.2181)	3.59% (0.1878)
MRS	−8.25% (0.4396)	−4.19% (0.3538)	−0.19% (0.3068)	2.16% (0.2503)	3.28% (0.2259)	3.81% (0.1951)
MRJDS	−0.69% (0.3390)	−7.70% (0.2366)	−8.41% (0.2091)	−5.18% (0.1843)	−1.73% (0.1661)	2.80% (0.1528)
ATM						
BS	−15.97% (0.2868)	−5.32% (0.1491)	−0.31% (0.1067)	2.09% (0.0819)	3.06% (0.0682)	1.07% (0.0645)
MR	−18.55% (0.2695)	−6.35% (0.0916)	−2.57% (0.0570)	−0.62% (0.0423)	0.59% (0.0357)	0.95% (0.0324)
MRJD	−22.60% (0.3671)	−4.80% (0.0911)	−1.58% (0.0540)	−0.10% (0.0395)	0.76% (0.0333)	0.91% (0.0313)
MRS	−14.91% (0.2314)	−4.12% (0.0766)	−1.22% (0.0518)	0.08% (0.0400)	0.72% (0.0355)	0.81% (0.0325)
MRJDS	−15.60% (0.3146)	−0.27% (0.0566)	1.05% (0.0401)	1.24% (0.0345)	1.07% (0.0319)	0.19% (0.0320)
ITM						
BS	1.32% (0.0294)	4.21% (0.0430)	4.09% (1.5583)	7.09% (0.0444)	7.99% (0.0442)	7.80% (0.0493)
MR	−0.43% (0.0248)	0.15% (0.0209)	−1.28% (1.6277)	0.93% (0.0203)	1.11% (0.0211)	0.56% (0.0243)
MRJD	−0.34% (0.0265)	−0.04% (0.0204)	−1.52% (1.6231)	0.67% (0.0199)	0.88% (0.0206)	0.53% (0.0233)
MRS	−0.46% (0.0245)	0.05% (0.0213)	−1.48% (1.6305)	0.65% (0.0202)	0.74% (0.0212)	0.64% (0.0227)

Table 5 (*Continued*)

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
MRJDS	−0.55% (0.0270)	−0.51% (0.0207)	−2.06% (1.6301)	0.13% (0.0175)	0.34% (0.0180)	0.32% (0.0205)
Absolute pricing error						
OTM						
BS	0.3051 (0.2421)	0.4024 (0.3394)	0.4478 (0.3906)	0.4659 (0.3954)	0.5490 (0.4447)	1.0002 (0.6830)
MR	0.2024 (0.1822)	0.2176 (0.2162)	0.2022 (0.2117)	0.1814 (0.1981)	0.1782 (0.1906)	0.2073 (0.2041)
MRJD	0.1842 (0.1570)	0.1966 (0.1773)	0.1992 (0.1820)	0.1854 (0.1677)	0.1951 (0.1788)	0.2251 (0.2015)
MRS	0.1357 (0.1324)	0.1260 (0.1385)	0.1243 (0.1434)	0.1236 (0.1374)	0.1335 (0.1570)	0.1773 (0.1941)
MRJDS	0.1928 (0.1735)	0.2106 (0.2004)	0.2057 (0.1982)	0.1881 (0.1827)	0.1886 (0.1810)	0.2127 (0.1994)
ATM						
BS	0.3692 (0.2797)	0.4866 (0.4020)	0.5133 (0.4808)	0.5204 (0.4958)	0.5746 (0.5427)	0.6284 (0.5401)
MR	0.3097 (0.2526)	0.2965 (0.2598)	0.2481 (0.2405)	0.2233 (0.2199)	0.2364 (0.2279)	0.2968 (0.2892)
MRJD	0.3631 (0.2615)	0.2973 (0.2503)	0.2501 (0.2314)	0.2347 (0.2221)	0.2647 (0.2331)	0.3076 (0.2894)
MRS	0.2665 (0.2097)	0.1894 (0.1931)	0.1932 (0.2132)	0.2117 (0.2071)	0.2391 (0.2249)	0.2828 (0.2852)
MRJDS	0.2796 (0.2088)	0.2709 (0.2180)	0.2467 (0.2223)	0.2249 (0.2042)	0.2487 (0.2189)	0.2945 (0.2838)
ITM						
BS	0.2888 (0.4424)	0.8788 (0.8188)	1.3089 (1.0400)	1.6627 (1.1495)	1.9809 (1.2119)	2.1686 (1.4980)
MR	0.2448 (0.2320)	0.2721 (0.2834)	0.3396 (0.5191)	0.3728 (0.3173)	0.4040 (0.3161)	0.4902 (0.4319)
MRJD	0.2670 (0.2429)	0.2788 (0.2710)	0.3196 (0.5103)	0.3473 (0.3083)	0.3770 (0.3103)	0.4690 (0.4228)
MRS	0.2886 (0.2509)	0.2806 (0.2616)	0.2809 (0.4939)	0.2691 (0.2872)	0.2932 (0.2938)	0.3769 (0.3756)
MRJDS	0.2463 (0.2327)	0.2779 (0.2789)	0.3139 (0.5119)	0.3277 (0.2959)	0.3667 (0.3022)	0.4537 (0.3814)

mula does not have the smallest absolute pricing error, its pricing error is close to the smallest for each period. Taking OTM options as an example, the absolute pricing error of the MRJDS pricing formula is larger than the MRJD absolute pricing error by approximately 0.005 in the 90- to 180-day time period. However, over the time period of 0–90 days, the absolute pricing error of the MRJDS pricing formula is smaller than the MRJD absolute pricing error, 0.0223 to 0.0365, respectively. Therefore, the study infers that the MRJDS pricing formula has the best overall results because it has the smallest pricing error during the financial collapse period.

Table 5 shows the complete analysis of absolute pricing errors during the non-financial collapse period. In the OTM scenario, the MRS pricing formula reveals the smallest absolute pricing error regardless of the time period. In the ATM scenario, the MRS pricing formula reveals the smallest absolute pricing error with 0–120 days time to maturity and more than 180 days to maturity, but the MR pricing formula reveals the smallest absolute pricing error with 120–180 days time to maturity. In the ITM scenario, the MR pricing formula reveals the smallest absolute pricing error with 0–60 days time to maturity, but the MRS pricing formula reveals the smallest absolute pricing error within the more than 60 days time to maturity. The observations from the non-financial collapse period shows that the MR and MRS pricing formulae without embedding jump events had smaller pricing errors, which is straightforward and consistent with common sense. Overall, during the non-financial collapse period, the MRS pricing formula generated better evaluation results with smaller pricing errors.

Table 5 also shows the relative pricing errors during the non-financial collapse period. In the OTM scenario, the pricing errors of Black-Scholes are negative, which indicates that OTM options market prices are lower than the theoretical prices, regardless of the time period. In the ATM scenario, the pricing errors are increasingly smaller than the OTM options. In the ITM scenario, the pricing error values are positive, which indicates that ITM options market prices are higher than the theoretical prices. Bakshi *et al.* (2010) define the aforementioned phenomena as systematic biases. Observing the systematic biases, we find that the MR and MR-related pricing formulae have much smaller biases than the BS model. This observation indicates that the MR and MR-related pricing formulae reveal a better fitness for the European energy options markets. The results coincide with Schwartz (1997) who found that changes in energy prices have an obvious MR-characteristic component. Bakshi *et al.* (2010) believe that the phenomenon of systematic biases in the stock market is related to the options volatility smile. Volatility smile is a common phenomenon in the options market; it indicates that the volatilities of OTM options are likely to be underestimated. At this point, the OTM market price is lower than the theoretical price whereas volatilities of ITM options are likely to be overestimated. Therefore, the market price is higher than the theoretical price.

This study finds that relative positive and negative pricing errors are consistent with those addressed previously in the non-financial collapse period, as shown in Table 5. These results demonstrate that systematic biases in the crude oil market during the non-financial collapse period coincide with similar findings in the stock market. Table 4 shows that the situation with systematic biases during the financial collapse period is the opposite of the normal market period, in which the OTM option-pricing error is positive and the ITM option-pricing error is negative. This study speculates that this phenomenon is the result of large price variations during the financial collapse period, which caused market investors to believe that there was a likelihood that an OTM call might morph into an ITM call. As a consequence, during this period, OTM option prices became even more likely to be overestimated.

4.5. The Impact of Mean-Reversion and Seasonality Characteristics

Figure 4 shows the performance of mean-reversion and seasonality characters, improving the absolute pricing error with respect to the BS model. BS-MR represents the amount of improvement in the MR model's absolute pricing error compared to the BS model, and BS-MRS denotes the value of the BS model absolute pricing error that is in excess of the MRS model. The graph indicates that mean-reversion and seasonality effects are strictly positively correlated with days to maturity in the non-financial collapse period regardless of the moneyness category. Thus, the developed MR and MRS pricing formula reveals better long-run pricing performance than in the short-run compared with the benchmark pricing formula. However, over the financial collapse period, the MR and MRS reveals better pricing performances in the ITM category but not in the OTM and ATM categories, particularly for the periods that are more than 90 days to maturity, which indicates that, during the financial collapse period, there are other dominant price factors that are causing the characters of the model to violate its usual tendencies, such as jump risks.

4.6. Dynamic Hedging Error

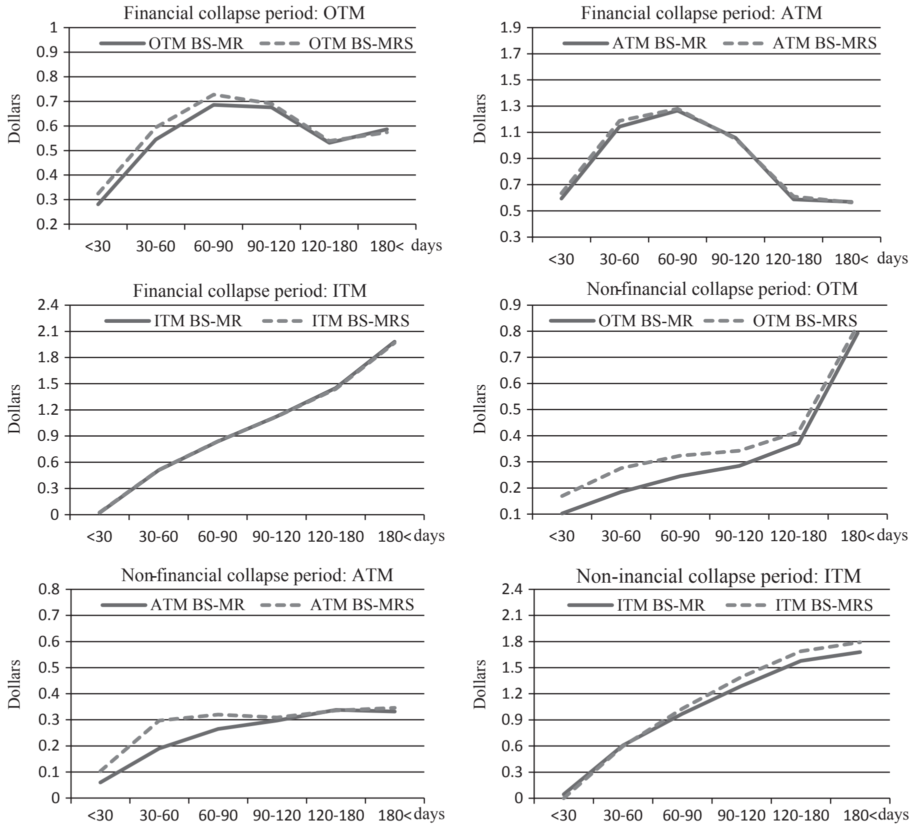
This study refers to hedge methods found in Bakshi *et al.* (2010) for constructing a self-financed hedging error – which contains spot pricing and cash positions – and for observing the dynamic hedging error from the model's delta-hedging ratio with per-day and per-five-day adjustments. The mathematical expression is as follows:

$$\Pi_t = X_t + Delta(t)S_t \tag{36}$$

where Π_t represents the replicating portfolio value at time t, which is used to replicate the options market price; S_t represents the spot price at time t; $Delta\ (t)$ is the delta hedging ratio at time t; and X_t is the cash position, which is the difference between the options market value and $Delta\ (t)$ units of the underlying asset price. If $X_t < 0$, this can be regarded as a cash lend out at a risk-free rate of r. If $X_t > 0$, this can be regarded as cash deposited in a bank at a risk-free rate of r. The algorithm for calculating dynamic hedging error in replicating portfolio is as follows:

First, screen out the options data for a specific data time series. Beginning from the first day of trading, t, calculate, in sequence, the period delta at time t and

Figure 4 The impact of mean-reversion and seasonality characteristics.



The left panels are the absolute pricing error of BS model minus the absolute pricing error of MR model and MRS model respectively for various maturities in the financial collapse period, from 1 July 2007 to 31 December 2008 and the right panels are the non-financial collapse period, from 1 January 2009 to 30 April 2012. The reported pricing error graphs from top to bottom are OTM, ATM, and ITM options respectively.

substitute it into the replicating portfolio. Moreover, the replicating portfolio value should be equal to the value calculated by equation (36).
When reaching the time $t + \Delta t$, the value of the replicating portfolio is:

$$\Pi_{t+\Delta t} = X_t e^{r\Delta t} + Delta(t) S_{t+\Delta t} \quad (37)$$

The price difference generated by hedging is expressed as:

$$H_{t+\Delta t} = C(t+\Delta t, T, K) - \left(X_t e^{r\Delta t} + Delta(t) S_{t+\Delta t}\right) \quad (38)$$

Repeat the procedure of replicating portfolio until each maturity date and strike price of the options have been calculated. By using the every daily hedging error, we are then able to calculate the mean and standard deviation of the hedging errors of all the sample data with different statuses of moneyness and different days to maturity.

This study used an average absolute hedging error for the average pricing error, which takes the absolute value of the error value before taking the average. Tables 6 and 7 show the results of delta-neutral hedging errors under the per-day ($\Delta t = 1$) adjusted delta and per-five-day ($\Delta t = 5$) adjusted delta of each model's replicating portfolio during the financial and non-financial collapse periods. Without taking into account transaction costs or observing the same models, moneyness and maturity dates, the hedging error from the per-day adjusted delta is smaller than that of the per-five-day adjusted delta. The performances of the hedging models are generally consistent with that of pricing errors. During the financial collapse period, the option-pricing formulae with jump risks performed better, whereas the options-pricing formulae without jump risks performed better during the non-financial collapse period. If these periods are separately compared, we find that – regardless of per-day or per-five-day adjusted hedging parameters – errors during the financial collapse period are larger than errors during the non-financial collapse period. On average, the hedging error of the every-five-day adjusted delta was twice as large as that of the per-day adjustment. This result allows market hedgers to select the parameters during an adjustment period that enhance risk-management efficiency by smart allocation of funds.

5. Conclusion

With recent increases in energy-price variation, there has been an increasing demand in the market to avoid price risk, which has been accompanied by an increase in trading volumes for energy derivatives. The choice of an appropriate model for calculating the price of derivatives depends in large part on the investment and hedging performance of market participants. The literature indicates that long-term energy prices are characterized by reversion to their averages; therefore, energy price dynamics can be explained by using the MR model. When significant events occur in the market, jump events are added to the MR model to capture the short-term commodity price change phenomenon. Finally, considering that commodity prices change regularly over time, seasonality is added to capture such price characteristics.

Based on these considerations, this study integrated five pricing models – the Black-Scholes, MR, MRJD, MRS, and MRJDS European option-pricing formulae – to derive the closed-form options solution under each model assumption. The study used these five models to analyze WTI crude oil options according to different moneyness levels, maturity dates, and pricing errors between theoretical and actual prices. The results show that, with the extreme price variation that occurred during the financial collapse period, the MRJDS model turned out to be the best fit. Pricing errors generated by the MRJDS model during this period were smaller than those of the other four models. However, during the non-financial collapse period, the MRS model generated pricing errors that were smaller, which indicates that it was sufficient to merely consider MR characteristics and seasonality because price volatilities were not as severe during this period.

Table 6 Delta-neutral hedging errors during financial collapse period

In calculating the hedging errors generated with daily hedge rebalancing, the all-adjust-absolute-delta hedges of calls use as many hedging instruments as there are sources of risk (except the jump risk) assumed in a given option model. The only exception is the BS delta-neutral strategy, which uses the underlying asset and a second call option to neutralize the delta risks of the target call, based on the Black-Scholes model. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedges, which are then liquidated the following day or five days later. For each target call option, its hedging error is, as of the liquidation day, the difference between its market price and the replicating portfolio value. The average absolute hedging error and the average dollar hedging error are reported for each model and for each moneyness-maturity category. The sample period is 1 July 2007–31December 2008 call option prices. BS, MR, MRJD, MRS, and MRJDS, respectively, stand for the Black-Scholes, the mean-reversion, the mean-reversion with jump diffusion, the mean-reversion with seasonality, and the mean-reversion with jump diffusion and seasonality models.

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
Absolute hedging error (per day)						
OTM						
BS	0.3999 (0.4806)	0.3641 (0.4575)	0.3788 (0.4602)	0.3347 (0.3980)	0.3078 (0.3348)	0.3279 (0.3955)
MR	0.1922 (0.3106)	0.1349 (0.1719)	0.1767 (0.2759)	0.1639 (0.2488)	0.1949 (0.2848)	0.2597 (0.3410)
MRJD	0.1869 (0.2760)	0.1392 (0.1796)	0.1803 (0.2669)	0.1662 (0.2390)	0.1962 (0.2691)	0.2705 (0.3427)
MRS	0.1921 (0.2955)	0.1396 (0.1834)	0.1753 (0.2377)	0.1620 (0.2155)	0.1857 (0.2251)	0.2785 (0.3383)
MRJDS	0.1945 (0.3066)	0.1333 (0.1706)	0.1630 (0.2216)	0.1518 (0.1978)	0.1756 (0.2094)	0.2509 (0.3021)
ATM						
BS	0.8523 (1.2592)	0.6851 (0.6162)	0.7413 (1.0778)	0.7414 (1.0063)	0.7575 (1.2086)	0.7666 (0.7542)
MR	0.3249 (0.6762)	0.2296 (0.2485)	0.2871 (0.5378)	0.2762 (0.4887)	0.2945 (0.4961)	0.3662 (0.4209)
MRJD	0.2959 (0.5124)	0.2349 (0.2557)	0.2904 (0.4759)	0.2822 (0.4566)	0.3007 (0.4644)	0.3810 (0.4315)
MRS	0.3072 (0.5586)	0.2348 (0.2610)	0.2766 (0.3741)	0.2533 (0.3503)	0.2718 (0.3718)	0.3700 (0.4063)
MRJDS	0.2906 (0.4659)	0.2257 (0.2475)	0.2663 (0.3762)	0.2721 (0.3653)	0.2902 (0.3840)	0.4103 (0.4529)
ITM						
BS	0.3145 (1.1106)	0.2967 (0.3566)	0.4740 (1.0524)	0.5333 (0.9217)	0.7180 (1.3889)	0.9826 (2.1806)
MR	0.2668 (0.9416)	0.2349 (0.2431)	0.3360 (0.6948)	0.3675 (0.5075)	0.4684 (0.6836)	0.6651 (0.9130)
MRJD	0.2721 (1.0010)	0.2422 (0.2493)	0.3455 (0.7113)	0.3780 (0.4873)	0.4830 (0.6800)	0.6896 (0.9327)

Table 6 (*Continued*)

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
MRS	0.2447 (0.7279)	0.2467 (0.2554)	0.3245 (0.4185)	0.3460 (0.3587)	0.4459 (0.5342)	0.6613 (0.8536)
MRJDS	0.2445 (0.6960)	0.2338 (0.2464)	0.3102 (0.4129)	0.3773 (0.3957)	0.4851 (0.5723)	0.7330 (0.9295)
Absolute hedging error (5-days)						
OTM						
BS	0.8876 (1.0385)	0.8294 (0.8832)	0.9111 (0.9228)	0.8634 (0.8848)	0.8207 (0.8088)	0.7905 (0.8366)
MR	0.4867 (0.5546)	0.3659 (0.4117)	0.4292 (0.5077)	0.4205 (0.4807)	0.4605 (0.5152)	0.6600 (0.7231)
MRJD	0.4769 (0.5667)	0.3710 (0.4163)	0.4387 (0.5055)	0.4322 (0.4798)	0.4713 (0.5123)	0.6858 (0.7485)
MRS	0.4900 (0.5521)	0.3669 (0.4146)	0.4218 (0.4785)	0.4211 (0.4670)	0.4511 (0.4952)	0.6504 (0.7251)
MRJDS	0.4741 (0.5629)	0.3611 (0.4045)	0.4111 (0.4536)	0.3986 (0.4216)	0.4160 (0.4442)	0.7196 (0.7941)
ATM						
BS	1.8637 (1.8270)	1.5344 (1.3881)	1.6148 (1.6794)	1.5843 (1.6299)	1.6195 (1.8529)	1.6366 (1.7542)
MR	0.7364 (0.9527)	0.6143 (0.6119)	0.7184 (0.7976)	0.7207 (0.7653)	0.7615 (0.8232)	1.0581 (1.0172)
MRJD	0.6829 (0.7350)	0.6222 (0.6332)	0.7322 (0.7588)	0.7399 (0.7647)	0.7853 (0.8115)	1.0916 (1.0492)
MRS	0.7056 (0.7711)	0.6178 (0.6116)	0.7144 (0.6636)	0.6865 (0.6612)	0.7293 (0.7415)	1.0727 (1.0404)
MRJDS	0.6847 (0.6676)	0.6065 (0.5981)	0.6819 (0.6253)	0.7199 (0.7058)	0.7806 (0.7645)	1.1559 (1.1200)
ITM						
BS	0.5968 (1.4216)	0.6398 (0.9224)	0.9328 (1.4410)	1.1059 (1.5996)	1.4694 (2.0757)	2.1980 (3.8077)
MR	0.5015 (1.1330)	0.5642 (0.7300)	0.7776 (0.9464)	0.9512 (1.0060)	1.1586 (1.1390)	1.6777 (1.6509)
MRJD	0.5141 (1.2389)	0.5798 (0.7660)	0.7958 (0.9924)	0.9755 (1.0173)	1.1872 (1.1629)	1.7269 (1.7030)
MRS	0.4701 (0.7615)	0.5620 (0.6904)	0.7482 (0.6429)	0.8886 (0.8626)	1.0842 (1.0135)	1.6505 (1.6242)
MRJDS	0.4695 (0.6789)	0.5448 (0.6770)	0.7121 (0.6188)	0.9507 (0.9218)	1.1733 (1.1025)	1.8188 (1.8413)

Table 7 Delta-neutral hedging errors during non-financial collapse period

In calculating the hedging errors generated with 5-day hedge rebalancing, the all-adjust-absolute-delta hedges of calls use as many hedging instruments as there are sources of risk (except the jump risk) assumed in a given option model. The only exception is the BS delta-neutral strategy, which uses the underlying asset and a second call option to neutralize the delta risks of the target call, based on the Black-Scholes model. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedges, which are then liquidated the following day or five days later. For each target call option, its hedging error is, as of the liquidation day, the difference between its market price and the replicating portfolio value. The average absolute hedging error and the average dollar hedging error are reported for each model and for each moneyness-maturity category. The sample period is 1 January 2009–30 April 2012 call option prices. BS, MR, MRJD, MRS, and MRJDS, respectively, stand for the Black-Scholes, the mean-reversion, the mean-reversion with jump diffusion, the mean-reversion model with seasonality, and the mean-reversion with jump diffusion and seasonality models.

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
Absolute hedging error (per day)						
OTM						
BS	0.2128 (0.3199)	0.1975 (0.2706)	0.1791 (0.2376)	0.1714 (0.2083)	0.1668 (0.2020)	0.1694 (0.2050)
MR	0.1349 (0.1431)	0.1166 (0.1612)	0.1095 (0.1461)	0.1048 (0.1236)	0.1143 (0.1432)	0.1386 (0.1860)
MRJD	0.1176 (0.1580)	0.1034 (0.1605)	0.1001 (0.1429)	0.0988 (0.1215)	0.1117 (0.1407)	0.1389 (0.1865)
MRS	0.1308 (0.1455)	0.1156 (0.1640)	0.1103 (0.1472)	0.1081 (0.1243)	0.1188 (0.1421)	0.1402 (0.1759)
MRJDS	0.1164 (0.1583)	0.1021 (0.1653)	0.0982 (0.1480)	0.0980 (0.1263)	0.1107 (0.1481)	0.1398 (0.1882)
ATM						
BS	0.4160 (0.4320)	0.3721 (0.3719)	0.3250 (0.3423)	0.2988 (0.3152)	0.2950 (0.3061)	0.3303 (0.3262)
MR	0.1783 (0.2072)	0.1570 (0.1763)	0.1581 (0.1723)	0.1665 (0.1766)	0.1821 (0.1975)	0.2352 (0.2631)
MRJD	0.1789 (0.2112)	0.1657 (0.1804)	0.1665 (0.1770)	0.1713 (0.1785)	0.1832 (0.1941)	0.2356 (0.2627)
MRS	0.1757 (0.2057)	0.1593 (0.1785)	0.1636 (0.1762)	0.1736 (0.1849)	0.1860 (0.1972)	0.2346 (0.2465)
MRJDS	0.1775 (0.2099)	0.1626 (0.1801)	0.1641 (0.1767)	0.1733 (0.1873)	0.1896 (0.2086)	0.2530 (0.2685)
ITM						
BS	0.1359 (0.2448)	0.2010 (0.2958)	0.2663 (0.5466)	0.3003 (0.3422)	0.3494 (0.3976)	0.4432 (0.4557)
MR	0.1174 (0.1779)	0.1443 (0.1984)	0.1805 (0.4637)	0.2013 (0.2260)	0.2353 (0.2715)	0.3153 (0.3647)
MRJD	0.1216 (0.1849)	0.1504 (0.2013)	0.1869 (0.4612)	0.2071 (0.2315)	0.2417 (0.2794)	0.3258 (0.3806)

Table 7 (*Continued*)

Days to maturity	~30	30–60	60–90	90–120	120–180	180~
MRS	0.1131 (0.1882)	0.1425 (0.2146)	0.1797 (0.4627)	0.2001 (0.2487)	0.2321 (0.2947)	0.3001 (0.3409)
MRJDS	0.1229 (0.1819)	0.1563 (0.2059)	0.1954 (0.4441)	0.2207 (0.2413)	0.2573 (0.2878)	0.3455 (0.3697)
Absolute hedging error (5-days)						
OTM						
BS	0.5040 (0.7575)	0.4603 (0.5860)	0.4326 (0.5241)	0.3977 (0.4561)	0.3729 (0.4369)	0.3502 (0.3899)
MR	0.4593 (0.3717)	0.3438 (0.3815)	0.3034 (0.3680)	0.2891 (0.3354)	0.2879 (0.3481)	0.3206 (0.3927)
MRJD	0.4052 (0.4214)	0.3176 (0.3787)	0.2849 (0.3578)	0.2761 (0.3206)	0.2802 (0.3370)	0.3184 (0.3866)
MRS	0.4485 (0.3825)	0.3442 (0.3953)	0.3095 (0.3795)	0.3011 (0.3508)	0.2996 (0.3466)	0.3281 (0.3614)
MRJDS	0.4023 (0.4237)	0.3140 (0.3910)	0.2833 (0.3758)	0.2789 (0.3434)	0.2820 (0.3545)	0.3142 (0.3773)
ATM						
BS	0.9984 (0.9580)	0.9023 (0.9031)	0.7949 (0.8173)	0.7422 (0.7792)	0.7084 (0.7640)	0.7731 (0.7735)
MR	0.5523 (0.4642)	0.4929 (0.4533)	0.4474 (0.4180)	0.4545 (0.4312)	0.4550 (0.4558)	0.5192 (0.5530)
MRJD	0.5587 (0.4790)	0.5125 (0.4644)	0.4684 (0.4204)	0.4692 (0.4259)	0.4602 (0.4449)	0.5223 (0.5387)
MRS	0.5526 (0.4726)	0.5040 (0.4781)	0.4784 (0.4612)	0.4929 (0.4859)	0.4901 (0.5058)	0.5344 (0.5205)
MRJDS	0.5605 (0.4945)	0.5126 (0.4897)	0.4785 (0.4542)	0.4894 (0.4677)	0.4889 (0.4965)	0.5530 (0.5443)
ITM						
BS	0.3624 (0.5462)	0.4762 (0.6842)	0.5947 (0.8457)	0.6837 (0.8320)	0.7787 (0.8771)	0.9933 (0.9952)
MR	0.2983 (0.3836)	0.3337 (0.3934)	0.3917 (0.5645)	0.4427 (0.4739)	0.5093 (0.5509)	0.6860 (0.7555)
MRJD	0.3081 (0.4034)	0.3482 (0.4045)	0.4069 (0.5718)	0.4562 (0.4868)	0.5214 (0.5683)	0.7047 (0.7817)
MRS	0.2977 (0.4162)	0.3467 (0.4711)	0.4104 (0.6211)	0.4651 (0.5747)	0.5058 (0.6083)	0.6365 (0.6768)
MRJDS	0.3149 (0.4086)	0.3670 (0.4209)	0.4394 (0.5739)	0.4946 (0.4919)	0.5464 (0.5549)	0.7152 (0.7017)

To explore the dynamic hedging performance of options, we constructed a self-financed portfolio. Without taking transaction costs into account, the study used an option-price-replication hedging strategy to analyze the impact of each evaluation model and the delta adjustment on the performance of hedging. The results show that the performance of the models in terms of hedging were generally consistent with those of pricing errors. In addition, regardless of which period was considered, the hedging error from the delta adjustment that was performed every five days was twice as large as that of the per-day adjustment. This result provides a reference for hedgers based on their own transaction cost considerations to reset their dynamic hedging parameter adjustment period for a more efficient risk-management strategy.

The study demonstrated that, regardless of the period, the MR-related European option-pricing formulae show better pricing and hedging abilities in the WTI crude oil options market than the Black-Scholes model, which indicates that the WTI crude oil price dynamic possesses MR characteristics. This conclusion is in response to findings by Schwartz (1997) and is consistent with his theory that commodity price dynamics are more receptive to the application of the MR model. In addition, by comparing multiple models, this study showed that a more complex model is not necessarily better; instead, it showed that the appropriate model must be selected based on spot price characteristics over different periods to obtain better valuation and hedging performance.

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Appendix A: The derivation of characteristic function of MRJDS model

The characteristic function of log-spot price corresponding to the MR, MRS, and MRJD models are specific cases of the MRJDS model. This study derived the more complicated characteristic function of the MRJDS model instead. Because the characteristic function of the dynamic of WTI crude oil under the MRJDS model is available, the option-pricing formula can be implemented with characteristic function by fast Fourier transformation.

The log-spot price, $Y_t = \ln S_t$, can be divided into the seasonality section, g_t, and a non-seasonality log-spot price section, X_t. The log-spot price dynamic under a risk-neutral measure is described as:

$$\mathrm{d}Y_t = \alpha\left(m^*_{\mathrm{MRJDS}} + g_t + \frac{1}{\alpha}\frac{\mathrm{d}g_t}{\mathrm{d}t} - Y_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t^Q + J\mathrm{d}N_t \tag{A.1}$$

where $m^*_{\mathrm{MRJDS}} = m - \eta - \frac{1}{\alpha}\left(\frac{\sigma^2}{2} + \lambda\bar{J}\right)$.

Given the dynamic process in the MRJDS model, the characteristic function can be written as:

$$f(t, T, Y_t; \phi) \equiv E^Q[\exp(i\phi Y_T)|Y_t = y] \tag{A.2}$$

where $i = \sqrt{-1}$, ϕ is the Fourier transform parameter.

Ito-Doeblin formula for jump processes gives the following partial integro- differential equation (PIDE) for the characteristic function:

$$\begin{aligned}&\frac{\partial f}{\partial t} + \alpha(m^*_{\mathrm{MRJDS}} + g_t + \frac{1}{\alpha}\frac{dg}{dt} - Y_t)\frac{\partial f}{\partial Y_t} + \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial Y_t^2}\\ &+ \lambda\int_R (f(t, T, Y_t + J) - f(t, T, Y_t))\psi(J)dJ = 0\end{aligned} \tag{A.3}$$

where $\psi(J)$ denotes the probability density function of random variable J. In the MRJDS model, $\psi(J)$ is set as the lognormal distribution.

Consider the exponential affine form for solving the characteristic function:

$$f(t, T, Y_t; \phi) = \exp\{i\phi Y_t + A(T-t) + Y_t B(T-t)\} \tag{A.4}$$

with boundary condition $A\ (T - t = 0) = B\ (T - t = 0) = 0$.

Substituting $\tau = T - t$ and the characteristic function into equation (A.3) yields:

$$\begin{aligned}&-\left(\frac{\partial A(\tau)}{\partial \tau} + \frac{\partial B(\tau)}{\partial \tau}Y_t\right) + \alpha\left(m^*_{\mathrm{MRJDS}} + g_t - \frac{1}{\alpha}\frac{\mathrm{d}g}{\mathrm{d}\tau} - Y_t\right)(i\phi + B(\tau)) + \frac{\sigma^2}{2}(i\phi + B(\tau))^2\\ &+ \lambda\left(\int_{-\infty}^{\infty} e^{(i\phi + B(\tau))J}\frac{1}{\sqrt{2\pi\delta^2}}e^{\frac{(J-\theta)^2}{-2\delta^2}}dJ - 1\right) = 0\end{aligned} \tag{A.5}$$

The PIDE (A.5) leads to the following system of ODEs:

$$\begin{cases} \frac{\partial A(\tau)}{\partial \tau} = \alpha\left(m^*_{\text{MRJDS}} + g_t - \frac{1}{\alpha}\frac{\text{d}g}{\text{d}\tau}\right)(i\phi + B(\tau)) + \frac{\sigma^2}{2}(i\phi + B(\tau))^2 \\ \qquad +\lambda\left(\int_{-\infty}^{\infty} e^{(i\phi+B(\tau))J}\frac{1}{\sqrt{2\pi\delta^2}}e^{\frac{(J-\theta)^2}{-2\delta^2}}dJ - 1\right) \\ \frac{\partial B(\tau)}{\partial \tau} = -\alpha(i\phi + B(\tau)) \end{cases} \tag{A.6}$$

It is simple to verify that $B(\tau) = i\phi\ (1 - e^{-\alpha\tau})$, and the jump component of the first ODE becomes:

$$\lambda\left(\int_{-\infty}^{\infty} e^{(i\phi+B(\tau))J}\frac{1}{\sqrt{2\pi\delta^2}}e^{\frac{(J-\theta)^2}{-2\delta^2}}dJ - 1\right) = \lambda\left(e^{i\phi e^{-\alpha\tau}\theta + \frac{-1}{2}\phi^2 e^{-2\alpha\tau}\delta^2} - 1\right) \tag{A.7}$$

Thus, the first ODE in (A.6) can be represented as:

$$\frac{\partial A(\tau)}{\partial \tau} = \alpha\left(m^*_{\text{MRJDS}} + g_t - \frac{1}{\alpha}\frac{\text{d}g}{\text{d}\tau}\right)i\phi e^{-\alpha\tau} - \frac{\sigma^2\phi^2 e^{-2\tau}}{2} + \lambda\left(e^{i\phi e^{-\alpha\tau}\theta + \frac{-1}{2}\phi^2 e^{-2\alpha\tau}\delta^2} - 1\right) \tag{A.8}$$

Integrate both sides of (A.8) to obtain $A(\tau)$:

$$\begin{aligned} A(\tau) = &\int_0^\tau \alpha\left(g_t - \frac{1}{\alpha}\frac{\text{d}g}{\text{d}s}\right)i\phi e^{-\alpha s}\text{d}s + \int_0^\tau \alpha m^*_{\text{MRJDS}} i\phi e^{-\alpha s}\text{d}s - \int_0^\tau \frac{\sigma^2\phi^2 e^{-2s}}{2}\text{d}s \\ &+ \lambda\int_0^\tau \left(e^{i\phi e^{-\alpha s}\theta + \frac{-1}{2}\phi^2 e^{-2\alpha s}\delta^2} - 1\right)\text{d}s \end{aligned} \tag{A.9}$$

As a result, we obtain the following characteristic function:

$$f(t, T, Y_t; \phi) = \exp\{i\phi Y_t + A(T-t) + Y_t B(T-t)\} \tag{A.10}$$

$$\begin{aligned} A(T-t) =& i\phi(g_T - g_t e^{-\alpha(T-t)}) + m^*_{MRJDS} i\phi(1 - e^{-\alpha(T-t)}) - \frac{\phi^2\sigma^2}{4\alpha}(1 - e^{-2\alpha(T-t)}) \\ & - \lambda\int_T^t \left(e^{i\phi e^{-\alpha(T-s)}\theta + \frac{-1}{2}\phi^2 e^{-2\alpha(T-s)}\delta^2} - 1\right)\text{d}s \\ B(T-t) =& -i\phi(1 - e^{-\alpha(T-t)}) \end{aligned} \tag{A.10}$$

Appendix B: Delta-hedging ratio

The delta-hedging ratio can be obtained by taking the partial differential of the closed-form option-pricing formula. An options strategy aims to reduce (hedge) the risk associated with price movements in the underlying asset by offsetting long and short positions. This strategy is based on the change in option price caused by a change in the price of the underlying security. The delta-hedging ratio in the framework of MRJDS dynamics is derived by taking a differential to the closed-form option-pricing model.

First, we rearrange the characteristic function (A.10), and then we take a differential with respect to the spot price:

$$f(t,T,y_t;\phi) = \exp\{(i\phi + B(T-t))\ \ln S_t + A(T-t)\} = S_t^{i\phi + B(T-t)} e^{A(T-t)} \quad \text{(B.1)}$$

and:

$$\frac{\partial f(t,T,y_t;\phi)}{\partial S_t} = (i\phi + B(T-t)) S_t^{i\phi + B(T-t)-1} e^{A(T-t)} = \frac{i\phi e^{-\alpha(T-t)}}{S_t} f(t,T,y_t;\phi) \quad \text{(B.2)}$$

Next, according to equations (28) and (B.2), the deviation of option price based on inverse Fourier transform can be derived as follows:

$$\begin{aligned}
\frac{\partial C_t}{\partial S_t} &\approx \frac{e^{-\omega k}\eta}{2\pi} \mathrm{Re}\left(\frac{\partial \psi(t,T,\xi_1)}{\partial f} \frac{\partial f}{\partial S_t} + e^{-i\xi_N k} \frac{\partial \psi(t,T,\xi_N)}{\partial f} \frac{\partial f}{\partial S_t} \right) \\
&+ \frac{e^{-\omega k}\eta}{\pi} \mathrm{Re}\left(\sum_{j=2}^{N-1} e^{-i\xi_j k} \frac{\partial \psi(t,T,\xi_j)}{\partial f} \frac{\partial f}{\partial S_t} \right) = \frac{e^{-\omega k}\eta}{2\pi} \\
&\mathrm{Re}\left(\psi(t,T,\xi_1) \frac{i\varphi(\xi_1) e^{-\alpha(T-t)}}{S_t} + e^{-i\xi_N k} \psi(t,T,\xi_N) \frac{i\varphi(\xi_N) e^{-\alpha(T-t)}}{S_t} \right) \\
&+ \frac{e^{-\omega k}\eta}{\pi}\ \mathrm{Re}\left(\sum_{j=2}^{N-1} e^{-i\xi_j k} \psi(t,T,\xi_j) \frac{i\varphi(\xi_j) e^{-\alpha(T-t)}}{S_t} \right)
\end{aligned} \quad \text{(B.3)}$$

where $\phi(\xi_j) = \xi_j - (\omega + 1)i$.

The aforementioned derivation process demonstrates the derivation of the delta-hedging rate to the European call option-pricing formula in the framework of the MRJDS dynamic. It maintains the same delta-hedge formula for the MR, MRS, and MRJD models with a corresponding characteristic function. Additionally, we know that the delta-hedge ratio for BS model is $N\ (d_1)$. Thus, we complete the derivation of hedge ratios for the models adopted in this study.

